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Estimation of the Mean-to-Surface-Velocity Ratio in Shallow Streams with Rough Beds

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Abstract

Estimating in a stream's cross-section the depth-averaged velocity, V , from the free-surface velocity, v_{surf} , is an efficient, non-invasive hydrometric method. The ratio $f_v = V/v_{surf}$ is typically assumed constant at $f_v = 0.86$ in field applications, despite observations to the contrary. Guidance is, therefore, needed in estimating actual f_v -ratios when velocity profile data are absent. This work provides field-verified guidance based on the hydromechanics of the logarithmic velocity law, which shows that f_v depends on the scaled resistance measure 'friction length/depth', y_o/h , with the $y_o(k)$ function of the equivalent sand grain roughness, k . The mean-to-surface-velocity ratio in rough-bed streams is estimated from the bed roughness and stream morphology by modifying *Nikuradze's* equation, $y_o = k/30$, to $y_o = ck$, with $c(h/k) \geq 1/30$, and $k \approx D_{84}$ —data fit: $c \approx 8.61(h/k)^{-1.821}$, $\sim 5 \leq h/k < \sim 30$. Field-verification of the ratio's modified hydromechanics, $f_v = f(h/y_o)$, with $y_o(h/k)$ evaluated from bed roughness estimated by inspection or sieve analysis shows this ratio holding within $\sim |10|$ % error for shallow streamflow over a coarse bed of gravels and rocks, giving submergences of $\sim 5 \leq h/D_{84} \leq \sim 30$; $y_o = k/30$ suits large streams with smooth beds ($h/k \geq \sim 30$, $f_v \geq \sim 0.86$). Variable roughness-estimated f_v -ratios appear to be more reliable than the fixed default, $f_v(h/y_o \approx 1000) = 0.86$. This flow-gauging concept is based on observable physical characteristics of a monitoring cross-section and facilitates the rating of hard-to-access streams draining small basins in ragged upland terrain.

Keywords: mean-to-surface-velocity ratio; logarithmic velocity profile; shallow streams; granular bed; relative roughness; submergence; friction length; hydrometry



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1. Introduction

Streamflow is the paramount quantity in surface hydrology, so considerable attention is paid to its monitoring, yet observations have decreased consistently since the 1980s [1]. Discharge through a stream's cross-section is monitored operationally by converting observed stages, h , to flows, Q , via a rating curve, $Q(h)$. Streamflow, however, is monitored regularly only in large rivers; in small streams, rating curves are often absent [2]. A rating curve is constructed by integrating velocities sampled in a cross-section of known bathymetry over a range of flows, ideally up to flood flows. However, practical issues exist; e.g., in-stream point-sampling with a current meter can be dangerous even at non-extreme flows (rule of thumb limit: $velocity(m/s) \times depth(m) \leq '1'$) and is prohibitive at high flows.

In contrast, a fixed side-looking Acoustic Doppler Current Profiler (ADCP), which measures speed from the frequency shift (*Doppler effect*) of a transmitted signal scattered back to the transceiver through particles in the water [3], is safe but too expensive and often unsuitable for routine use in small, shallow streams [4], which may be also difficult to access. A portable ADCP can also be employed in shallow streams via wading; however, the hydrographer is again exposed to high-velocity risks. Rating curves are thus often extrapolated to high flows, despite such extrapolations being prone to significant errors.

Measuring velocities on the free surface of a stream can be a suitable hydrometric alternative if surface velocities can be related to depth-averaged velocities at the sampling locations. This approach rests on the ability to sense surface velocities remotely. Image velocimetry (e.g., Refs. [5–8]) and hand-held radar [9] allow us to carry out such non-invasive observations efficiently. Estimating the mean velocity, V , as a constant fraction, f_v , of the surface velocity, v_{surf} , $V = f_v v_{surf}$, is convenient. The default value, $f_v \approx 0.86$ [10], fits a logarithmic velocity profile with the dispersion quantified by the momentum distribution coefficient $\beta = 1 + (\sigma/\mu)^2 \approx 1.03$, $\sigma/\mu \approx 0.16$; $\mu = V$ is the profile's mean velocity and σ its standard deviation [11]; a power-law velocity profile with the exponent $m = 6$ also yields $f_v = m/(m + 1) \approx 0.86$. Through hydraulic reasoning, however, a constant f_v -ratio is an approximation [12], and its variability with the discharge has been observed in the field, e.g., at the gauging station of a large river [13]. Analysis of 3611 streamflow datapoints from mostly medium to small French rivers (in 90% of cases: $Q < 10.2 \text{ m}^3/\text{s}$, hydraulic radius: $R_h < 0.63 \text{ m}$, and width: $b < 24.8 \text{ m}$) found f_v -values ranging from ~ 0.7 to 0.95 (5% to 95% quantiles) [14]. A large set of streamflow records from rivers across the USA was evaluated (mean values: $Q = 483 \text{ m}^3/\text{s}$, top width: $B = 94.4 \text{ m}$, and depth: $h = 2.32 \text{ m}$) in [15], cf. figure 2b, and significant variability in the maximum-to-mean-velocity ratio was found. At gauging stations on the Tiber River, f_v -ratios ranged in the interval 0.86 (wide section) $\leq f_v \leq 1.05$ (narrow section, profile *dip*) [16], while [17] found $V/v_{max} \approx 0.7$ in three streams.

However, f_v -values generally vary also across a stream, particularly in upland catchments, due to the variable depth and bed roughness in irregular natural cross-sectional geometry; the relative roughness (=roughness/depth) thus varies across the stream. The logarithmic velocity profile (see Section 2) has been used in [13] to show that f_v is a function of the ratio friction length/depth, and, since the friction length depends on the roughness, the relative roughness is generally variable, and so is the f_v -ratio. This reasoning, supported by the data mentioned above, as well as by observations such as those in appendix C in [18], and [19] and also of the authors of [11], motivated this reassessment of the hydromechanics underpinning the f_v -ratio.

The emphasis here is on streamflow in shallow channels with rough beds. Accepting the logarithmic law, we modify the friction length from *Nikuradze's* original value [20–22], relating it to the relative roughness, to fit the flow conditions in shallow channels with rough beds. The key to this work is determining f_v -ratios by estimating relative roughness from sampling/observations of the bed material and from the bathymetry of smaller streams with irregular cross-sections and rough beds. We use field-verified hydromechanics to estimate the mean-to-surface-velocity ratio when velocity profile data are absent, aiming to provide the guidance that hydrographers lack. Then, gauging the discharge from surface velocities would especially facilitate setting up rating curves at difficult-to-access ungauged streams with rough bed draining small basins in ragged upland terrain.

2. Concepts and Fundamental Equations

In the absence of a measured discharge, or of at least a measured velocity profile, an analytical model of the velocity variation in an open channel must be used in order to

estimate an f_v -ratio that parameterizes the conversion of surface velocity to depth-averaged velocity. Such analytical models idealize steady free-surface streamflow as fully developed (longitudinally uniform) in an infinitely wide channel (2-D). Assuming an infinitely wide channel is a major approximation, as the velocity field in natural streams depends on cross-sectional shape and roughness. However, also traditional hydrometry applies quasi-2D analysis: it assumes a streamflow field as represented by velocities sampled at verticals on which the selection of sampling points is guided by the *Prandtl–von Kármán* logarithmic velocity profile in an infinitely wide channel, at 60% of a shallow (≤ 0.76 m) depth below the free surface, or at 20% and 80% or at 20%, 60% and 80% of depth; a midsection's mean velocity is obtained by weighing the observations appropriately [3].

The *Prandtl–von Kármán* logarithmic law [20–22] and the mathematically simpler, but empirical, power law [20] are often used to fit measured velocities [9,23], doing so about equally well (as does the parabolic profile of [17]). The logarithmic law has been selected for this analysis because its parameters are observable, while the power-law exponent is not; see also [24].

The logarithmic *Prandtl–von Kármán* velocity law describes steady, uniform turbulent flow in an infinitely wide channel (approximately in channels of finite width, b , and depth, h , with $h/b \ll 1$):

$$v(y) = (v_*/\kappa) \ln(y/y_o), \quad y_o \leq y \leq h \quad (1)$$

v_* is the shear velocity, $\kappa \approx 0.4$ is the *von Kármán* constant (clear water), and y is the normal distance from the bed; the friction length, y_o , positions the zero-velocity, $v(y_o) = 0$, evading the logarithm's infinite tail at zero [20–22]. The logarithmic law is derived by approximating the shear stress near the wall as constant (*Prandtl's* mixing length model, $l = \kappa y$ [25]) but has been found to also describe the velocity profile well in the outer region [26]. The shear velocity, $v_* = \sqrt{\tau_o/\rho}$ (ρ is the water density), expresses the wall shear, τ_o , i.e., the resistance force that macroscopically balances the gravity force in steady uniform flow. The derivative of Equation (1), $dv/dy = (v_*/\kappa)/y$, shows that the variation in the velocity increases with increasing bed resistance (higher v_*) and decreases with increasing depth. Hence, velocity profiles over smooth beds in deep cross-sections, common to large rivers, are expected to be fairly uniform, i.e., high f_v . Conversely, velocity profiles in shallow streams with rough beds, common to headwaters, are expected to be more variable, i.e., low f_v .

According to *Nikuradze's* [27] flow experiments on pipes, y_o depends on the turbulence level and the wall roughness, except in the hydraulically smooth regime, where the roughness elements are submerged in the viscous sublayer, $y_o = \nu/9v_*$, and ν is kinematic viscosity (e.g., [28], p. 201); such conditions may occur in certain laboratory settings and commercial pipes, but they rarely exist in natural streams. *Nikuradze* gives the friction length in hydraulically rough flow as a fixed fraction of the equivalent sand grain roughness, k , which he defines using the median grain diameter, D_{50} :

$$y_o = k/30. \quad (2)$$

This result holds for the roughness type and range of *Nikuradze's* [27] experiments, in which sand grains of nearly uniform diameter, packed in the densest formation, were glued to an inner pipe wall [22] so that only a sector of those 'spherical' grains protruded above the glue layer into the flow. The large size of natural channels gives high Reynolds numbers, $R = V(4R_h)/\nu \geq 10^5$ – 10^6 (R_h is the hydraulic radius, and $\nu \approx 10^{-6}$ m²/s is the kinematic viscosity of water), which, combined with a fairly rough wetted perimeter, for example, a relative roughness of $k/(4R_h) \geq 0.01$, places the flow in the hydraulically rough regime, where resistance depends on the relative roughness alone [21,22,26]. *Nikuradze's* relationship, however, must likely be modified for shallow channels with rough granular

boundaries. Tentative results from *Smart* (reported in [18,19]) show that f_v decreases markedly for submergence values of $h/D_{84} < 10$; h/D_{84} is an inverse relative roughness measure, with D_{84} the size of the 84th largest percentile of the grains of the bed material. The authors of [29] report similar results.

According to the *Prandtl–von Kármán* logarithmic law, f_v varies with the relative roughness length $= y_o/(\text{depth } h)$ [13], and experiments by [29] using a laboratory flume demonstrate that V/v_{max} (log-profile: $v_{max} = v_{surf}$) depends on the submergence, h/k . The observed variability of f_v with discharge [13] is explained by the variation in relative roughness with depth, while the inherent variability of f_v across a natural stream is acknowledged in [18]. In ref. [13], an ensemble profile was formed from velocity data sampled at several verticals, from which they extracted a cross-sectional f_v -value by fitting a log-profile to the normalized data ensemble; this method weighs profiles equally and suits streams of fairly uniform cross-sectional characteristics. An over-the-cross-section average f_v -value was also used in [9]. A key idea in our work is to determine *local* f_v -ratios by estimating *variable* relative roughness from bathymetry and by sampling or observing the bed material of smaller streams with irregular cross-sections and rough beds: $f_v = f(k/h)$.

3. Review of the Hydromechanics of the Surface Velocity to the Depth-Averaged Velocity Ratio

3.1. The Equations of Hydromechanics, with Laboratory and Field Evidence

The presence of two parameters, v_* and y_o , makes the logarithmic law appear unsuitable for a parsimonious discharge estimation. However, upon forming the ratio of the mean velocity to the surface velocity, $f_v = V/v_{surf}$ (Equation (5), below), the friction velocity is eliminated, and the only parameter remaining to be estimated is the friction length.

The profile's mean velocity, V , is obtained by integrating Equation (1) over the nominal water column, from y_o to the free surface at $y = h$,

$$V = \left(\frac{v_*}{\kappa}\right) \left[\frac{h}{h-y_o} \ln\left(\frac{h}{y_o}\right) - 1 \right], \quad (3)$$

and the surface velocity is given by evaluating Equation (1) at $y = h$,

$$v(h) = (v_*/\kappa) \ln(h/y_o). \quad (4)$$

Dividing Equation (3) by Equation (4) gives the f_v -ratio as a function of h/y_o , as already derived by Le Coz et al. (2010) [13] and graphed in Figure 1a,

$$f_v = V/v_{surf} = \frac{h}{h-y_o} - 1/\ln(h/y_o) = \frac{1}{1-y_o/h} - \frac{1}{\ln\left(\frac{h}{y_o}\right)}, \quad (5)$$

($f_v \approx 1 + y_o/h - 1/\ln(h/y_o)$ for $h/y_o \geq 100$, error $< 1\%$); typical values of the reciprocal of the depth-scaled friction length in streams are $10^2 \leq h/y_o \leq 10^4$ [13]. Equation (5) may be the basis for a parsimonious discharge estimation, provided that the friction length is linked to the bed roughness across the h/y_o -range of interest, so that V/v_{surf} is quantified as $f_v(k/h)$; this is shown in Figure 1b for *Nikuradze's* $y_o = k/30$, per Equation (2). Further, the logarithm in Equation (5) 'forgives' misestimating y_o by compressing uncertainties about the roughness.

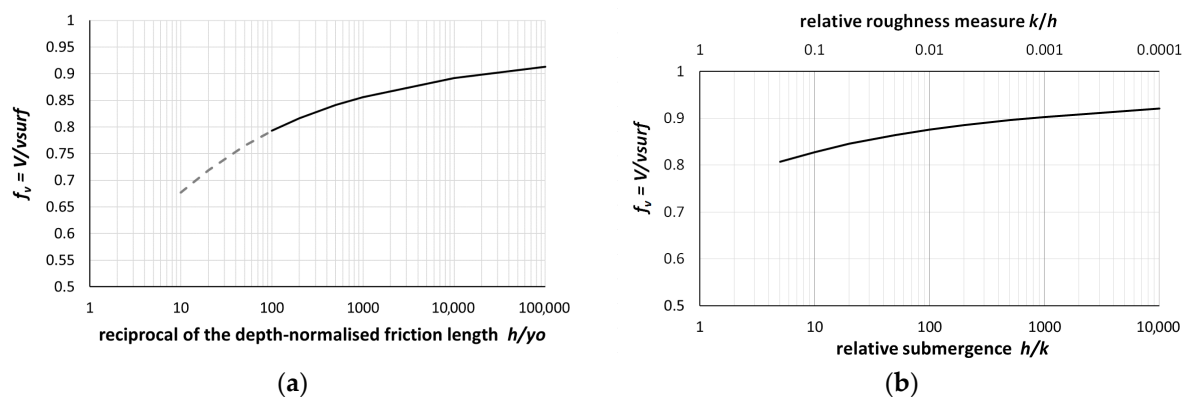


Figure 1. $f_v = V/v_{surf}$ as a function of (a) h/y_0 , Equation (5), and (b), again with Equation (5), and for $y_0 = k/30$, based on the Prandtl–von Kármán log-profile for uniform turbulent flow in an infinite-width channel.

As mentioned above, the laboratory experiments in [29] demonstrate that $\phi = V/v_{max}$ (for log-profile, $v_{max} = v_{surf}$) depends on the submergence, h/k . (NB: In pipe flows, the roughness scales with the hydraulic diameter, $D_h = 4R_h$, i.e., $k/4R_h \approx k/4d$ in shallow channels, $h/b \ll 1$); therefore, the reciprocal of the submergence is four times the relative bed roughness, $4k/(4R_h) \approx 4k/4h = k/h$, in an infinitely wide channel.) Drawing on additional laboratory and field data, Greco and Martino [29] developed an empirical graph, $\phi(h/k)$, similar to Figure 1b but somewhat simplified. It consists of two straight lines: the horizontal, $\phi = 0.9$, corresponds to beds with $h/k \geq 20$, while for $2 < h/k < \sim 7$ (quite rough beds), ϕ varies in the range 0.52–0.73; the intermediate zone, $\sim 7 < h/k < \sim 30$, is nearly unpopulated (the graph has a gap).

Field data in [13,14] (also citing a flume study) show that the logarithmic profile fits streamflow up to $y/h \approx 0.65$, above which the data are approximated by a constant velocity. Koussis et al. [11] truncated a maximum-entropy (ME) velocity profile at $y/h = 0.84$ and fitted the constant velocity $v_{surf} = v_{ME}(0.84) = V + \sigma$ above it; as it turns out, $v_{ME}(0.84) = v_{log}(y/h = 1)$ (see Figure A1 in Appendix A for the log-constant and max-entropy-constant profiles). The authors of [17] found the v_{max} located at $0.71 \leq y/h \leq 0.8$. Equation (6) (analogous to Equation (5)) was derived for the log-constant profile in [13], for which, typically, $\eta = (y/h)_{dip} \approx 0.6$ –0.7:

$$f_v = V/v_{surf} = \frac{h}{h - y_0} - \frac{\eta h - y_0}{h - y_0} \frac{1}{\ln(\eta h/y_0)} \quad (6)$$

Values of h/y_0 resulting from fitting a logarithmic profile to all observations and just to data of the logarithmic part of a log-constant profile are different, but only the friction length of the former has physical meaning [13]. Obviously, Equations (5) and (6) yield different values of h/y_0 for the same f_v -value, with the log-constant profile indicating an apparently reduced friction length.

Biggs et al. [18] developed an equation that brackets the f_v -ratio estimates at ungauged sites from the bed slope, S_0 , from the logarithmic law. Equation (4), solved for the friction length and substituted in Equation (5), gives

$$y_0 = h / \exp(\kappa v_{surf}/v_*), \quad (7)$$

$$f_v = V/v_{surf} = \frac{h}{h - y_0} - \frac{1}{\ln(h/y_0)} = \frac{1}{1 - \exp\left(-\kappa \frac{v_{surf}}{v_*}\right)} - \frac{v_*}{\kappa v_{surf}}. \quad (8)$$

The macroscopic momentum balance in the flow direction for uniform flow in an infinitely wide channel, $\tau_o = \rho g S_o$, and the definition of the shear velocity, $v_* = \sqrt{\tau_o/\rho}$, give $v_* = \sqrt{ghS_o}$; $S_o = \sin\theta$, with θ the bed's angle against the horizontal. Entering that result into Equation (8) eliminates the shear velocity and expresses the flow ratio in terms of the bed slope:

$$f_v = V/v_{surf} = \frac{1}{1 - \exp\left(-\kappa \frac{v_{surf}}{\sqrt{ghS_o}}\right)} - \frac{\sqrt{ghS_o}}{\kappa v_{surf}}. \quad (9)$$

This f_v estimator trades off the uncertainty about the friction length in y_o/h for the approximation of the friction slope by the bed slope that is assumed to hold under all flow conditions. Related drawbacks are (a) profiles in streams that may not be logarithmic (applies to either method), (b) a friction slope that generally differs from the bed slope, and (c) roughness that is not considered explicitly.

3.2. Flow Conditions in Shallow Channels with Rough Granular Boundaries

If flow conditions resemble those in Nikuradze's lab experiments (uniform $k/h \ll 0.1$), estimates of y_o from Equation (2) likely yield plausible f_v -values. The equivalent sand grain roughness and the shear velocity appear in the *wall Reynolds number*, $R_w = v_* k/\nu$, which classifies resistance in boundary layer flow: $R_w < 4$ hydraulically smooth, $R_w > 100$ hydraulically rough, and $4 < R_w < 100$ the transition zone [30]. On the other hand, if the bed in a shallow stream is coarse, the data in [17,29] indicate $f_v < \sim 0.7$, which translates, via Equation (5), to $h/y_o \leq \sim 20$ and then via Nikuradze's Equation (2), $y_o = k/30$, to the uncommonly extreme $h/k \leq \sim 2/3$. We therefore argue, in analogy to the work in [31,32] on flow resistance in streams with rough gravel beds, that a relationship different from Equation (2) is appropriate for estimating the friction length from a bed-material roughness size for, e.g., $h/k < \sim 6$, assuming $k = D_{84}$, or for $h/k < 10$, assuming $k = D_{50}$. These limits exceed the top of the intermediate roughness scale of [33] by ca. 1/3, defined as $1.2 < h/D_{84} < 4$ or $2 < h/D_{50} < 7.5$.

Various statistical roughness measures of the form $k = aD_x$, $a \geq 1$ and x percent finer than D define k in this range of natural roughness in the literature. In [18], Smart characterized the roughness regime via h/D_{84} , and Greco and Martino [29] used h/D with D the diameter of sand grains forming the bed plus the diameter of the spheres fixed on it. In [27,34–36] h/D_{50} was chosen. In [32] gravel-bed roughness was characterized with D_{84} , a size compatible with the observation that the bed of natural streams tends to be armored with the larger roughness elements ([30], Chapter 10). The h/D values in [29] are somewhat smaller than h/D_{84} ; however, the two measures are comparable because the spheres are identical and the sand grains $d_g \ll D$; in those rough-bed experiments, h/D falls in the intermediate to the rough scales, $1.89 \leq h/D \leq 6.43$, which explains values of $V/v_{max} (\approx f_v)$ in the range 0.52–0.73. Fairly large ranges of h/D_{84} are assigned to single f_v -values in table 4.2 in [18]; e.g., $10 < h/D_{84} < 30$, $f_v = 0.84$, and $2 < h/D_{84} < 10$, $f_v = 0.63$; therefore, these correspondences to the roughness classification of [33] are quite approximate.

In principle, the bed roughness can be characterized on the basis of the standard deviation, σ_{bed} , of the distance from a reference plane to the top of a solid boundary's granular elements [37] (an approach appropriate for laboratory research but impractical for field work). Monzavi's method was applied in flume experiments of free-surface flow over a permeable rough bed of 20 mm average-diameter natural gravel in [38], and $\sigma_{bed} = 7.83$ mm was determined as the roughness; the submergence was calculated as H_o/σ_{bed} , with $H_o = \text{flow depth} + \sigma_{bed}$ the nominal depth above the reference level, for two runs with depths $h = 7.5$ cm and 9.2 cm: $H_o/\sigma_{bed} = 10.58$ and 12.75, respectively.

4. Estimation of the f_v -Ratio in Rough-Bed Channels Based on the Relative Roughness

The velocity profile over a rough bed is linear in the thin laminar sublayer, logarithmic in the turbulent boundary layer above it, and slightly non-logarithmic in the outer layer [26]. In low-submergence flows, however, the velocity profile may deviate from the logarithmic form; nevertheless, several studies reported in [31] show that log-based resistance equations still afford adequate predictions. For irregular, large macro-scale roughness, *Nikuradze's* roughness height (for hydraulically rough flows in pipes with inner walls covered with near-uniform sand grains) is not a suitable lower integration limit for the log-profile, as data show that this profile does not extend over the depth, h . A zero-displacement, d_0 , was introduced in [35] to parameterize the space-and-time-averaged logarithmic profile, $v(y) = (\frac{v_*}{\kappa}) \ln[(y - d_0)/(y_{low} - d_0)] + v_{slip}$, from y_{low} , $v(y_{low}) = v_{slip} \neq 0$, to $y_{high} < h$; this equation has a solid hydromechanics basis but is too complex for hydro-metric practice. Therefore, we modify the f_v -relationship derived from the logarithmic law to apply it down to the start of the intermediate roughness range, with the generalized friction length expression

$$y_o = ck, \text{ variable } c \geq 1/30 \text{ and } k \approx D_{84} \quad (10)$$

such that Equation (10) fits the data in [29] well and that of Smart (in [18]) for $\sim 5 \leq h/D_{84} \leq \sim 10$, merging smoothly with Equation (5) at $h/k = 33$ (*Nikuradze's* $y_o = k/30$, $f_v \approx 0.86$):

$$f_v = V/v_{surf} = \frac{h}{h - y_o} - 1/\ln(h/y_o) = \frac{1}{1 - ck/h} - 1/\ln(h/ck) \quad (11)$$

Figure 2 shows Equation (11) with $c = 0.013$ at $h/D_{84} = 33$, $c = 0.05$ at $h/D_{84} = 20$, $c = 0.1$ at $h/D_{84} = 10$ and $c = 0.5$ at $h/D_{84} = 5$, which yield $f_v = 0.86, 0.836, 0.793$ and 0.677 , respectively; $c = 0.013$ derives from *Nikuradze's* $c = 1/30$, after adjusting the k from D_{50} to D_{84} by assuming an exponential distribution. These $(c, h/D_{84})$ -data are regressed by the relationship

$$c = 8.61(h/D_{84})^{-1.821}, r^2 \approx 0.973, \sim 5 \leq h/D_{84} \leq \sim 33 \quad (12)$$

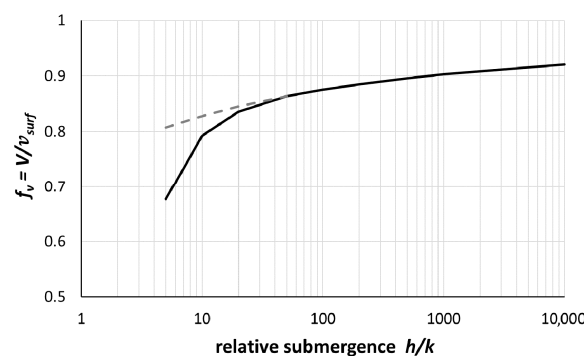


Figure 2. Ratio $f_v = V/v_{surf} = f(h/k)$ based on log-profile: Equation (11) with $c(h/D_{84})$; Equation (11) with $c = 1/30$.

Increasing friction lengths are reflected in larger c -values as h/D_{84} decreases, particularly for $h/D_{84} < 10$, accounting, approximately, for the prominent form drag in the total resistance [21,26] in shallow flow over a rough bed [33]. The findings of [38], that logarithmic profiles fit the data in their two aforementioned runs with the equivalent sand

grain roughness multiplied by 1.5 to 2, i.e., $c = 0.05\text{--}0.067$, are relevant in this context. In Section 4, we test Equations (10)–(12) with field data. We remind the reader that the constants in the log-profile, $v(y)/v_* = 2.5\ln(y/k) + 8.5$, derive from $y_o = k/30$ and $\kappa^{-1} = 2.5$ ($8.5 = 2.5\ln 30$) and should be adjusted in the case of fairly rough walls.

The sensitivity, S , of the f_v -ratio to variation (or to misestimation) in the roughness, k , is $S = df_v/dk = (df_v/dy_o)(dy_o/dk)$, with $y_o = c(k)k = N(h/k)^\zeta k$, $N = 8.61$ and $\zeta = -1.821$, Equations (11) and (12). Evaluation of this expression gives $S = \{h^{-1}(1 - y_o/h)^{-2} - y_o^{-1}[\ln(y_o/h)]^{-2}\}[N(1 - \zeta)(h/k)^\zeta]$, which is restated as

$$S = \{(y_o/h)(1 - y_o/h)^{-2} - [\ln(y_o/h)]^{-2}\}[N(\zeta - 1)/k] = F(y_o/h)/k \quad (13)$$

The function, F , on the right-hand side of Equation (13) is $F(y_o/h) = F(k/h)$; hence, given k/h and Δk , $\Delta f_v = F(k/h)\Delta k/k$. We consider, as an example, the case $k/h = 0.1$, $f_v \approx 0.79$; if $k = 0.05$ m and $\Delta k = 0.01$ m, $\Delta k/k = 0.2$ or 20%, and then $\Delta f_v \approx 0.03$ and $\Delta f_v/f_v \approx 4\%$; i.e., the uncertainty in estimating the roughness, k , causes a lesser uncertainty in f_v , and thus in estimating the discharge.

Before applying the method in the field, we prove the *concept* with high-accuracy data in [39] that were sampled nearly uniformly in the laboratory channel of a trapezoidal cross-section, with a base width of $b_b = 6$ ft (1 ft = 0.305 m), a depth across the base of $h \approx 0.5$ ft (the free surface is tilted due to the meandering flow) and side slopes of 1:1. The Cartesian co-ordinates are indicated in Table 1, which lists the observations shown as velocity profiles in Figure 3. The channel's meandering course induced a spiral secondary flow in the trapezoidal cross-section that was more pronounced than would have been solely due to finite width and inclined sides; this is contrary to the uniform longitudinal flow in the infinite-width cross-section assumed in the 2-D-flow theory.

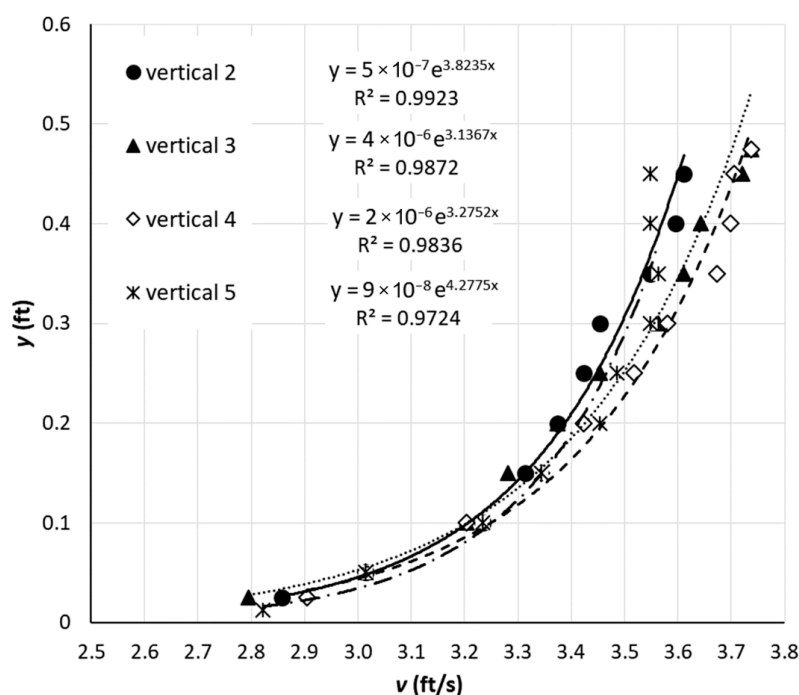
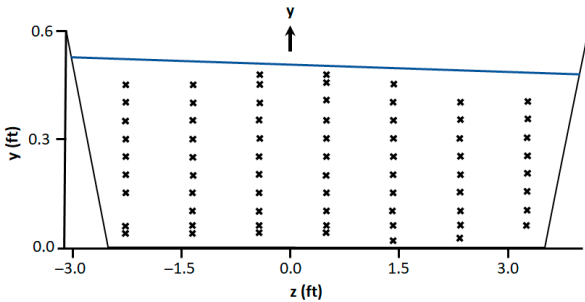


Figure 3. Velocity data and log-profiles in the trapezoidal cross-section of a laboratory channel.

Table 1. Velocities (ft/s) measured in a trapezoidal cross-section (Yen, 1965, [39]).

	0.4750	-	-	3.737	3.737	-	-	-
	0.4500	3.140	3.611	3.721	3.705	3.548	-	-
	0.4000	3.218	3.595	3.642	3.699	3.548	3.266	2.779
	0.3500	3.250	3.548	3.611	3.674	3.564	3.266	2.889
	0.3000	3.281	3.454	3.564	3.580	3.548	3.297	2.967
	0.2500	3.297	3.423	3.454	3.517	3.485	3.328	2.999
	0.2000	3.266	3.375	3.375	3.423	3.454	3.328	3.048
	0.1500	3.203	3.313	3.281	3.344	3.344	3.288	3.030
	0.1000	NaN	3.234	3.203	3.203	3.234	3.140	2.952
	0.0500	2.889	3.014	3.014	3.014	3.014	3.014	2.828
	0.0250	2.795	2.857	2.795	2.905	-	-	-
	0.0125	-	-	-	-	2.822	2.590	-
	depth (ft)/ distance (ft)	−2.35	−1.40	−0.45	0.50	1.45	2.40	3.35
	verticals	1	2	3	4	5	6	7

The profiles measured on the verticals at $z = -1.40$, -0.45 and 0.5 ft conform very well to the logarithmic law idealization ($r^2 \approx 0.98$), with friction lengths of 5×10^{-7} ft $\leq y_o \leq 4 \times 10^{-6}$ ft (1 ft = 0.305 m); the two central profiles give $y_o \approx 3 \times 10^{-6}$ ft. The profile at $z = 1.45$ ft is log-constant, while the profiles at $z = -2.35$, 2.40 and 3.35 ft exhibit a maximum below the free surface (a dip) (see Table 1). The texture of the solid boundary is ‘well finished cement mortar’ ([39], p. 32), for which ([22], table 8.3.5.1) lists k in the range 0.012–0.170 mm; we use $k = 0.1$ mm (~average). Then, $y_o = k/30$ gives $y_o \approx 1.1 \times 10^{-5}$ ft, so $h/y_o \approx 4.575 \times 10^4$ and, via Equation (5), $f_v = 0.906$. The log-profiles on the two central verticals fit the data with $y_o \approx 3 \times 10^{-6}$ ft, so $h/y_o \approx 1.67 \times 10^5$ and $f_v \approx 0.916$. The flat f_v -curve for $h/y_o \geq 10^4$, Figure 1a, explains the close agreement of these f_v estimates. Evaluation of the raw data on the two central verticals gives $f_v \approx 0.91$. The resistance regime for equivalent sand grain roughness, $k = 0.1$ mm and $\kappa/v_* \approx 3.2$ s/ft (see Figure 3), with $\nu = 1 \times 10^{-6}$ m²/s, $v_* \approx 0.125$ ft/s ≈ 0.038 m/s, is borderline smooth-transitional, $R_w = v_*k/\nu \approx 4$, which also explains $f_v \approx 0.9$.

The validity of the proposed concept is supported by the agreement, within ~1%, of the mean to the surface velocity ratios based on the raw data and derived using Equation (5) with the friction length estimated: (a) from the observed (central) velocity profiles and (b) according to Nikuradze’s Equation (2) by adopting the value of the equivalent sand grain roughness from a table. These results are also fair, given that the flow was meandering in this trapezoidal channel (non-2D, $h/b \approx 1/12$). Considering only the logarithmic part of the non-logarithmic profile on the off-center vertical, $z = 1.45$ ft (near-vertical for $y > 0.35$ ft, $\eta \approx 0.7$, Figure 3, like those observed in [13]), yields the (lacking physical meaning) apparently reduced friction length, $y_o = 1 \times 10^{-6}$ ft (one-third of the estimates from the central log-profiles); Equation (5) then gives $f_v \approx 0.925$ (little changed, due to the flat f_v -curve for $h/y_o > 10^4$, relative to $f_v = 0.916$ derived from the profiles on the central verticals). But if, for comparison, Equation (6) is applied with $h = 0.5$ ft, $\eta = 0.7$ and the inconsistent value, $y_o = 1 \times 10^{-6}$ ft, the estimate is $f_v \approx 0.95$, while $y_o = 3 \times 10^{-6}$ ft gives $f_v \approx 0.94$; these inconsistent results overestimate the ratio $f_v = 0.916$ from the observed data of the central profiles by ca. 3% and that from the raw-data-derived $f_v \approx 0.9$ by almost 5%. In any case, however, $f_v \neq 0.86$.

5. Field Applications of the Velocity Ratio $V/v_{\text{surf}} = f(h/(ck))$ in Open-Channels with Rough Bed

The procedure for applying the proposed methodology first consists of *preparation*: (a) ascertain the bathymetry from the water level and the cross-sectional geometry, and (b) estimate the roughness by either inspecting the stream bed, or better, through sieve-analysis of the sampled bed material. It then involves *operation*: measure (i) the stage and (ii) the surface velocity (remotely, e.g., with hand-held radar) at a number of positions to capture the lateral velocity variation, considering the variable macro-geometry, h , and bed roughness k , and (c) use Equations (11) and (12) to calculate f_v and $V = f_v v_{\text{surf}}$.

First, however, we assess how Equation (11) for $f_v(k/h)$ applies in the field using streamflow data from monitoring stations in the Peloponnese, Greece: (i) at Atsicholos Bridge on the river Loussios, near Karytaina, Arcadia; (ii) near Karavas village, Laconia, on the Evrotas River; and (iii) at the Rentifi Mill on Alagonia Creek, Messinia. Detailed velocity profiles sampled with a current meter are used for verification; the data are stored in the database HYDRONET [40], <https://hydronet.noa.gr> (accessed on 17 January 2024), established in the OpenHi.net project [41]. The bed material was inspected carefully while recording the bathymetry of the monitoring cross-sections. The quality characteristics of the log-profiles fitting to the data are listed in the Supplementary Material B.

The cross-section of the Loussios River at Atsicholos Bridge is shallow with a depth/width $\approx 1:20$ (Figure 4). The bed material for $0 \text{ m} \leq x \leq \sim 8.5 \text{ m}$ is initially coarse sand mixed with gravel, progressively turning into larger pebbles and stones; for $\sim 8.5 \text{ m} \leq x \leq \sim 11.5 \text{ m}$, large, irregular but smooth rocks form the bed; for $x > 11.5$, the steep river bank is rough. The goodness of fit of the velocity data from the log-profiles is important information in assessing the reliability of the f_v -predictions. The data from which the contour plot in Figure 4 has been produced are in Supplementary Material A. The discharge at the monitoring cross-section was $Q = 5.264 \text{ m}^3/\text{s}$ [11].

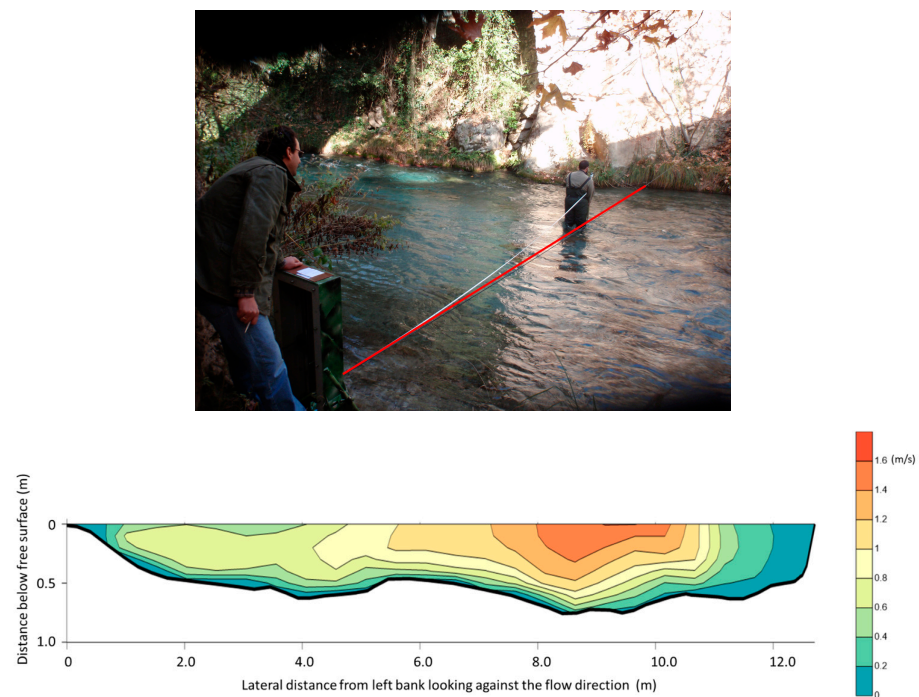


Figure 4. Loussios River monitoring station at Atsicholos Bridge: upper panel—measuring cross-section (red line); lower panel—contours of the velocity field (display exaggerated 2H:1V). Reproduced with permission from [11] lower panel).

Up to approximately $x = 5.5 \text{ m}$, the velocity profiles are not logarithmic (strong dip and $r^2 < 0.5$, except at $x = 5 \text{ m}$ where $r^2 \approx 0.74$). A major reason for this is the influence of the left bank (looking upstream), while the mild hump at $5 \text{ m} \leq x \leq 6.5 \text{ m}$ may also cause deviations from the logarithmic law. The deceleration of the flow due to the proximity of the right bank is prominent for $x \geq 11 \text{ m}$. In the range $\sim 6 \text{ m} \leq x \leq \sim 11 \text{ m}$, all velocity profiles fit the logarithmic model with $r^2 > \sim 0.7$; however, on the verticals at $x = 10, 10.5$ and 11 m (as at $x = 5 \text{ m}$), the profile-fit may be acceptable *statistically*: $0.7 \leq r^2 \leq 0.81$, rather than *hydraulically*, i.e., conforming visually well to the logarithmic form. This leaves the span $6 \text{ m} \leq x \leq 9.5 \text{ m}$ for testing the concept, where the logarithmic model fits the data shown in Figure 5 with $0.72 \leq r^2 \leq 0.95$ (average $r^2 \approx 0.85$). Over this span, the bed consists, near the middle of the cross-section, of smooth pebbles of a gradually increasing size of $\sim 5 \text{ cm}$ long by $\sim 1.5 \text{ cm}$ thick to $\sim 6 \text{ cm}$ long by $\sim 3 \text{ cm}$ thick, with a rough patch of about $x = 7.5 \text{ m}$ of irregularly lying $\sim 5\text{--}7 \text{ cm}$ large stones in a gravelly bed, and in the deepest part, $x > 8.0 \text{ m}$, of irregularly lying but fairly smooth rocks, $\sim 20 \text{ cm}$ long by $\sim 10 \text{ cm}$ thick. Despite the smooth surface of many individual roughness elements for $x \geq 7.0 \text{ m}$, the macroscopic bed geometry of this shallow streamflow is fairly rough (submergence $h/k < 10$).

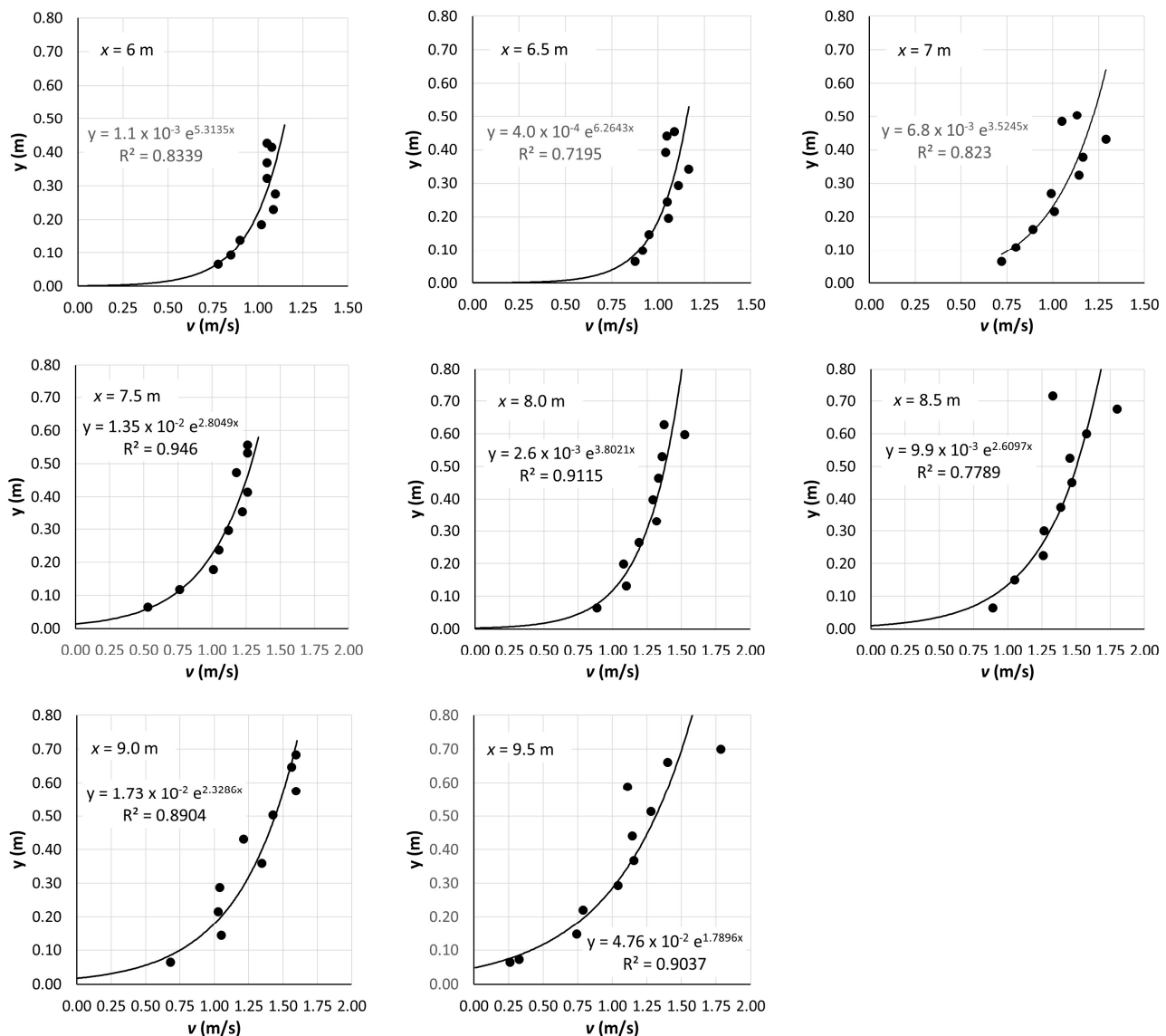


Figure 5. Measured velocities on the verticals $x = 6$ to 9.5 m in the monitoring cross-section at Atsicholos Bridge over the Loussios River and fitted logarithmic profiles; velocity (m/s) is given on the horizontal axis and distance from bed (m) on the vertical axis.

In the monitoring cross-section of the Evrotas River (Figure 6), a pronounced unevenness at $x = 3.5$ m creates two distinct sub-regions. The bed is largely rough—20 cm rough stones and 2–3 cm gravel over $0 \text{ m} < x \leq \sim 5.5 \text{ m}$ —but less rough about the hump, as the larger stones lie on the flat part of the bed: over $\sim 5.5 \text{ m} \leq x \leq 9 \text{ m}$, rocks and gravel first and then sand and mud. The irregular geometry and diverse bed material promote the formation of very different velocity profiles.

Figure 6 presents the velocity field in contour form; see Supplementary Material A for the data from the monitoring station of the Evrotas River near the village of Karavas. The discharge at the monitoring cross-section was $Q = 1.995 \text{ m}^3/\text{s}$ [11]. Logarithmic profiles were fitted to the data on the verticals at $x = 0.5, 2.0, 2.5, 3.0, 4.0, 5.0, 5.5, 6.0$ and 7.5 m , from the left bank looking downstream (Figure 7). As noted, these profiles vary considerably across the stream, also indicated by markedly varying exponents and y_0 -values of the regression curves. Velocities are expectedly higher in the deeper subsection, $1.5 \text{ m} \leq x < \sim 3.0 \text{ m}$; those in the shallower subsection, $\sim 4.0 \text{ m} < x \leq 8.0 \text{ m}$, are noticeably lower (also, and especially close to the banks, $x \leq 1.0 \text{ m}$ and $x \geq 8.0 \text{ m}$). Note that the

velocity scale (horizontal axis) is ~ 1 m/s in the graphs for $2.0 \text{ m} \leq x \leq 4.0 \text{ m}$ but merely ~ 0.3 m/s in the other graphs, yet the depths differ only by a factor of 1.5–2. The uniformly high velocities of ~ 1.2 m/s at the hump at $x = 3.5 \text{ m}$, $h = 0.407 \text{ m}$, are expectedly regressed poorly ($r^2 \approx 0.32$, not graphed). The poor fit at $x = 4.0 \text{ m}$ ($r^2 = 0.77$) is reasonably explained by the proximity to the hump and by the abrupt flow deceleration in the shallower subsection. The logarithmic law fits the data next to the bank, $x = 0.5 \text{ m}$, surprisingly well ($r^2 = 0.856$), considering that the ‘infinitely wide channel, 2D flow’ assumption is poorly met, yet the log-profile fails to fit the data well at $x = 1.0 \text{ m}$, $r^2 \approx 0.3$. Near the opposite bank ($x \geq 6.0 \text{ m}$, approximately), the inclined boundary (1V:5H) slows down the shallow flow markedly ($\max v \approx 0.2$ m/s). In general, the logarithmic law appears to fit several profiles over $\sim 0.5 \text{ m} \leq x \leq \sim 7.5 \text{ m}$ well enough to test the proposed concept ($r^2 > 0.77$).

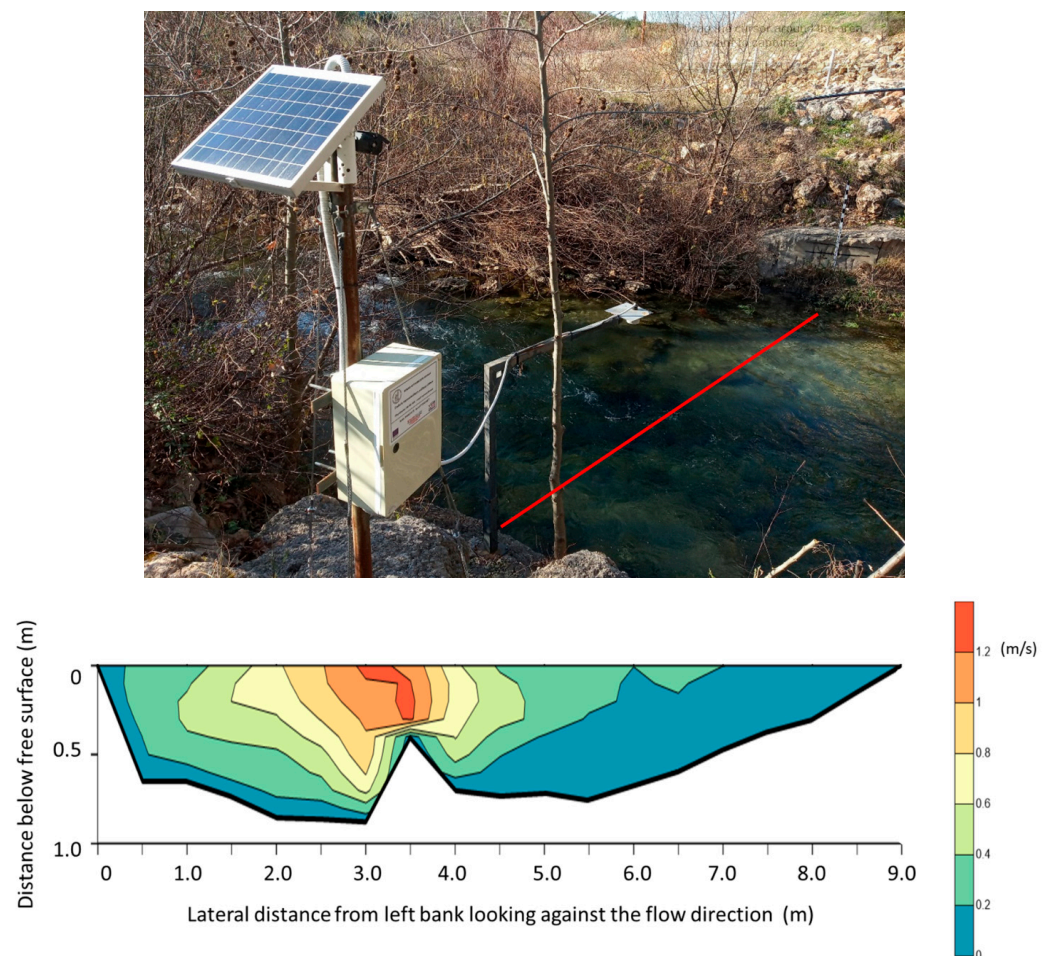


Figure 6. Evrotas River monitoring station near the village of Karavas, Laconia: upper panel—location of the station and measuring cross-section (red line); lower panel—contours of the velocity field at the monitoring station (display is exaggerated 2V:1H dimension). Reproduced with permission from [11] (lower panel).

Alagonia Creek is a small tributary of the Upper Nedon River that originates on Mount Taygetos and discharges into the Messinian Bay, traversing the city of Kalamata, SW Peloponnese. During the sampling, the maximum depth in the cross-section near the Rentifi Mill was $h = 0.584 \text{ m}$, and the surface width was $b \approx 2.95 \text{ m}$. Detailed sampling was carried out in intervals of $\sim 10\%$ depth, at verticals 0.2 to 0.5 m apart, plus at 0.2 m from the steep left bank and at 0.75 m from the intersection of the free surface with the sloping right bank (looking downstream). The data measured on the vertical with the maximum depth, $h = 0.584 \text{ m}$, at the lateral distance $x = 0.9 \text{ m}$ from the left bank, are graphed in Figure 8,

along with the fitted logarithmic profile ($r^2 \approx 0.92$). Over the first meter from the left bank, the bed is armored with rough stones/rocks of 5–10 cm size.

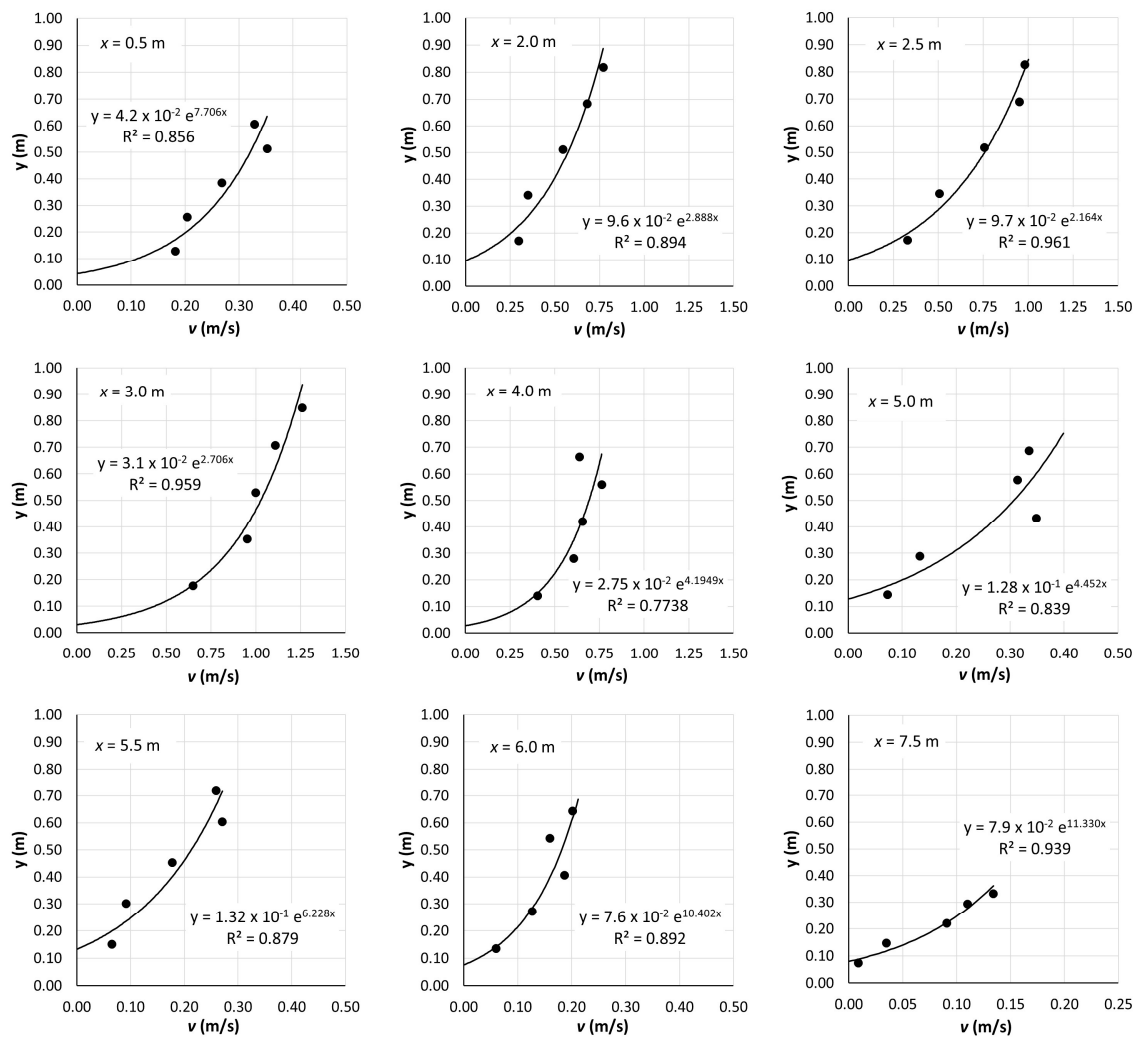


Figure 7. Velocities measured at 9 verticals in the monitoring cross-section of the Evrotas River, with fitted logarithmic profiles; velocity is on the horizontal axis and distance from bed on the vertical axis.

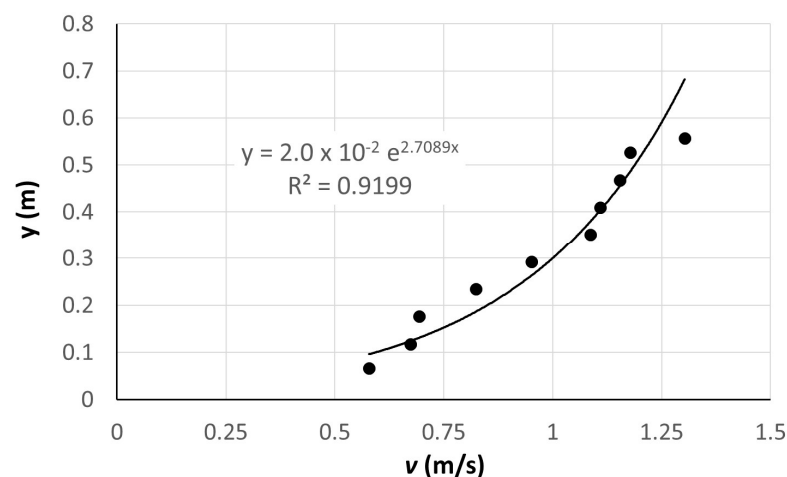


Figure 8. Velocities measured at the $x = 0.7$ m vertical in the Rentifi Mill cross-section of Alagonia Creek and fitted log-profile; velocity is on the horizontal axis and distance from bed on the vertical axis.

Finally, we sourced data for the Venoge River (canton Vaud, Switzerland) from [35]. The measuring cross-section is located in a straight reach, has a mean depth of $h = 0.21$ m, a width of $b = 6.30$ m, a bed slope of $S_o = 0.33\%$, and $D_{50} = 40$ mm. Detailed data were collected with an Acoustic Doppler Velocity Profiler (ADVP) over one half of the cross-section; a depth-scaled profile, $\langle v \rangle$ ($0 < y/h \leq \sim 0.85$), was generated from the data *ensemble*—figure 8a in [35] (note the *dip* at $y/h \approx 0.85$), reproduced here as Figure 9—excluding the top point and points with $y/h < \sim 0.2$. The ADVP's blanking distance did not allow for measurement in the top $\sim 15\%$ of the water column, and the surface velocity was not measured, so the f_v -ratio cannot be estimated from the data directly (but see the Discussion Section below). However, we can test the friction velocity, length, and *von Karman's* κ derived from the data against those equivalent parameters of the double-averaged logarithmic profile, as parameterized in [35].

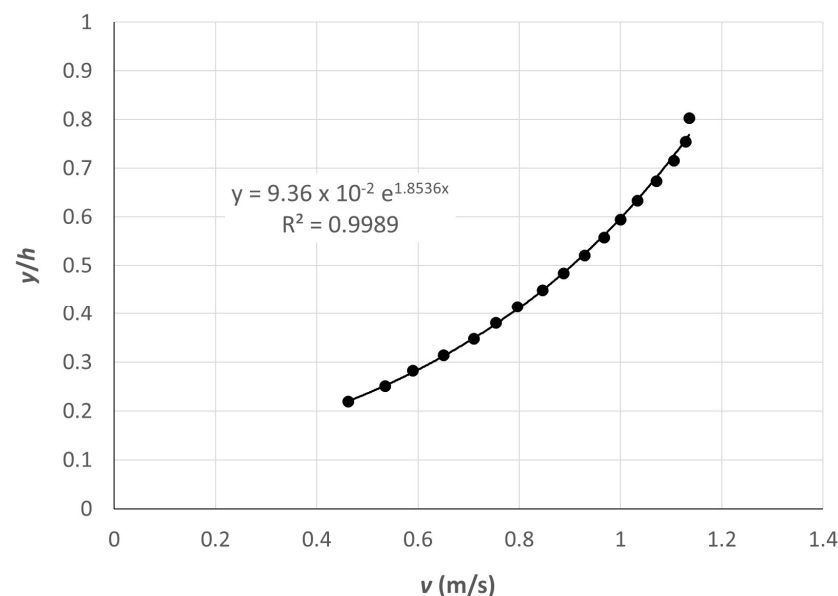


Figure 9. Ensemble logarithmic velocity profile for Venoge River, adapted from [35].

6. Discussion

Loussios River. The f_v -ratio computed with the friction length determined from the observed data varies considerably across the stream [0.704, 0.860] (Table 2), with values of $f_v < 0.8$ corresponding to the region with $h/y_o < 100$; hence, the default ratio in hydrometry, $f_v = 0.86$, is a poor estimate for these profiles. The results differ when f_v is computed based on the inspected roughness, k , via (a) Equation (11) and $y_o = ck$; $c(h/D_{84})$ via Equation (12); and (b) Equation (5) with $y_o = k/30$. The former better approximates the f_v -ratios from the observed friction length. These two sets of predicted f_v -ratios differ substantially in the two areas of the cross-section where $k = 0.1$ m and $h/k \leq \sim 10$, in which the results of Equation (11) are close (within $\pm \sim 6\%$) to those of Equation (5), with y_o estimated from the observed profiles, while those of Equation (5) with *Nikuradze's* friction length, $y_o = k/30$, are higher by $\sim 9.6\%$, on average. It follows, therefore, that the amended friction-length formula— $y_o = ck$ and $c = 8.61(h/D_{84})^{-1.821}$ (Equations (10) and (12))—could be applied to shallow streams with a rough bed, for example, $h/k \leq 30$. Also, all f_v values at $x = 6.5$ m, despite their agreement, seem less reliable due to the mediocre fit of the log-profile, $r^2 \approx 0.72$, which can be attributed to the presence of a maximum-velocity dip to $y \approx 0.35$ m, $y/h \approx 0.7$.

Table 2. Loussios River: $f_v = V/v_{surf}$ from Equation (5) based on (a) observed profiles and (b) Nikuradze's $y_o = k/30$ and from Equation (11) based on estimated bed roughness and $y_o = ck$, $c \approx 8.61(h/D_{84})^{-1.821} \geq 1/30$.

Location Depth		From Velocity Data				From Bed Inspection				With Nikuradze's Formula		
x [m]	h [m]	y_o [m]	h/y_o	f_v Equation (5)	k [m]	h/k	$y_o = ck$ [m]	h/y_o	f_v Equation (11)	$y_o = k/30$ [m]	h/y_o	f_v Equation (5)
6.0	0.460	0.0011	418	0.837	0.015	30.7	0.0003	1764	0.867	0.0005	920	0.855
6.5	0.489	0.0004	1222 ₅	0.860	0.015	32.6	0.0002	2096	0.870	0.0005	978	0.856
7.0	0.539	0.0068	79.3	0.784	0.03	18.0	0.0014	390	0.835	0.0010	540	0.843
7.5	0.591	0.0135	43.8	0.759	0.05	11.8	0.0049	120	0.799	0.0017	355	0.832 ₅
8.0	0.664	0.0026	255 ₅	0.824	0.03	22.1	0.0009	703	0.849	0.0010	664	0.848
8.5	0.751	0.0099	75.9	0.782	0.1	7.5	0.0225	33.3	0.746	0.0033	225	0.820
9.0	0.719	0.0173	41.6	0.756	0.1	7.2	0.0244	29.5	0.740	0.0033	216	0.819
9.5	0.735	0.0476	15.4	0.704	0.1	7.4	0.0234	31.4	0.743	0.0033	220 ₅	0.819

Evrotas River. The data listed in Table 3 show that the f_v -ratios computed with Equation (5) using friction lengths determined from the observed profiles vary markedly [0.623, 0.738]. It is worth noting that all ratios are substantially less than the default $f_v = 0.86$; this is due to their association with the values of the inverted scaled friction length, $h/y_o < 100$, well below those suggested in [13] for streams: $10^2 \leq h/y_o \leq 10^4$. The h/y_o -values that are based on the inspected bed roughness are also in a relatively low range [9, 678] because they derive from large roughness elements in this shallow stream, so the amended friction length, $y_o = ck$, $c = 8.61(h/D_{84})^{-1.821}$ (Equations (10) and (12)), yields fairly low h/y_o values. The range of the resulting f_v -ratios computed with Equation (11) is [0.666, 0.850] (Table 3), +10% on average relative to those obtained with observed friction lengths, but at $x = 3, 4$, and $6, 7.5$ m ratios are overestimated by 10–20%. The corresponding f_v -ratios in Table 3 determined with Nikuradze's friction length, $y_o = k/30$, are higher by ~22%, on average [0.802, 0.848], even though the h/y_o -values are relatively low too (128 to 678); therefore, Nikuradze's formula cannot account adequately for the actual, high-roughness field conditions.

Table 3. Evrotas River: $f_v = V/v_{surf}$ from Equation (5) based on (a) observed profiles and (b) Nikuradze's $y_o = k/30$ and from Equation (11) based on estimated bed roughness and $y_o = ck$, $c \approx 8.61(h/D_{84})^{-1.821} \geq 1/30$.

Location Depth		From Velocity Data				From Bed Inspection				With Nikuradze's Formula		
x [m]	h [m]	y_o [m]	h/y_o	f_v Equation (5)	k [m]	h/k	$y_o = ck$ [m]	h/y_o	f_v Equation (11)	$y_o = k/30$ [m]	h/y_o	f_v Equation (5)
0.5	0.641	0.042	15.3	0.703	0.15	4.3	0.0750 *	8.5	0.666	0.005	128	0.802
2	0.853	0.096	8.9	0.669	0.15	5.7	0.0750 *	11.4	0.685	0.005	171	0.811
2.5	0.862	0.097	8.9	0.669	0.15	5.7	0.0750 *	15.8	0.686	0.005	172	0.812
3	0.883	0.031	28.5	0.738	0.05	17.7	0.0024	373	0.834	0.00167	530	0.842
4	0.699	0.0275	25.4	0.732	0.05	14	0.0036	193	0.815	0.00167	419	0.837
5	0.722	0.128	5.6	0.637	0.15	4.8	0.0764	9.5	0.673	0.005	144	0.806
5.5	0.754	0.132	5.7	0.638	0.15	5	0.0709	10.6	0.681	0.005	151	0.807
6	0.678	0.076	8.9	0.669	0.03	22.6	0.0009	745 ₅	0.850	0.001	678	0.848
7.5	0.367	0.079	4.6	0.623	0.03	12.2	0.0028	131	0.803	0.001	367	0.833

* Values from inspected $k = 0.15$ m in $4.3 \leq h/k \leq 5.7$ are estimated as $y_o = ck = 0.075$ m with c fixed at $c(h/k = 5) = 0.5$.

Alagonia Creek. The logarithmic profile fitted to the data ($r^2 \approx 0.92$) (Figure 8) is $v(y) \approx 0.37 \ln(y/0.02) = 0.37 \ln(50y)$, so $y_o \approx 0.02$ m (and $v_*/\kappa = 0.37$, $v_* \approx 0.152$ m/s for $\kappa = 0.41$), which yields as mean velocity is $V \approx 0.88$ m/s, and $v_{surf} \approx 1.248$ m/s, and thus, $f_v \approx 0.705$. Then, with $h = 0.584$ m, (a) $h/y_o = 0.584/0.02 = 29.2$, and Equation (5) gives $f_v = 0.74$; (b) if the roughness is estimated as the mean, $\langle k \rangle = (0.1 \text{ m} + 0.05 \text{ m})/2 = 0.075$ m, $h/\langle k \rangle \approx 7.8$. and $c \approx 0.2$, Equations (10)–(12) yield $f_v \approx 0.75$: $f_v \approx 0.74$ – 0.75 is fair for a rugged headwater stream and much better than the default $f_v = 0.86$. In contrast,

$y_o = \langle k \rangle / 30 = 0.0025$ gives $h/y_o \approx 232$ and $f_v \approx 0.82$ (a poorer but still more useful estimate than $f_v = 0.86$).

Venoge River. We used the ensemble profile with the average granularity of the bed material over the bed (no local variation). Fitting a depth-scaled logarithmic velocity profile, $v(y/h) = (v_*/\kappa) \ln[(y/h)/(y_o/h)]$, to the data ensemble yields $v = 0.536 \ln[(y/h)/0.0936]$, $r^2 \approx 0.999$ (Figure 9); given $h = 0.21$ m [35], $y_o \approx 0.02$ m with $v_* = \sqrt{ghS_o} \approx 0.082$ m/s, $\kappa \approx 0.152$. The corresponding results in table 2 of [35] are $y_o \approx 0.066$ m, $v_* = 0.078$ m/s and $\kappa = 0.17$. There is thus fair agreement between corresponding values of v_* and κ , but not of y_o . However, y_o can be estimated directly as follows. Figure 4 in [35] at $D_{84} \approx 85$ mm yields $h/D_{84} \approx 2.3$. As this submergence value lies outside of the h/D_{84} -range [5,20] used to assign c -values (then fitted by Equation (12)), we extrapolate linearly from the $(h/D_{84}, c)$ -pairs (10, 0.1) and (5, 0.5) to $h/D_{84} = 2.3$ and obtain $c = 0.716$ and the good estimate $y_o = 0.061$ m (nevertheless, extrapolation beyond the calibration range of Equation (12) warrants caution). The f_v -ratio estimated from the ensemble log-profile, $f_v = 0.7367/1.269 \approx 0.58$, and by Equation (5) with $y_o(h/D_{84} = 2.3) = 0.061$ m, $f_v \approx 0.61$, are just 5% apart and in excellent agreement with the estimate of Greco and Martino ([29], cf. Figure 4)) $f_v \approx 0.6$, and all far from $f_v = 0.86$; (the error relative to the data-derived f_v is $(0.86 - 0.58)/0.58 = 0.48$, i.e., a 48% discharge overestimation). For comparison, Equation (9) ([18]), with $v_* = 0.082$ m/s, $\kappa = 0.152$ and $v_{surf} \approx 1.269$ m/s, gives $f_v \approx 0.437$ (for $v_{surf} \approx v(y/h = 0.8) \approx 1.15$ m/s, $f_v \approx 0.665$). The wall Reynolds number, $R_w \approx 7000$, classifies the flow regime of the Venoge River as hydraulically rough.

7. Conclusions

This work set out to provide guidance to hydrographers in estimating the ratio of mean to surface velocity in rough-bed streams when velocity profile data are absent. Considerations of hydromechanics and assessment of the reported laboratory and field cases support the following conclusions:

1. The mean-to-surface-velocity ratio $f_v(y_o/h)$ derived from the *Prandtl–von Kármán* logarithmic velocity profile with the friction length, y_o , evaluated based on an estimated bed roughness—through inspection or via sieve analysis of the bed material (cf. Venoge River case)—is more reliable than the *default* $f_v = 0.86$. This assessment rests on data fitted by log-profiles with $r^2 \geq 0.72$ and holds regardless of whether y_o is described by Equation (2) or by Equations (10) and (12). The dependence of f_v on the depth-scaled friction length enables accounting for cross-sectional variability and better flow estimation after appropriate segmentation.
2. Specifically, if the streambed is granular and rough, i.e., consisting of gravels and rocks, and the depth is shallow, giving submergence values $\sim 5 \leq h/D_{84} < \sim 30$, comparing f_v -ratios derived from velocity field data against f_v -ratios evaluated with the inspected roughness show that $y_o = ck$, $0.013 < c \leq 0.5$, $k \approx D_{84}$, $c \approx 8.61(h/k)^{-1.821}$, Equations (11) and (12) more closely estimate the field-data-derived f_v -ratios than *Nikuradze's* $y_o = k/30$ with Equation (5). Practically, $f_v(y_o/h)$ with $y_o = ck$, $c \approx 8.61(h/k)^{-1.821}$, Equation (12), holds within $\sim 10\%$ error, on average (range 3–20%), for granular stream boundaries down to the start of the intermediate roughness scale of [33] and is robust against misestimating roughness; $y_o = k/30$ (Equation (2)) is suitable for shallow and large streams with smooth beds, for example, $h/k \geq 30$, giving $f_v \geq 0.86$.
3. The classification of resistance in boundary-layer flow by the *wall Reynolds number*, $R_w = v_*k/\nu$, for the two extreme test cases, *Yeh's* laboratory experiment, $R_w \approx 4$, and the Venoge River, $R_w \approx 7000$, suggests that the proposed formulation describes flow from the start of the transition zone, $R_w = 4$, to well in the hydraulically rough regime, $R_w > 100$ [30].

4. The proposed method merits consideration in hydrometry, as the present study assessed the suitability of the friction-length ansatz, $y_o/k = c(h/k) \geq 1/30$, $c \approx 8.61(h/k)^{-1.821}$ (Equation (12)), in challenging streamflow cases over a fairly wide range of relative roughness. With additional validation, this concept may also prove useful for $h/k < 5$, as shown by extrapolation in the case of the Venoge River. In the intermediate roughness scale of Bathurst et al. (1981) [33], fitting a logarithmic velocity profile to the data may require a *von Kármán* constant smaller than $\kappa = 0.4$ (cf. [35]), but care is due, as ' κ operates in an inverse manner to v_* , an error in v_* can be concealed by adjusting κ and vice versa' [42].

Suggestions for further research: The mean-to-surface-velocity ratio derived from the Prandtl–von Kármán logarithmic velocity law, by definition, cannot account for the variable macro-geometry of a natural stream's cross-section, as the log-law assumes an infinitely wide channel (an assumption shared by current hydrometric practice). The influence of the macro-geometry could be studied initially by conducting laboratory experiments in rectangular flumes of various widths, but at the same depth (variable h/b) and with the same granular roughness (constant relative roughness).

Supplementary Materials: The following supporting information can be downloaded at <https://www.mdpi.com/article/10.3390/w18080985/s1>, Supplementary Material A: Streamflow monitoring data. Figure S1. Loussios River: Cross-section at Atsicholos Bridge, near the town of Karytaina, Arcadia; Table S1. Velocity measurements in Loussios River at the Atsicholos Bridge monitoring station; Figure S2. Evrotas River: Cross-section near the village of Karavas, Laconia; Table S2. Velocity measurements in Evrotas River near Karavas Village. Supplementary Material B: Statistical evaluation of fitting logarithmic profiles to the field data.

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Appendix A

Max-Entropy Erf-Type Velocity Profile at $y/h = 0.841$: $v_{ME}(y/h = 0.841) = v_{log}(y/h = 1)$

The f_v -ratio, as a function of the momentum distribution coefficient $\beta = E[v^2]/V^2$, was derived from the max-entropy concept, which led to an error-function-type velocity profile, $v_{ME}(y/d)$ [11]. To curtail unphysically large values near the free surface, that profile was truncated at $y/h = 0.84$, maintaining the velocity constant for $y/h > 0.84$ at $v_{surf} = (V + \sigma)$ of v_{ME} (similar to the log-law's constant-velocity adjustment above $y/d \approx 0.6$ – 0.7 in [13,43] to fit observations in large, shallow and wide streams) (Figure A1). Here, we check those results against those corresponding to the logarithmic law.

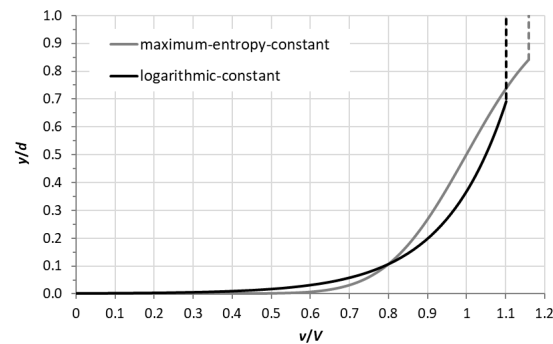


Figure A1. Logarithmic-constant and maximum-entropy-constant velocity profiles for $\beta = 1.16$.

Two statistics of the velocity distribution are needed in this context: the standard deviation, σ , which measures the variation of the velocity about its mean $\mu = V = E[v]$, with $E[.]$ signifying the expectation operation (averaging or integration), and the coefficient of variation, $C_v = \sigma/\mu$:

$$E[v^2] = \frac{1}{A} \int_A v^2 dA = \beta (E[v])^2 = \beta V^2 \quad (A1)$$

$$C_v = \sigma/\mu = \sqrt{(E[v^2] - (E[v])^2)/E[v]} = \sqrt{\beta - 1} \quad (A2)$$

Solving Equation (A2) for the momentum distribution coefficient yields

$$\beta = 1 + C_v^2 \quad (A3)$$

Chow [28]—in a brief paragraph and a footnote directing the reader to an exercise in Chapter 8—the following relationship was derived for the logarithmic profile:

$$\beta = 1 + \varepsilon^2, \quad \varepsilon = v_M/V - 1; \quad (A4)$$

V is the mean velocity, and v_M is the maximum velocity, which occurs at the free surface, so $v_M/V = v_{surf}/V = 1/f_v$. It is informative to compare the momentum distribution coefficient, $\beta = 1 + C_v^2$ (Equation (A3)) to Chow's [28] Equation (A4), $\beta = 1 + \varepsilon^2$, with $\varepsilon = v_{surf}/V - 1 = 1/f_v - 1$,

$$\beta = 2 + 1/f_v^2 - 2/f_v \quad (A5)$$

and the f_v value estimated from the truncated maximum-entropy velocity profile with $v_{surf} = v_{ME}$ ($y/d = 0.841$) ([11], Table 1) (Table A1). In most turbulent profiles, $1 < \beta \leq 1.15$, $0 < C_v = \sigma/\mu \leq 0.387$; in fairly straight, prismatic channels, $\beta \approx 1.01$ – 1.12 ; and in non-uniform channels, $\beta \approx 1.01$ – 1.2 , as the value of β in natural cross-sections increases due to their irregular geometry [28].

Table A1. Variation in the ratio of the mean to the surface velocity, $V/v_{surf} = f(C_v(\beta))$.

Coefficient of variation $C_v = \sigma/\mu$	0	0.1	0.16	0.2	0.316
Momentum correction factor $\beta = 1 + C_v^2$	1	1.0100	1.0256	1.0400	1.0999
$f_v = V/v_{surf}$ from truncated max-entropy profile	1	0.909	0.862	0.833	0.760
$\beta = 2 + 1/f_v^2 - 2/f_v$	1	1.0100	1.0256	1.0402	1.0997

The excellent agreement of corresponding β -values ($\Delta\beta/\beta < 0.003$, $< 0.3\%$), across the range of C_v -values, means that the velocity of the max-entropy profile at the truncation point $v_{ME}(y/d \approx 0.84) = v_{ME-surf}$ is nearly identical to the surface velocity of the logarithmic profile; e.g., $v_{log-surf}/V = v_{ME-surf}/V \approx 0.862$ for $\beta \approx 1.0256$. In turn, this implies that, for

a given β (or, for a given $\varepsilon = C_v$), the f_v -ratios computed from either of these profiles are essentially identical.

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