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**THE BERTH ALLOCATION PROBLEM: A FORMULATION REFLECTING TIME
WINDOW SERVICE DEADLINES**

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Abstract: The berth-allocation problem (BAP) aims to optimally schedule and assign vessels to berthing areas along a quay. The vessels arrive at the port over a period of time and normally request service and departure within a time window. These time windows are usually determined through contractual agreements between the port operator and the carrier, in terms of time of departure after the vessel's arrival at the port. Formulations presented in the current literature, reduce the time window to a point in time. In this paper the discrete dynamic BAP (DDBAP) is formulated as a linear MIP problem with the objective to simultaneously minimize the cost from vessels' late departures (departure past the time window) and maximize the benefits from vessels' early departures and timely departures (departure before and within the requested time window). Two different models along with numerical examples and a comparison to other BAP models are presented to demonstrate the benefits of the proposed berth scheduling formulation.

1. INTRODUCTION

The Berth Allocation Problem (BAP) in a container terminal can be simply described as the problem of allocating berth space to vessels. The problem has two planning/control levels: the strategic/tactical, and the operational. At the strategic/tactical level the number and length of berths/quays that should be available at the port to service the anticipated traffic are determined.. At the operational level, the allocation of berthing space to a set of vessels scheduled to call at the port within a few days time horizon has to be decided upon. This paper deals with the operational level of the BAP.

Although, initially queuing approaches were used to model the BAP (Edmond and Maggs, 1978), at the operational level the BAP is typically formulated as combinatorial optimization problem (i.e. machine scheduling problem, 2D packaging problem). Several berth allocation models have appeared in the literature, differing in the assumptions made, the mathematical formulation, and solution approach. Usually, the formulation of the problem leads to NP-hard or NP-complete problems requiring the use of heuristics and meta-heuristics to obtain solutions in a computationally acceptable time.

The BAP can be modeled as a discrete problem where the quay is viewed as a finite set of berths, each serving one vessel at a time, or as a continuous problem where vessels can berth anywhere along the quay. In the discrete case, the BAP can be modeled as an unrelated parallel machine-scheduling problem, where a vessel is treated as a job and a berth as a machine, whereas in the continuous case as a packaging or the two dimensional cutting stock problem, where one dimension is time and the other the size of the vessels.

The BAP can be also modeled as a static problem (SBAP), if all vessels to be serviced are already in the port at the time the scheduling begins, or as a dynamic problem (DBAP), if some of the vessels have not yet arrived but their estimated time of arrival (ETA) is known in advance. Service priorities and “preferred” berth position for a certain vessel are two issues addressed in some of the BAP published

work. Service priorities have been addressed by assigning weights to vessels. The second issue has been treated through penalizing berthing assignments by a factor proportional to the distance from a preferred point, relating to the distance of the berthing area from the storage yard area, where containers to be loaded on board the given vessel are stored (also known as the preferred berthing point). According to Cordeau et al. (2005) though, planners prefer to handle this aspect by increasing the expected handling time according to the quay segment where the vessel moors. Finally, technical restrictions such as berthing draft, inter-vessel and end-berth clearance distance, that bring the problem formulation closer to real world conditions, are other issues that have been considered.

The problem of the SBAP and DBAP along with the discrete and continuous BAP has been widely studied in different combinations. Most of the studies, as will be seen in more detail in the next section, try to minimize the total service and waiting time (total completion time-TCT) and/or the deviation from the preferred berth, since it is expected that minimization of the deviation from the preferred berthing position will reduce service time and operator's cost. These objectives, however, do not take into account the issue of meeting contractual agreements for the vessels' scheduled start of cargo handling operations and/or departure (which can be denoted as the time of departure after the scheduled arrival in the port). Contractual arrangements can vary from berthing (and start of cargo handling operations) upon arrival, to guaranteed service time window and/or guaranteed service productivity (UNCTAD, 1986). Earliness or lateness of a vessel's handling operations completion time (loading/unloading of containers) implies benefits or costs to both the port operator and the ocean carrier. If these operations are completed after an agreed upon time, the port operator may pay a penalty to the ocean carrier, while if these operations are completed before that time, the carrier may pay a premium fee to the port operator, subject to the contractual arrangements, although in practice premium may be compensated with past or future penalties assigned to the port operator due to failure to meet the terms of the contract. Early departures can help ocean carriers in managing the time factor of their service schedules, by providing time buffer to cope with time lost in other ports (Notteboom, 2006). Early premiums can be offset by

reducing voyage operating cost through reducing the voyage speed and therefore the fuel consumption. In fact, recently, ocean carriers seek to reduce operating cost through voyage speed reduction, while maintaining service reliability (Savvides, 2006 and Lloyds List, 2006).

To our knowledge, research deviating from the general formulations and considering a penalization approach has only been presented by Kim and Moon (2003), Park and Kim (2003), and lately by Wang and Lim (2006). Kim and Moon studied the continuous SBAP with the objective to minimize the cost from non-optimal berthing and the penalty from delaying the departure of a vessel. Their formulation considered handling times as independent of the berth assignment and benefits by early departures were not considered. Park and Kim (2003) minimized the weighted sum of the handling cost of containers, the penalty cost incurred by berthing earlier or later than the expected time of arrival, and the penalty cost incurred only by the delay of the departure beyond the promised due time. Wang and Lim (2006) also consider only the penalty costs from delayed departures.

A study presented by Imai et al. (2003) could also be considered relevant to the one presented in this paper. In Imai et al. (2003) the DBAP with vessel service priorities is formulated as a MIP problem, giving different priorities to vessels using weights assigned based on the volume of each vessel to be handled at the port.

In this paper we consider the discrete DBAP formulation of the problem with scheduled/agreed departures. In addition to minimizing the costs from late departures, we also maximize premiums from early departures to the port operator. Furthermore, handling times depend on the berthing location for each vessel, making the problem more realistic.

This penalization and premium assignment approach for late or early departures can be used when considering the port operator's and ocean carrier's relation when the terminal is operated by a company (publicly or privately owned) different than the carrier, since it provides a method for considering benefits/costs endured from (not) meeting departure deadlines, usually prearranged by contract. The formulation can also be valuable to ports operated by the ocean carrier (dedicated terminals), since

different vessels may have different priorities for the carrier and consequently different departure deadlines by which they must complete cargo handling operations and leave for the next destination port.

The rest of the paper is organized as follows. The next section presents a literature review of existing published studies on the BAP. The problem is described in Section 3 and formulated in section 4. Section 5 presents a numerical example. The paper ends with a brief discussion of the results and proposed future research.

2. LITERATURE REVIEW

Several papers have appeared in the literature dealing with the BAP. One of the first relevant papers was presented by Lai and Shih (1992). The authors assumed that a wharf is represented as a continuous line that could be partitioned into several sections, to each of which only one vessel could be allocated at any specific time. A heuristic algorithm was developed considering a first-come-first-served (FCFS) rule. Brown et al. (1994, 1997) treated the BAP in naval ports. They identified the optimal set of vessel-to-berth assignments that maximizes the sum of benefits for vessels while in port.

Imai et al. (1997) first introduced the idea that for high port throughput, optimal vessel-to-berth assignments should not be based on the First Come First Served (FCFS) rule. However, their formulation may result in some customer's dissatisfaction regarding order of service. To deal with the two evaluation criteria, i.e., berth performance and dissatisfaction due to the order of service, they developed a heuristic algorithm to find a set of non-inferior solutions, while maximizing the former and minimizing the latter. Lim (1998) addressed the continuous BAP with the objective of minimizing the maximum amount of quay space used at any time with the assumption that once a vessel is berthed, it will not be moved to any place else along the quay before it departs. Berthing upon arrival was also assumed.

Li et al. (1998) formulated the SBAP as a scheduling problem with a single processor through which multiple jobs can be processed simultaneously. The objective was the minimization of the make-span. Similar to Li et al. (1998), Guan et al. (2002) considered the berth allocation problem as a multiprocessor task scheduling. They developed a heuristic to minimize the total weighted completion time of vessel service and performed worst-case analysis. Weights were assigned to each job depending on the vessel's size.

Imai et al. (2001) addressed the DBAP with the objective to minimize the sum of a vessel's waiting and handling time (total completion time). Handling time was assumed deterministic and dependent on the berth. In the same context Nishimura et al. (2001) addressed the same problem but for a public berth system. In this paper the authors extended the work presented by Imai et al. (2001) to include physical restrictions (water-depth and quay length). They also dropped the assumption that each berth can handle one vessel at a time. Service priority relied on the FCFS rule. The objective was to minimize total completion time. Imai et al. (2003) modified and extended the discrete DBAP formulation of Imai et al (2001) and Nishimura et al.(2001) in order to include service priority constraints. The objective was to minimize the total completion time while differentiating priorities to vessels by variation of their service time in the solution. In 2005, Imai et al. extended their previous work by solving the DBAP in a continuous berth space with the objective of minimizing the total completion time.

Guan and Cheung (2004) presented a berth allocation model that allows multiple vessels to moor at a berth, considers vessel arrival time and optimizes the total weighted flow time. Following the idea by Imai et al. (2003) they apply a weight coefficient to each vessel. They develop a tree procedure and a heuristic that combines this procedure with the heuristic in Guan et al. (2002). Kim and Moon (2003) studied the continuous SBAP and formulated a MIP model and used simulated annealing to find near optimal solutions. The objective was to minimize delays and handling cost by non-optimal locations of the vessels' berthing.

Park and Kim (2002) consider the continuous BAP. Their objective is to estimate the berthing time and location by minimizing the total waiting and service time and the deviation from the preferred berthing location. They are the first ones to include penalization of the deviation from the optimal berth. Park and Kim (2003), extend their previous work to combine the BAP with consideration of quay crane capacities. Their study determined the optimal start times of vessel services and associated mooring locations while at the same time determined the optimal assignment of quay cranes to vessels.. The handling time was considered independent from the mooring location of the vessel. Lee et al. (2006) following the work of Park and Kim (2003) presented a method for scheduling berth and quay cranes, which are critical resources in container ports. A bi-level programming model with the objective of minimizing the sum of total completion time of all the vessels and the completion time for all the quay cranes is formulated by considering various practical constraints such as interference between the quay cranes. To solve this model, a genetic algorithm is used to determine the near optimal solution. A computational experiment is conducted to examine the performance of the proposed bi-level programming model and algorithm.

Cordeau et al. (2005) considered the discrete case of the DBAP and provided two formulations: a formulation similar to Imai et al. (2001) and a formulation similar to the Multi Depot Vehicle Routing Problem with Time Windows. To avoid simplifications, contrary to Park and Kim (2003), the authors did not solve the BAP and the Quay Crane Assignment Problem (QCAP) simultaneously. The objective was the minimization of the total (weighted) service time for all vessels, defined as the time elapsed between the arrival in the port and the completion of handling.

Imai et al. (2006a) addressed the berth allocation problem at a multi-user container terminal with indented berths for fast handling. A new integer linear programming formulation was presented, which was then extended to model the berth allocation problem at a terminal with indented berths, where both mega-container vessels and feeder vessels are to be served for higher berth productivity. A genetic algorithm heuristic was used to solve the problem to optimality. From derived computational results it

was concluded that while the indented terminal served the mega-vessel faster than the conventional terminal, the total service time for all vessels was longer than the one in the conventional terminal. Imai et al. (2006b) addressed a variation of the BAP at multi-user terminals, as vessels for which the expected wait time exceeds a given time limit were assigned to an external terminal. The objective of the problem was to minimize the total service time of vessels at the external terminal. A GA based heuristic was developed and numerical experiments showed that the heuristic performed well in reducing external terminal usage.

Wang and Lim (2006) solve the DBAP by minimizing un-allocation, position and delay costs, using a Stochastic Beam Search Heuristic that outperformed both the current state-of-the-art metaheuristics and the traditional beam search. The authors concluded that the formulation and solution approach is fast, easy to modify and implement, and can be directly applied to solving multi-stage decision making problems.

3. PROBLEM DESCRIPTION

Assume that a set of vessels are set to arrive at a port over a period of time and serviced at a number of berths. We assume that each berth can handle one vessel at a time regardless of the vessel's size and that there are no physical/technical restrictions. The vessel's handling time is assumed to be dependent on the berth where it will be assigned and on the number of containers to be handled at that port.

Allocating vessels to berths by minimizing the total completion time leads to vessels with smaller handling volumes receiving higher priorities than vessels with larger handling volumes (Pinedo, 2002). The later end up serviced at the end of the queues at each berth. Therefore, given the situation that two vessels with different handling volumes arrive at the same time, the large vessel will wait for the smaller vessel to get serviced, if they are both assigned at the same berth. Assignment policies based on this objective have the consequence of higher waiting times for larger vessels. Some large vessels though,

for a number of reasons (call at another port, time sensitive cargo, contract requirements, etc), might need to be assigned for service and/or finish their service as soon as possible, after their arrival at the port. In fact, high carrying capacity vessels are more likely to belong to mainliner services. These services are much more time sensitive, as compared to feeder services, which, normally, use lower carrying capacity vessels. Additionally, this assignment policy implies that the berth will be unutilized for certain periods of time waiting for the small vessels to arrive, resulting in reduced berth productivity and extra cost to the port operator.

Information on vessel's agreed/prearranged departure can be included in a scheduling policy by modifying existing DBAP formulations to either: a) minimize the total time differential between arrival and service completion (completion time) for a subset of the vessels to be scheduled, b) introduce a set of constraints that satisfy the requirement(s) for early service completion time for a subset of the scheduled vessels, or c) introduce priority considerations through weights.

The first formulation presents the shortcomings mentioned earlier, inherent in minimizing the total completion time for the given subset of vessels.

The second formulation, though intuitive, has not been treated in the literature and it could eventually lead to infeasible solutions when the number of the customers requesting early start or service completion time becomes large in comparison to the total number of customers requesting service.

The later formulation (Imai et al.; 2003) has the difficulty of selecting the appropriate weights, since service priority varies in a fuzzy way with the different values of the weights, unlike penalties/premiums for late/early departures that are usually predefined by contract. Selecting the set of weights to achieve the goal previously described, (i.e. move large vessels forward of small vessels that have arrived at the same time at the port), could lead to cumbersome iterative computations. Furthermore, using weights based on cargo volumes does not capture priorities based on the type of service (mainliner or feeder service) and does not provide any information on the cost that vessel operators or the port operator will

endure by assigning small weights to some vessels that will then be moved to the end of the service queues.

The objective of this paper is to modify the DBAP formulation by Imai et al. (2001) so that the scheduling policy is not determined by the total completion time, but from taking into account and responding to the requests/agreements for early service/departure. A generic non linear formulation is proposed in this paper, along with the proof that this formulation is the general case of two existing DBAP formulations, originally proposed by Imai et al. (2001, 2003). Consequently, the generic formulation is reformulated to a MILP formulation.

The proposed approach differs from the one presented by Kim and Moon (2003), in that: a) handling times vary with the berth assignment, and b) the assignment policy is based on benefits from both late and early departures. Unlike Kim and Moon (2003) though, the discrete, instead of the continuous BAP is considered herein.

4. MODEL FORMULATION

4.1 Original Formulation

The [DBAP] problem was initially presented and formulated by Imai et al. (2001) and improved by Imai et al. (2003) [DBAPW with the following parameters and variables defined: $i=(1,\dots,I) \in B$ set of berths, $j=(1,\dots,T) \in V$ set of vessels, $k=(1,\dots,T) \in O$ set of service orders, S_i =time when berth become idle, A_j =arrival time, C_{ij} = handling time of vessel j at berth i , $X_{ijk}=1$ if vessel j is serviced at berth i as the k^{th} vessel, y_{ijk} = idle time of berth i between departure of vessel j and its immediate predecessor. The problem formulation by Imai et al. (2001, 2003) is shown below (equations 1 through 5).

$$[\text{DBAP}]: \min \sum_{i \in B} \sum_{j \in V} \sum_{k \in U} \{(T - k + 1)C_{ij} + S_i - A_j\} X_{ijk} + \sum_i \sum_j \sum_k (T - k + 1)y_{ijk} \quad (1a)$$

$$\text{Subject to: } \sum_{i \in B} \sum_{k \in U} X_{ijk} = 1, \forall j \in V, \quad (2)$$

$$\sum_{j \in V} X_{ijk} \leq 1, \forall i \in B, k \in O, \quad (3)$$

$$\sum_{m \neq j \in T} \sum_{h < k \in O} (C_{im} X_{imh} + y_{imh}) + y_{ijk} - (A_j - S_i) X_{ijk} \geq 0, \forall i \in B, j \in T, k \in O, \quad (4)$$

$$X_{ijk} \in \{0,1\}, \quad y_{ijk} \geq 0 \quad (5)$$

$$\text{[DBAPW]: } \min \sum_{i \in B} \sum_{j \in V} \sum_{k \in U} (C_{ij} + S_i - A_j + \sum_{l \neq j} \sum_{m < k} C_{il} X_{ilm}) a_j X_{ijk} + \sum_i \sum_j \sum_k (y_{ijk} + \sum_{l \neq j} \sum_{m < k} y_{ilm}) a_j \quad (1b)$$

Subject to: (2)-(5)

The objective function of [DBAP] minimizes the total of waiting and handling times for every vessel. The objective function of [DBAPW] minimizes the weighted total of waiting and handling times for every vessel. Constraints (2) ensure that vessels must be serviced once; constraints (3) that each berth services one vessel at a time; and constraints (4) that each vessel is serviced after its arrival. In the objective function the term C_{ij} is weighted to ensure that if a vessel is served with k successors the appropriate waiting time will be added to these successors.

4.2 Proposed Formulation

In formulating the DBAP problem with departure deadlines and time windows the value of lateness penalty and earliness premium need to be introduced as well as the requested departure time window. We also need to introduce a fourth index to the decision variable. Thus we define the following: $X_{ijk1} = 1$ if vessel j serviced at berth i as the k^{th} vessel and departs before time window (Departure time $< t_{j1}$) and zero otherwise, $X_{ijk2} = 1$ if vessel j serviced at berth i as the k^{th} vessel and departs after scheduled time window (Departure time $> t_{j2}$) and zero otherwise, $X_{ijk3} = 1$ if vessel j serviced at berth i as the k^{th} vessel and departs within time window ($t_{j1} < \text{Departure time} < t_{j2}$) and zero otherwise, $a_j =$ per hour earliness

premium¹ for vessel j , b_j = per hour lateness penalty² for vessel j , γ_j = per hour premium for timely departure, t_{j1} = requested early departure time (usually given in hours after the vessel's scheduled arrival), t_{j2} = requested early departure time (usually given in hours after the vessel's scheduled arrival).

Equations (4) estimates the service start time for each vessel (for vessels arriving after the beginning of the planning horizon but can be extended to include all vessels, with idle time zero for vessels already in port at the beginning of the planning horizon) and is used in the formulation presented herein to obtain the difference of the actual and requested departure time. If y_{ijk} is greater than zero then the vessel starts service at arrival. If y_{ijk} is zero then the vessel starts service after its arrival. The [DBAP] with departure deadlines in the form of time windows may be formulated as a MIP problem as follows (equations 6 through 14):

$$\text{[DBAPDTW1]: } \min \sum_i \sum_j \sum_k -(a_j X_{ijk1} - b_j X_{ijk2} + \gamma_j X_{ijk3}), \quad (6)$$

$$\text{Subject to: } \sum_i \sum_j \sum_k X_{ijk1} + X_{ijk2} + X_{ijk3} = 1, \forall j, \quad (7)$$

$$\sum_{j \in V} X_{ijk1} + X_{ijk2} + X_{ijk3} \leq 1, \forall i \in B, k \in O, \quad (8)$$

$$\sum_{m \neq j \in T} \sum_{h < k \in O} (C_{im} (X_{imh1} + X_{imh2} + X_{imh3}) + y_{imh}) + y_{ijk} - (A_j - S_i)(X_{ijk1} + X_{ijk2} + X_{imh3}) \geq 0, \quad (9)$$

$\forall i \in B, j \in T, k \in O$

$$t_{j1} X_{ijk1} \geq \sum_{j \neq m \in T} \sum_{h < k \in O} (C_{im} (X_{imh1} + X_{imh2} + X_{imh3}) + y_{imh}) + y_{ijk} + (C_{ij} + S_i) X_{ijk1} - M(1 - X_{ijk1}), \quad (10)$$

$\forall i \in B, j \in T, k \in O$

$$t_{j2} X_{ijk2} \leq \sum_{j \neq m \in T} \sum_{h < k \in O} (C_{im} (X_{imh1} + X_{imh2} + X_{imh3}) + y_{imh}) + y_{ijk} + (C_{ij} + S_i) X_{ijk2}, \forall i \in B, j \in T, k \in O, \quad (11)$$

$$t_{j1} X_{ijk3} \leq \sum_{j \neq m \in T} \sum_{h < k \in O} (C_{im} (X_{imh1} + X_{imh2} + X_{imh3}) + y_{imh}) + y_{ijk} + (C_{ij} + S_i) X_{ijk3}, \forall i \in B, j \in T, k \in O, \quad (12)$$

$$t_{j2} X_{ijk3} \geq \sum_{j \neq m \in T} \sum_{h < k \in O} (C_{im} (X_{imh1} + X_{imh2} + X_{imh3}) + y_{imh}) + y_{ijk} + (C_{ij} + S_i) X_{ijk3} - M(1 - X_{ijk3}), \quad (13)$$

$\forall i \in B, j \in T, k \in O$

¹ a_j is a positive number

² b_j is a positive number

$$X_{ijk1}, X_{ijk2}, X_{ijk3} \in \{0,1\}, y_{ijk} \geq 0, M \text{ is a large positive number,} \quad (14)$$

The objective function (6) seeks to minimize the total cost from delayed departures and maximize the benefits from early and timely departures. Constraints (7) ensure that vessels must be serviced once (either earlier or later than its requested departure time); constraints (8) ensure that each berth services one vessel at a time; and constraints (9) ensure that each vessel is serviced after its arrival. Constraints (10) and (14) enforce the declaration of the decision variables.

The previous formulation [DBAPDTW1] does not take into consideration the exact time of a vessels' finish time, i.e. the same cost is applied if a vessel finished service 5 or 10 hours later than the end of the desired time window. This can lead to the problem when all vessels can be serviced in one of the time slots, i.e. all vessels served before or within or after time window. [DBAPDTW1] though may be expanded to portray hourly costs and benefits as follows: Define: $DT_{ijk}^+ = t_{j1} - FT_j \geq 0$, and $DT_{ijk}^- = t_{j2} - FT_j \leq 0$, where FT_j is the finish time of vessel j . Unlike [DBAPDTW1], in this formulation the benefit and cost parameter a_j and b_j have units of monetary value per hour. The benefit parameter γ_j can be calculated by the following formula: $\gamma_j = (t_{j2} - t_{j1}) * a_j$. The new formulation is presented in equations (15)-(20).

$$\text{[DBAPDTW2]: } \min \sum_i \sum_j \sum_k -(a_j DT_{ijk}^+ + b_j DT_{ijk}^- + \gamma_j X_{ijk3}), \quad (15)$$

Subject to: (7)-(14) and,

$$DT_{ijk}^+ \leq (t_{j1} - C_{ij} - S_i) X_{ijk1} - y_{ijk} - \sum_{j \neq m \in T} \sum_{h < k \in O} (C_{im} (X_{imh1} + X_{imh2} + X_{imh3}) + y_{imh}) + \alpha_{ijk}, \quad (16)$$

$$\forall i \in B, j \in T, k \in O$$

$$DT_{ijk}^- \leq (t_{j2} - C_{ij} - S_i) X_{ijk2} - y_{ijk} - \sum_{j \neq m \in T} \sum_{h < k \in O} (C_{im} (X_{imh1} + X_{imh2} + X_{imh3}) + y_{imh}) + M(1 - X_{ijk2}), \quad (17)$$

$$\forall i \in B, j \in T, k \in O$$

$$\alpha_{ijk} \leq M(1 - X_{ijk1}), \forall i \in B, j \in T, k \in O \quad (18)$$

$$\alpha_{ijk} \leq \sum_{j \neq m \in T} \sum_{h < k \in O} (C_{im} (X_{imh1} + X_{imh2} + X_{imh3}) + y_{imh}) + y_{ijk}, \forall i \in B, j \in T, k \in O, \quad (19)$$

$$DT_{ijk}^+ \leq MX_{ijk1}, \forall i \in B, j \in T, k \in O, \quad (20)$$

Further explanations are needed for constraints (16) through (20). If vessel j is not serviced at berth i then (16) and (17) should be zeroed out from the objective function. On the other hand, if vessel j is serviced at berth i before t_{j1} (16) should be forced to zero and vice versa for (17). In what follows is a mathematical explanation of the previous statements. If $X_{ijk1} = X_{ijk2} = X_{ijk3} = 0$ then $DT_{ijk}^+ \leq 0$ and $DT_{ijk}^- \leq M$, which means that both $(DT_{ijk}^+ \& DT_{ijk}^-)$ will be forced down to zero.

On the other hand if: $X_{ijk1} = X_{ijk3} = 0$, then $X_{ijk2} = 1$, $alpha_{ijk} = \sum_{j \neq m \in T} \sum_{h < k \in O} (C_{im} X_{imh2} + y_{imh}) + y_{ijk}$,

$$DT_{ijk}^+ = 0, \text{ and } DT_{ijk}^- = (t_j X_{ijk2} - \sum_{j \neq m \in T} \sum_{h < k \in O} (C_{im} X_{imh2} + y_{imh}) - y_{ijk} - (C_{ij} + S_i) X_{ijk2})$$

If: $X_{ijk2} = X_{ijk3} = 0$, then $X_{ijk1} = 1$, $alpha_{ijk} = 0$, $DT_{ijk}^- = 0$, and

$$DT_{ijk}^+ = t_j X_{ijk1} - \sum_{j \neq m \in T} \sum_{h < k \in O} (C_{im} X_{imh1} + y_{imh}) - y_{ijk} - (C_{ij} + S_i) X_{ijk1}$$

Finally if: $X_{ijk1} = X_{ijk2} = 0$, then $X_{ijk3} = 1$, $DT_{ijk}^+ = 0$, and $DT_{ijk}^- = 0$, as shown previously.

Proposition I: [DBAPDTW2] is a linear formulation of a special case of [DBAPW]

Proof: Assume that the time window is reduced to a point in time, where $t_{j1} = t_{j2} = A_j$. Also assume that $a_j = 0$, and $b_j > 0$. In this case it follows that: $DT_{ijk}^+, X_{ijk1}, X_{ijk3} = 0, \forall i \in B, j \in T, k \in O$, and thus the fourth

index is no longer needed. Thus [DBAPDTW2] is reformulated as:

$$\min \sum_i \sum_j \sum_k -b_j DT_{ijk}^- \quad (21)$$

$$\text{Subject to: } \sum_i \sum_j \sum_k X_{ijk} = 1, \forall j, \quad (22)$$

$$\sum_{j \in V} X_{ijk} \leq 1, \forall i \in B, k \in O. \quad (23)$$

$$\sum_{m \neq j \in T} \sum_{h < k \in O} (C_{im} X_{imh} + y_{imh}) + y_{ijk} - (A_j - S_i) X_{ijk} \geq 0, \forall i \in B, j \in T, k \in O, \quad (24)$$

$$DT_{ijk}^- \leq M(1 - X_{ijk}) - (C_{ij} + S_i - A_j)X_{ijk} - y_{ijk} - \sum_{j \neq m \in T} \sum_{h < k \in O} (C_{im} X_{imh}) - \sum_{j \neq m \in T} \sum_{h < k \in O} y_{imh} \quad (25)$$

$$\forall i \in B, j \in T, k \in O$$

$$X_{ijk} \in \{0,1\}, \text{ Integer}, y_{ijk} \geq 0, DT_{ijk}^- \leq 0. \quad (26)$$

It is easy to prove that this formulation is the linear version of [DBAPW]. The difference between the two formulations is the addition of the last set of constraints in the former one that achieves the linearity.

DT_{ijk}^- will be zero if vessel j is not serviced at berth i and equal to the finish time of service of vessel j otherwise. \square

Proposition I: [DBAPDTW2] is the general case of [DBAP]

Proof: Since [DBAPDTW2] is the general case of [DBAPW], and [DBAPW] is the general case of [DBAP], it follows that [DBAPDTW2] is the general case of [DBAPW]. \square

[DBAPDTW1] and [DBAPDTW2] is a linear MIP. Compared to the original [DBAP] the new formulations increase the number of decision variables. [DBAP] was solved by a heuristic procedure based on the Lagrangian relaxation of the problem. The more complicated formulation of [DBAPW], where weights were used to decide on a vessels' priority, was solved by a genetic algorithm based heuristic for the problem resulting in a non-linear formulation by Imai et al. (2003) (unlike [DBAPDTW1] and [DBAPDTW2] which are linear formulations), while a similar but simpler formulation, to the one presented herein, was solved by the use of Simulated Annealing by Kim and Moon (2003). Furthermore, another formulation of the DBAP presented by Imai et al. (2006), which was more complex than both [DBAPDTW1] and [DBAPDTW2], was also solved by the use of the same genetic algorithm (GA) based heuristic as Imai et al. (2003). It can be concluded that the GA approach implemented by Imai et al. (2003, 2006) should be sufficient for solving medium to large-scale

instances. On the other hand, custom MIP solvers can be used for small scale instances {in this paper the NEOS³ solver (Grop & Moré, 1997)} was used to carry out the computations for the numerical example presented in the next section).

5. Numerical Example

The models previously presented were coded in GAMS on a Dell 670 Workstation for a small instance of the problem (2 berths and 10 vessels). Tables 1 and 2 present a comparison for the small instance of the problem to illustrate possible advantages of the proposed formulation. Results from [DBAPDTW1] and [DBAPDTW2] are presented along with results from [DBAP] and [DBAPW] on the same dataset. It should be expected that as the instance of the problem (number of berths and vessels) becomes larger the differences become more noticeable, and it is up to the port operator to select which assignment best satisfies its requirements and goals.

Table 1. Objective values of models

Objective / Model	Total Time (in hours)	Total Weighted Time (in hours*weight)	Cost from Delays/ Benefit from Timely/Early Departures – Lump (in \$)	Cost from Delays/ Benefit from Timely/Early Departures – Hourly (in \$)
DBAP	1,193*	82,177	-44300	-45,884
DBAPW	1,221	78,123*	-47500	-45,152
DBAPDTW1	1,640	84,450	-23100*	-46,734
DBAPDTW2	1,641	83,068	-41100	-44,620*

Note: Minimum value of each objective per model formulation

Table 2. Vessel assignment from each model

DBAP		DBAPW		DBAPDTW1		DBAPDTW2	
Berth 1	SO	Berth 1	SO	Berth 1	SO	Berth1	SO
Vessel6	1	Vessel6	1	Vessel4	1	Vessel4	1
Vessel2	2	Vessel9	2	Vessel10	2	Vessel9	2
Vessel3	3	Vessel7	3	Vessel7	3	Vessel7	3
Vessel1	4	Vessel5	4	Vessel3	4	Vessel8	4
Vessel10	5	Vessel10	5	Vessel1	5	Vessel10	5

³ <http://www-neos.mcs.anl.gov/>

Berth2	SO	Berth2	SO	Berth2	SO	Berth2	SO
Vessel4	1	Vessel4	1	Vessel6	1	Vessel6	1
Vessel9	2	Vessel3	3	Vessel5	2	Vessel2	2
Vessel7	3	Vessel2	4	Vessel2	3	Vessel3	3
Vessel8	4	Vessel1	5	Vessel9	4	Vessel1	4
Vessel5	5	Vessel8	5	Vessel8	5	Vessel5	5

6. CONCLUSIONS

Berth allocation policies with service priority are important in terminal operations and are applicable in situations involving various vessel sizes and different handling volumes. Service priority reflects a series of service contractual arrangements between ocean carriers and port operators. A critical question, though, is what should be the basis on which these priorities will be quantified to reflect realistically contractual arrangements usually adopted by carriers and port operators. Minimizing the total completion time (including waiting time) by weighting vessels, based on handling volumes, presents certain limitations. Furthermore, while scheduling berths minimizing the total completion time of operations might provide a measure of the ports' efficiency and attractiveness, it offers no information on dissatisfaction of the individual customer and the resulting direct or indirect cost to the port operator (delay penalties, customer dissatisfaction that may lead to loss of a contract, etc).

On the other hand quantifying monetary costs from late departures and benefits from early departures could be more valuable in determining future contractual agreements between port operators and ocean carriers and more straightforward in determining the most beneficial berth allocation policy. The formulation presented in this paper is of equal complexity as previous formulations, and has the advantage of being a weighted berth allocation policy where weights are represented as monetary costs derived directly from contractual agreements between terminal operators and ocean carriers. This formulation makes decisions on vessel priority and the corresponding benefits/losses more quantifiable and straightforward.

The berth allocation policy adopted in this paper could be beneficial for ports operated by a company (publicly or privately owned) different than the ocean carrier (common user terminals), since it provides

information on costs/benefits endured from (not) meeting departure deadlines. The proposed approach could also be valuable to ports operated by the carrier (dedicated terminals) as different vessels may have different priorities for the carrier and consequently different departure deadlines, by which they must complete cargo handling operations and leave for the next destination port.

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