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FACULTY OF ENGINEERING AND TECHNOLOGY

MSc Thesis

A FREE BASIC EDQNM CODE FOR ISOTROPIC TURBULENCE

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ΤΕΧΝΟΛΟΓΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΚΥΠΡΟΥ

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ΚΩΔΙΚΑΣ EDQNM ΓΙΑ ΕΠΙΛΥΣΗ ΙΣΟΤΡΟΠΙΚΗΣ ΤΥΡΒΗΣ

του

Αλέξανδρου Νικολαΐδη

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Η έγκριση της μεταπτυχιακής διατριβής από το Τμήμα Πολιτικών Μηχανικών και Μηχανικών Γεωπληροφορικής του Τεχνολογικού Πανεπιστημίου Κύπρου δεν υποδηλώνει απαραιτήτως και αποδοχή των απόψεων του συγγραφέα εκ μέρους του Τμήματος.

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ABSTRACT

Isotropic turbulence is an ideal state where the motion properties, in the statistical sense, do not show any directional dependence. More strictly, they satisfy rotational and translational invariance. Isotropic turbulence is the simplest form of turbulent flow still maintaining all its fundamental characteristics. It has been the playground for theoretical work through the last century. Isotropy as a statistical notion is not manifest at the level of the velocity field dynamics but at the level of the correlation functions, where the averaging has removed all the irrelevant direction dependencies. Isotropic turbulence lives strictly in an infinite physical space. Via this energy cascade, turbulent flow can be realized as a superposition of a spectrum of flow velocity fluctuations and eddies upon a mean flow. The eddies are loosely defined as coherent patterns of flow velocity, vorticity and pressure. However, the isotropic flow, requiring first of all an infinite physical space to live in, is an idealization that cannot be strictly realized even in the clinical environment of numerical simulations. In the direct numerical simulations (DNS) of isotropic turbulence one usually solves the Navier-Stokes equation imposing periodicity. The idea is to introduce finiteness in space in a smooth manner. Isotropic flows are replaced by another kind of ideal flows that can be handled numerically.

In the infinite space the infinite sequence of equations has to be closed at some finite order, this is being done semi-empirically. In other words, we must truncate this set of equations by a model, each reasonable model is called a closure model. The Eddy Damped Quasi-Normal Markovian (EDQNM) is a subfilter closure model applied in spectral wavenumber space rather than physical space which considers interactions between resolved and subfilter wavenumbers by considering the statistics of their possible interactions. The EDQNM achieves closure by modeling the 4th spectral moments.

An EDQNM code for resolving forced isotropic turbulence was created in this work. The work may be considered as continuation of previous work done by Michalis Pieris (2016) who wrote an EDQNM code for equally distant wave-numbers, however permitting the possibility of resolving large Reynolds numbers. The new code was applied in the case of for forced turbulence at Reynolds number around 1000, resembling high Reynolds DNS from the past. The comparison shows significant similarities in the macroscopic characteristics of the stationary state, however, it reveals a different behavior at the dissipative range with sharper tails and lower palinstrophy values at the EDQNM spectra.

CHAPTERS

FIGURES

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1. Introduction

From our kitchen top to the skies turbulence is everywhere! A complex and beautiful type of flow characterized by chaotic property changes. Humans are a pattern-seeking species. We like to seek order in apparent chaos, and this, perhaps, is what makes turbulence such a captivating subject. Leonardo da Vinci described turbulence (Fig. 1) in a very motivated way through his words: *Observe the motion of the surface of the water, which resembles that of hair, which has two motions, of which one is caused by the weight of the hair, the other by the direction of the curls; thus the water has eddying motions, one part of which is due to the principal current, the other to random and reverse motion*. (Trans. Piomelli in Lumley, J.L., 1997. Some comments on turbulence, *Phys. Fluids A* **4**, 203-211)

Figure 1. The formation of turbulence under a water flow, as it was sketched by Leonardo da Vinci.

In the present work we give a rough description of the present theory about isotropic turbulence. We present the principal equations Navier-Stokes and the Direct Numerical Simulations in order to solve them numerically. Finally we present and analyze in detail an EDQNM code for isotropic turbulence and we solve numerically a forced stationary case.

1.1 Turbulence and its characteristics

In fluid dynamics, **turbulence** or **turbulent flow** is a flow regime characterized by chaotic property changes (Figures 2-3). This includes (Pope,) low momentum diffusion, high momentum convection, and rapid variation of pressure and flow velocity in space and time. Flow in which the kinetic energy dies out due to the action of fluid molecular viscosity is called laminar flow. The different regimes that can take place (laminar, turbulent, ...) are controlled by the dimensionless number (now called the Reynolds number)

Figure 2. Turbulence formation at the smoke of a cigarete.

$$Re = LV/v, \qquad (1.1)$$

where L and V are a typical length scale and a typical velocity scale of the flow, and v is the viscosity. While there is no theorem relating the non-dimensional Reynolds number to turbulence, flows at Reynolds numbers larger than 5000 are typically (but not necessarily) turbulent, while those at low Reynolds numbers usually remain laminar. In Poiseuille flow, for example, turbulence can first be sustained if the Reynolds number is larger than a critical value of about 2040; moreover, the turbulence is generally interspersed with laminar flow until a larger Reynolds number of about 4000. In turbulent flow, unsteady vortices appear on many scales and interact with each other. Drag due to boundary layer skin friction increases. The structure and location of boundary layer separation often changes, sometimes resulting in a reduction of overall drag. Although laminar-turbulent transition is not governed by Reynolds number, the same transition occurs if the size of the object is gradually increased, or the viscosity of the fluid is decreased, or if the density of the

Figure 3. Turbulence in a cup of coffee.

fluid is increased. Turbulence is characterized by the following features (Davidson, 2004).

- Turbulence is chaotic with intrinsic spatio-temporal irregularity
- Loss of predictability, but stable statistical properties
- Extremely wide range of strongly and nonlocally interacting degrees of freedom ('scales' in time and space)
- Highly dissipative, statistically irreversible
- Turbulent flows are three-dimensional and rotational with continuous self-production of vorticity
- Strongly diffusive with enhanced transport of momentum, energy, and passive scalars
- Strongly nonlinear, non-integrable, nonlocal and non-Gaussian.

More specifically, turbulence is characterized mainly by

Irregularity: Turbulent flows are always highly irregular. For this reason, turbulence problems are normally treated statistically rather than deterministically. Turbulent flow is chaotic. However, not all chaotic flows are turbulent.

Diffusivity: The readily available supply of energy in turbulent flows tends to accelerate the homogenization (mixing) of fluid mixtures. The characteristic which is responsible for the enhanced mixing and increased rates of mass, momentum and energy transports in a flow is called "diffusivity". Turbulent diffusion is usually described by a turbulent diffusion coefficient. This turbulent diffusion coefficient is defined in a phenomenological sense, by analogy with the molecular diffusivities, but it does not have a true physical meaning, being dependent on the flow conditions, and not a property of the fluid itself. In addition, the turbulent diffusivity concept assumes a constitutive relation between a turbulent flux and the gradient of a mean variable similar to the relation between flux and gradient that exists for molecular transport. In the best case, this assumption is only an approximation. Nevertheless, the turbulent diffusivity is the simplest approach for quantitative analysis of turbulent flows, and many models have been postulated to calculate it.

Rotationality: Turbulent flows have non-zero vorticity and are characterized by a strong three-dimensional vortex generation mechanism known as vortex stretching. In

fluid dynamics, they are essentially vortices subjected to stretching associated with a corresponding increase of the component of vorticity in the stretching direction, due to the conservation of angular momentum. On the other hand, vortex stretching is the core mechanism on which the turbulence energy cascade relies to establish the structure function. In general, the stretching mechanism implies thinning of the vortices in the direction perpendicular to the stretching direction due to volume conservation of fluid elements. As a result, the radial length scale of the vortices decreases and the larger flow structures break down into smaller structures. The process continues until the small scale structures are small enough that their kinetic energy can be transformed by the fluid's molecular viscosity into heat. This is why turbulence is always rotational and three dimensional. For example, atmospheric cyclones are rotational but their substantially two-dimensional shapes do not allow vortex generation and so are not turbulent. On the other hand, oceanic flows are dispersive but essentially non rotational and therefore are not turbulent.

Figure 4. The night sky (Van Gong)

Dissipation: To sustain turbulent flow, a persistent source of energy supply is required because turbulence dissipates rapidly as the kinetic energy is converted into internal energy by viscous shear stress. Turbulence causes the formation of eddies of many different length scales. Most of the kinetic energy of the turbulent motion is contained in the large-scale structures. The energy "cascades" from these large-scale structures to smaller scale structures by an inertial and essentially inviscid mechanism. This process continues, creating smaller and smaller structures which produces a hierarchy of eddies. Eventually this process creates structures that are small enough that molecular diffusion becomes important and viscous dissipation of energy finally takes place. The scale at which this happens is the Kolmogorov length scale which is analyzed below.

1.2 The scales of isotropic turbulence and the formation of the energy spectrum

Isotropic turbulence is an ideal state where the motion properties, in the statistical sense, do not show any directional dependence (Figure 5). More strictly, they satisfy rotational and translational invariance. Isotropic turbulence is the simplest form of turbulent flow still maintaining all its fundamental characteristics. It has been the playground for theoretical work through the last century (Richardson, 1922; Taylor, ;Karman, ;Kolmogorov, 1941; Kraichnan,). Isotropy as a statistical notion is not manifest at the level of the velocity field dynamics but at the level of the correlation functions, where the averaging has removed all the irrelevant direction dependencies. Isotropic turbulence lives strictly in an infinite physical space. Via this energy cascade, turbulent flow can be realized as a superposition of a spectrum of flow velocity fluctuations and eddies upon a mean flow. The eddies are loosely defined as coherent patterns of flow velocity, vorticity and pressure.

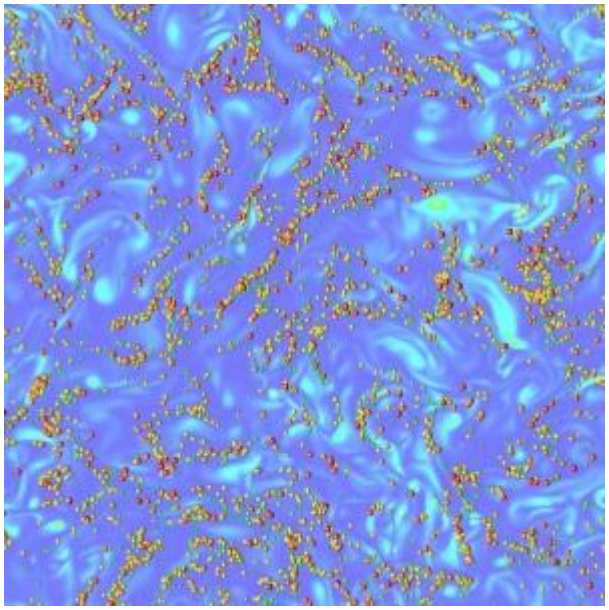

Figure 5. Isotropic turbulence structures resolved by DNS

Turbulent flows may be viewed as made of an entire hierarchy of eddies over a wide range of length scales and the hierarchy can be described by the energy spectrum (Figure 6) that measures the energy in flow velocity fluctuations for each length scale -wavenumber (Akylas, 2015). There is a continuous energy flux from larger vortexes towards the smaller, which dissipates the energy up to the molecular thermodynamic level (Figure 7). This is the energy cascade procedure, which according to Richardson (1922) was presented as: "*Big whirls have little whorls that feed on their velocity, and little whorls have lesser whorls and so on to viscosity*". This is a complex, non-linear procedure, resulting to scales that are generally uncontrollable and highly non-symmetric. Nevertheless, based on the macroscopic characteristics of turbulence (Akylas, 2015), kinetic energy, ***K***, dissipation (rate of

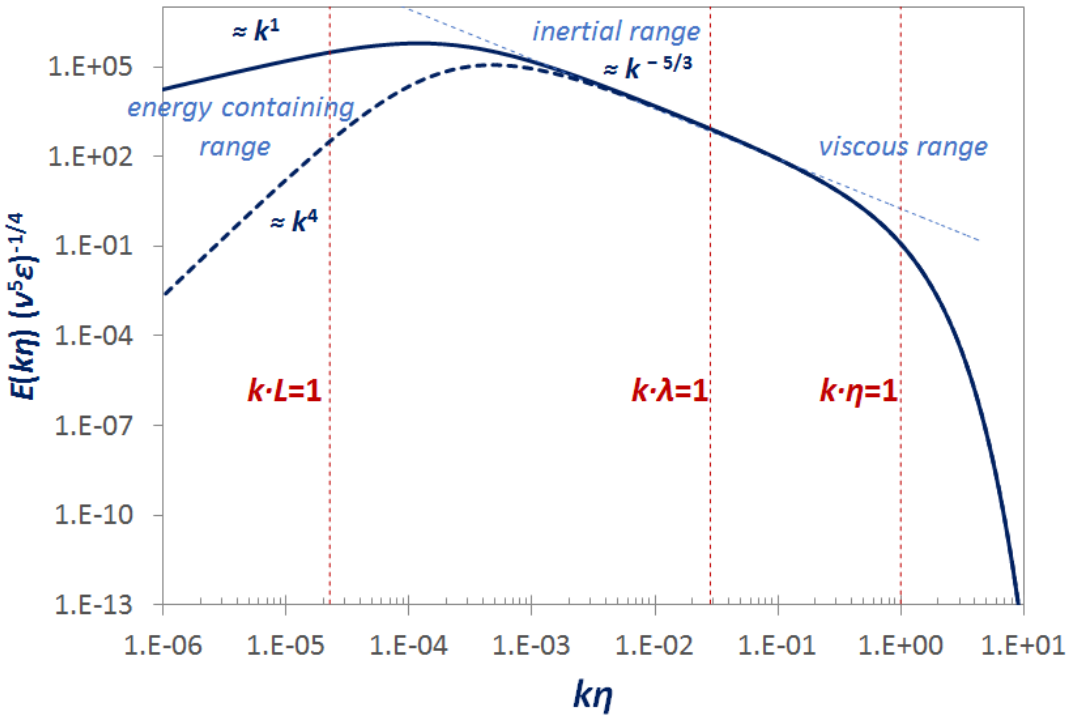

Figure 6. The spectrum of isotropic turbulence with its characteristic scales

energy loss), ε, and viscosity (internal characteristic of the fluid), ν, three well separated scales can be formed

$$L = \frac{K^{3/2}}{\varepsilon}, \qquad \lambda = \frac{(\nu K)^{1/2}}{\varepsilon}, \qquad \eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4} \tag{1.2}$$

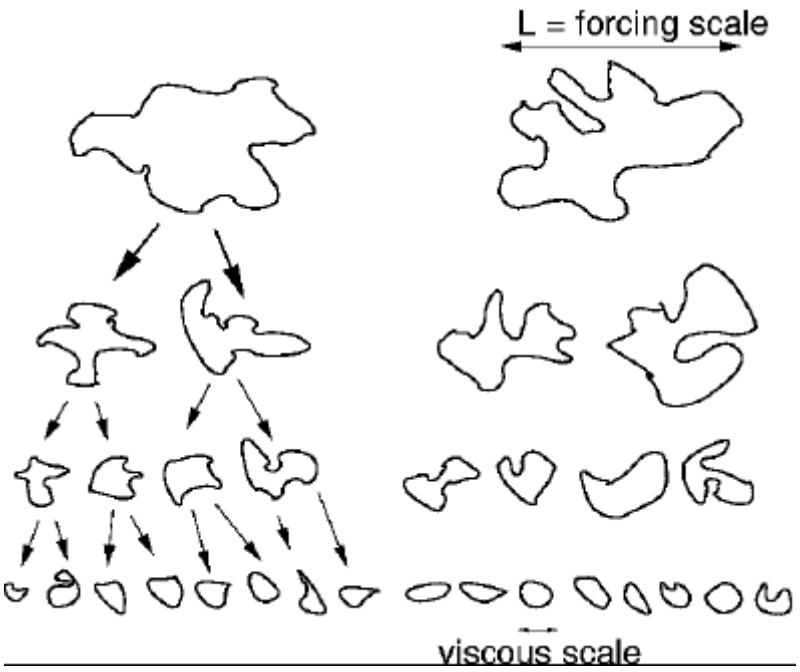

Figure 7. The cascade process schematized by Richardson, 1922.

Integral length scales, L: Largest scales in the energy spectrum. These eddies obtain energy from the mean flow and also from each other. Thus, these are the energy production eddies which contain most of the energy. They have the large flow velocity fluctuation and are low in frequency. Integral scales are highly anisotropic and are defined in terms of the normalized two-point flow velocity correlations. The maximum length of these scales is constrained by the characteristic length of the apparatus. For example, the largest integral length scale of pipe flow is equal to the pipe diameter. In the case of atmospheric turbulence, this length can reach up to the order of several hundred kilometers.

Kolmogorov length scale, η: Smallest scales in the spectrum that form the viscous sub-layer range. In this range, the energy input from nonlinear interactions and the energy drain from viscous dissipation are in exact balance. The small scales have high frequency, causing turbulence to be locally isotropic and homogeneous.

Taylor microscales, λ: The intermediate scales between the largest and the smallest scales which make the inertial subrange. Taylor microscales are not dissipative scale but pass down the energy from the largest to the smallest without dissipation. Some literatures do not consider Taylor microscales as a characteristic length scale and consider the energy cascade to contain only the largest and smallest scales; while the latter accommodate both the inertial subrange and the viscous sublayer. Nevertheless, Taylor microscales are often used in describing the term “turbulence” more conveniently as these Taylor microscales play a dominant role in energy and momentum transfer in the wavenumber space.

1.3 Structure of the work

In the present work we givve a rough description of the present theory about isotropic turbulence. In chapter 2, we present the principal equations Navier-Stokes and discuss the Direct Numerical Simulations in order to solve them numerically. Finally we present and analyze in detail an EDQNM code for isotropic turbulence and we resolve numerically a forced stationary case that resembles similar DNS data.

2. The Navier- Stokes equations

The **Navier-Stokes equations** (Davidson, 2004) govern the motion of fluids and can be seen as Newton's second law of motion for fluids. In the case of an incompressible Newtonian fluid, this yields

$$\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_k \partial x_k}, \tag{2.1}$$

where **u** is the fluid velocity, ***p*** is the fluid pressure, ***ρ*** is the fluid density, and ***ν*** is the fluid viscosity. The Navier-Stokes equations were derived by Navier, Poisson, Saint-Venant, and Stokes between 1827 and 1845. These equations are always solved together with the continuity equation:

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{2.2}$$

The Navier-Stokes equations represent the conservation of momentum, while the continuity equation represents the conservation of mass. Due to the complexity of the system the physically relevant information about the flow is of statistical nature (Gravanis and Akylas, 216). Denoting a suitable statistical ensemble average by angle brackets, the tensor field $\left\langle u_{i_1}(\vec{x}_1)u_{i_2}(\vec{x}_2)\cdots u_{i_n}(\vec{x}_n)\right\rangle$ is the same-time, order *n* correlation function. Some of the points *x* may coincide, so that there are only *m*<*n* different *x*'s involved and these fields may be designated as *m*-point, order *n* correlation functions. We consider flows with zero mean velocity, $\left\langle u_i(\vec{x})\right\rangle = 0$. The simplest non trivial single-point correlation functions of the velocity field are the total kinetic energy $K = \frac{1}{2}\left\langle u_i u_i\right\rangle$ and the dissipation $\varepsilon = \nu\left\langle \partial_k u_i \partial_k u_i\right\rangle$. Consider ideal homogeneous turbulent flows. Then the single-point correlation functions do not depend on the position in space. Therefore correlation functions which are total derivatives vanish identically. This allows us to show that *K* and *ε* are related by the exact energy balance equation

$$\frac{dK}{dt} = -\varepsilon \tag{2.3}$$

Although it is possible to find some particular solutions of the Navier-Stokes equations governing fluid motion, all such solutions are unstable to finite perturbations at large Reynolds numbers (Falcovich, 2006). Sensitive dependence on the initial and boundary conditions makes fluid flow irregular both in time and in space so that a statistical description is needed. The Russian mathematician Andrey Kolmogorov (Frisch, 1995) proposed the first statistical theory of turbulence, based on the aforementioned notion of the energy cascade (an idea originally introduced by Richardson) and the concept of self-similarity. As a result, the Kolmogorov microscales were named after him. It is now known that the self-similarity is broken so the statistical description is presently modified. Still, a complete description of turbulence remains one of the unsolved problems in physics.

2.1 Direct Numerical Simulations

A **direct numerical simulation (DNS)** is a simulation in computational fluid dynamics in which the Navier–Stokes equations are numerically solved without any turbulence model (Orszaq, 1970). This means that the whole range of spatial and temporal scales of the turbulence must be resolved. However, the isotropic flow, requiring first of all an infinite physical space to live in, is an idealization that cannot be strictly realized even in the clinical environment of numerical simulations. In the direct numerical simulations (DNS) of isotropic turbulence one usually solves the Navier-Stokes equation imposing periodicity. The idea is to introduce finiteness in space in a smooth manner. Isotropic flows are replaced by another kind of ideal flows that can be handled numerically (Gravanis and Akylas, 2016). Explicitly, the DNS turbulent flows are governed by the velocity field equations (2.1-2) satisfying the conditions

$$u_i(x,y,z) = u_i(x+l,y,z) = u_i(x,y+l,z) = u_i(x,y,z+l) \tag{2.4}$$

for the axes x, y, z where we explicitly write down (2.1) in components.

Condition (2.4) introduces a tiling, or tessellation, of the infinite physical space (Gravanis and Akylas, 2016). The tiling is made up of an infinite number of cubic domains of side l. Each such cubic domain is a unit cell of the tessellation repeated

infinitely many times. In practice, one usually thinks in terms of a single such unit cell speaking of 'periodic boundary conditions'. This is slightly misleading. Each unit cell of the tessellation is a notional subset of the infinite space that does not bound the flow in its interior. The condition (2.4) does not change if we pick another origin for the axis *x*, *y* and *z* as long as we do not rotated those axes. The condition (2.4) does not distinguish between tessellations whose unit cells are parallel. The boundary of the unit cell of a given tessellation contains points that lie in the interior of a unit cell of another equally good tessellation.

We may adopt a more abstract view. The condition (2.4) says that the field configuration looks the same at all points in space whose coordinates differ by integer multiples of *l*. This is why we can restrict ourselves into a unit cell. We lose no bit of information about the field if we identify all points in space whose coordinates differ by integer multiples of *l*. That means that all unit cells are mapped on a single one whose boundary points have been identified. The result is a compact space without boundary. The condition (8) is reduced merely to the single-valued-ness of the field on the compact space. Clearly this treatment can be applied to any cubically tessellated space. Compactification is the term we shall use for this procedure, implying both the operation on the space itself as well as on the field theory that lives in it. If a field theory lives in an infinite line then the unit cell is an interval whose end-points are identified. The result is a field theory that lives in a circle. If a theory lives on a plane then the unit cell is a square whose opposite sides are identified resulting in a theory that lives in a compact space which topologically is a torus. DNS flows are described by the compactification of the Navier-Stokes (2.1) in a space which topologically is a three-torus (Figure 8).

The more abstract view just described lies in the difference between the local and the global properties of a space. The tori are intrinsically flat spaces. Locally they are not different than the original infinite spaces. Globally, they are clearly non-trivial. The field theories of interest are written in terms of local equations, such as the Navier-Stokes equation. Therefore compactification does not affect the form of the equations but rather determines the boundary conditions and other global specifications.

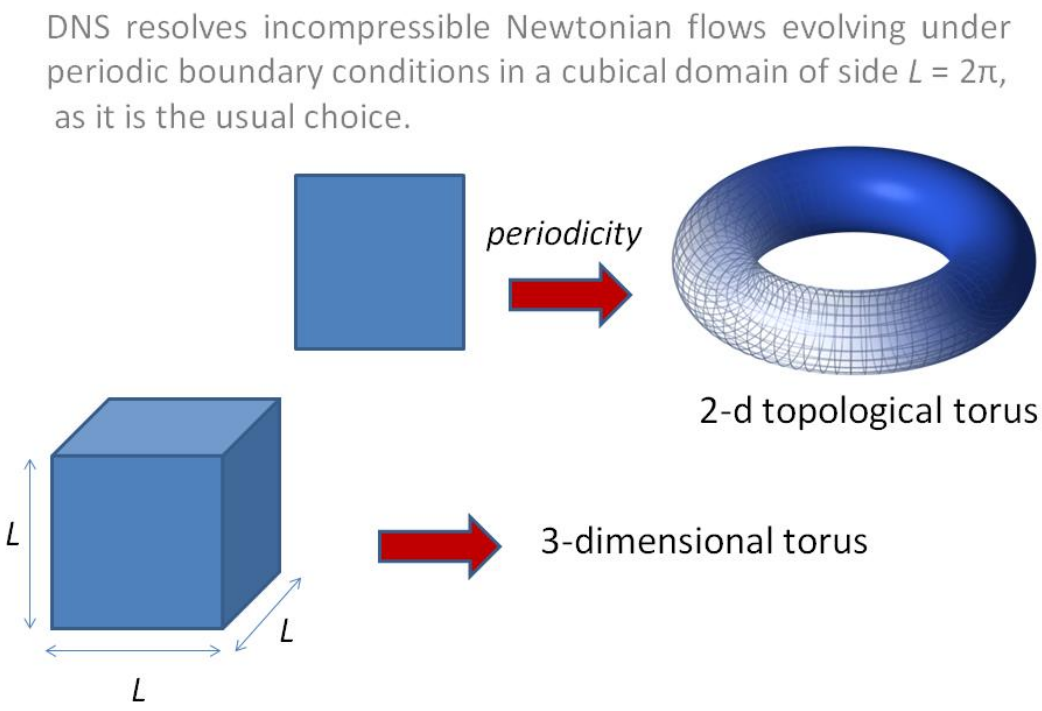

Figure 8. The compactification of infinite space by the DNS (Akylas, 2015).

As Gravanis and Akylas (2016) state, DNS turbulence can be regarded as statistically homogeneous turbulence. From the point of view of compactification it is easy to see that there is no a priori difficulty with homogeneity: The compact space is perfectly homogeneous and the flow encounters no special points. Regarding isotropy things are different. The compact space does have special directions; to assist imagination one may use the two-torus as a model of that space. The space is perfectly isotropic locally, allowing the flow to evolve and adjust itself accordingly. Globally it is not, affecting the overall state of isotropy in the flow as scales are coupled to each other through cascade. One may note that the solely global breaking of isotropy is no surprise since compactification is a strictly global operation on a perfectly homogeneous and isotropic space. In all, DNS turbulent flows are certainly not ideally isotropic but due to the compromise between the local and global properties of the compact space these flows acquire a considerable degree of isotropy.

Now the DNS flows are supposed to simulate ideal isotropic flows in the first place, and we want to use the formulas valid in the latter with all their irreplaceable simplicity, and have at our disposal the suitable framework for studying fundamental conjectures such as Kolmogorov's theory or deviations from it. In practice, one proceeds by eliminating the direction-dependence of the numerical results by averaging

over the directions; specifically this is done in the Fourier space in quantities such as the energy spectrum. The resulting scalar quantities are then regarded as adequate approximations of the analogous quantities in ideal isotropic flows for scales adequately smaller than l (Figure 9). Indeed, in practice, standard measures of isotropy show small, fluctuating deviations around their ideal values.

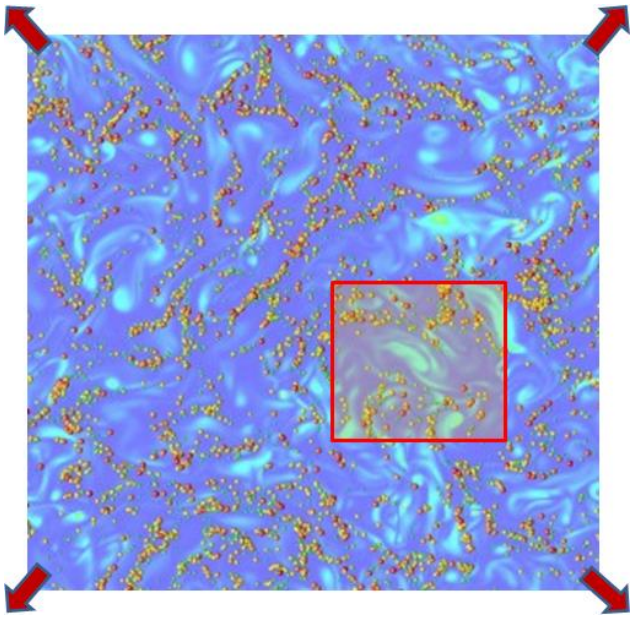

Figure 9. Local isotropy in DNS.

All the spatial scales of the turbulence must be resolved in the computational mesh, from the smallest dissipative scales (Kolmogorov microscales), up to the integral scale L, associated with the motions containing most of the kinetic energy. On the other hand, the integral scale depends usually on the spatial scale of the boundary conditions. To satisfy these resolution requirements, the number N of points along a given mesh direction with increments h, must be $Nh > L$, so that the integral scale is contained within the computational domain, and also $h \leq \eta$, so that the Kolmogorov scale can be resolved. Since the dissipation, $\varepsilon \approx u'^3/L$, where u' is the root mean square (RMS) of the velocity, the previous relations imply that a three-dimensional DNS requires a number of mesh points N^3 satisfying $N^3 \geq Re^{9/4}$, where Re is the turbulent Reynolds number, $Re = u'L/\nu$. Hence, the memory storage requirement in a DNS grows very fast with the Reynolds number. In addition, given the very large memory necessary, the integration of the solution in time must be done by an explicit method. This means that in order to be accurate, the integration, for most discretization methods, must be done with a time step, Δt, small enough such that a

fluid particle moves only a fraction of the mesh spacing *h* in each step. One can estimate that the number of floating-point operations required to complete the simulation is proportional to the number of mesh points and the number of time steps, and in conclusion, the number of operations grows as Re^3. Therefore, the computational cost of DNS is very high, even at low Reynolds numbers. For the Reynolds numbers encountered in most industrial applications, the computational resources required by a DNS would exceed the capacity of the most powerful computers currently available. However, direct numerical simulation is a useful tool in fundamental research in turbulence. Using DNS it is possible to perform "numerical experiments", and extract from them information difficult or impossible to obtain in the laboratory, allowing a better understanding of the physics of turbulence. Also, direct numerical simulations are useful in the development of turbulence models for practical applications. This is done by means of "a priori" tests, in which the input data for the model is taken from a DNS simulation, or by "a posteriori" tests, in which the results produced by the model are compared with those obtained by DNS.

2.2 K41 Theory

A successful phenomenological theory for turbulence is the K41 Theory. Kolmogorov studied the energy spectrum of the homogeneous isotropic turbulence in 1941, he found that dissipation only occurs in small scale, and there is an inertial range in k space for turbulence (see Fig. 10), in which range the energy is neither injected nor dissipated, energy is only transferred from large scale to small scale (Stroh, 2013). He obtained the famous −5/3 exponent of the turbulence energy spectrum in its inertial range: $E(k) = C\,\varepsilon^{2/3}\,k^{-5/3}$, where E(k) is the energy spectrum, ε is a constant energy dissipation rate, k is the wave number, and C is a universal constant which value can not be derived from K41 theory itself. K41 Theory is a big success, because it fits the experiments very well and the results are universal and robust. However there are two major concerns: 1. it doesn't consider a phenomenon called intermittency at all, since this is not related to the topic, it won't be discussed here; 2. K41 theory is too simple, it is a phenomenological theory about energy spectrum of turbulence, mainly derived by dimensional analysis, and is only able to give us limited information, it does not give us insight into the dynamics or transport properties of turbulences. Besides, some people believe we should be able to develop a theory to calculate the universal

constant *C* in K41 Theory from first principle without any free parameter. So more theoretical studies about the dynamics and transport of turbulences are done beyond the K41 Theory. The two most successful methods are Renormalization Group (RNG) Theory and Closure Theory. In this work we will focus on the latter.

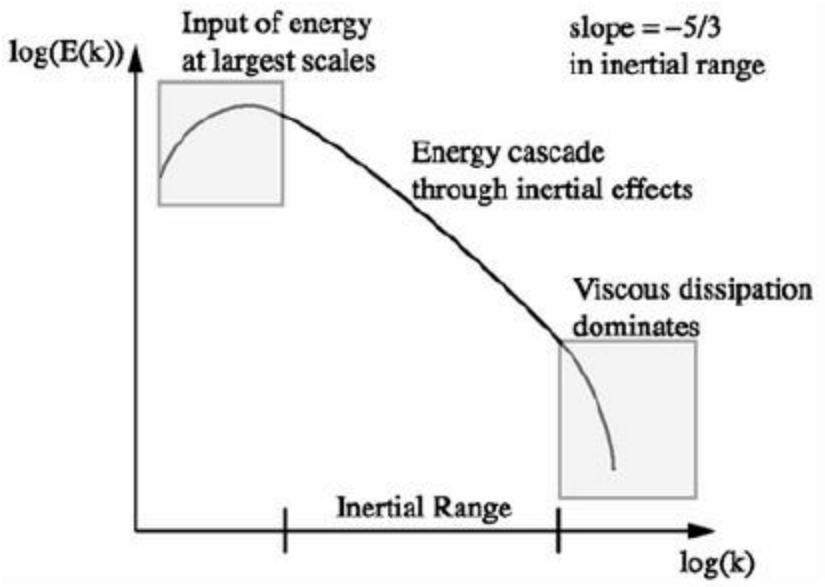

Figure 10. K41 Theory and inertial range (Stroh, 2013).

We will first do a brief introduction to Closure Theory and finally presentreview the currently most accepted Damped Quasi-Normal Markovian (EDQNM) model.

2.3 Closure problem and EDQNM

Consider ideal isotropic turbulent flows. There is no intrinsic direction in the flow, locally or globally. These flows are automatically homogeneous. Ideally homogeneous and therefore isotropic flows exist necessarily in an infinite space. The correlation function tensor fields can be reduced purely geometrically to a set of scalar fields that depend only on rotation-invariant quantities, distances and angles. In the place of scalar fields one usually uses pseudo-scalar fields with specific parity properties. For simplicity we may refer also to them as 'scalar fields'.

Any *m*-point correlation function $\left\langle u_{i_1}(\vec{x}_0)u_{i_2}(\vec{x}_1)\cdots\right\rangle$ depends on the separations $\vec{r}_a = \vec{x}_a - \vec{x}_0$, where a = 1,..,*m*–1. This function can be reduced to the associated longitudinal correlation scalar fields

$$(\vec{e}_a)_{i_1}(\vec{e}_b)_{i_2}\cdots\left\langle u_{i_1}(\vec{x}_0)u_{i_2}(\vec{x}_1)\cdots\right\rangle \tag{2.5}$$

where $\vec{e}_a$ is a unit vector parallel to $\vec{r}_a$. Transverse fields are obtained by replacing some of the *e*'s with vectors perpendicular to them. The scalar fields depend only on the lengths of the $\vec{r}_a$ and the angles between them. It is not hard to see that the natural domain of an *m*-point correlation function is an $(m-1)$-dimensional hyperplane.

The correlation functions obey a set of infinite, strongly coupled equations of motion. The equations of motion of the *m*-point, order *n* function involve the $(m+1)$-point and order $(n+1)$ function restricted on the $(m-1)$-dimensional domain of the order *n* function. This applies for all $n\geq 2$. This is the infinite hierarchy of correlation function equations. The coupling follows from the quadratic term of the Navier-Stokes equation. In order to get some knowledge of the system, we can't solve the infinite set of equations, just as we are not able to solve the original Navier-Stokes Equation. This gives rise to the 'closure problem'. Closing off the infinite sequence of equations at some finite order has to be done semi-empirically. In other words, we must truncate this set of equations by a model, each reasonable model is called a closure model.

The Eddy Damped Quasi-Normal Markovian (EDQNM) is a subfilter closure model applied in spectral wavenumber space rather than physical space which considers interactions between resolved and subfilter wavenumbers by considering the statistics of their possible interactions. The EDQNM achieves closure by modeling the 4th spectral moments. EDQNM (Orszaq, 1970) is based on theoretical work of Kraichnan who developed his turbulence theories over many decades and was one of the prominent American theorists in this area. Following earlier work of Andrei Kolmogorov (1941), Lars Onsager (1945), Werner Heisenberg (1948), Carl Friedrich von Weizsäcker and others on the statistical theory of turbulence, Kraichnan developed a field-theoretic approach to fluid flow in 1957 -1965 derived from approaches to the quantum many-body problem- the *Direct Interaction Approximation*. An EDQNM code for solving isotropic turbulence will be presented and tested throughout the next chapter.

3. A free EDQNM code for isotropic turbulence

In the following paragraphs we present in detail our EDQNM code for resolving forced isotropic turbulence. The work may be considered as continuation of previous work done by Michalis Pieris (2016) who wrote an EDQNM code for equally distant wave-numbers, however permitting the possibility of resolving large Reynolds numbers. Most of the ideas and the coding that will be presented is based in my personal communication with Prof. E. Akylas (2016) at CUT.

3.1 Why free?

During the last years, we faced the need to produce EDQNM results in order to test and validate some stages of our research in isotropic turbulence theory (Akylas, 2015). Interestingly, although there is wide literature related to the EDQNM theory and formulation, concerning a real EDQNM application and its related coding it seems that everything is rather left to the researcher's imagination. Although quite easy in general the application of an EDQNM closure is not so straight forward, when one starts from scrap. Thus, we decided to write a simple code, explaining in detail its basic "tricky" features, in an instructive way and give it away to anyone interested into the subject. By any means we would had been very happy if some years ago we were able to read an article like this one (Akylas, 2015).

3.2 The discretized space of our EDQNM code

The general EDQNM closure formulation which is originally written for the continuous spectral space can be found in a very elementary form in Lessieur and Ossia, (2000), in their equations 1-5. In order to proceed with the numerical application of the EDQNM closure the continuous spectral space should be discretized appropriately. The basic idea in any EDQNM application (which is one of their main advantages) is to stretch successively the spectral bandwidths in order to cover the "infinite" spectral space with a limited and thus numerically soluble number of wave number points. Such a technique allows for resolving very high wave numbers in order to achieve drastically large Reynolds numbers. However, the application of the EDQNM formulation in such a "stretched" geometry is not so

straight forward and special care needs to be given to deal with problems that may arise.

We present here, in detail the techniques we used in order to construct a very basic EDQNM code. In our application we have chosen to discretize the wave numbers on a geometric sequence of the form

$$k_i = k_1 \lambda^{i-1}, \tag{3.1}$$

splitting the continuous spectral space (the existence of a minimum wave number larger than zero will be discussed later) in wave number bands with widths

$$c = \frac{dk_i}{di}\Delta i = k_i\, d\ln(k_i) = k_i \ln\lambda = k_1 \lambda^{i-1}\ln\lambda\,. \tag{3.2}$$

By doing so, the integrals of the spectral moments in the continuous space can be approximated by EDQNM sums as

$$n-moment = \sum_{i=1}^{i\max} k_i^n E(k_i)\,\Delta k_i\,. \tag{3.3}$$

For instance the kinetic energy, the dissipation and the palinstrophy of the EDQNM spectra are calculated as sums over the index i, $KE = \sum E(k_i)\,\Delta k_i$, $\varepsilon = 2\nu\sum k_i^2 E(k_i)\Delta k_i$, xu and so on.

Table 1. The perfect approximation of the spectral moments by equation (3.3).

moment	**0**	**2**	**4**	**6**
analytical	1	1920	6881280	39636172800
EDQNM sum (3)	1	1920	6881280	39636172800

One can easily verify (table 1) with a spectrum of the form $E(k) = k^4 e^{-k/8}/786432$, that the approximation of the integrals by the EDQNM series is very accurate.

In order to proceed with the proper sampling during the formation of the wavenumber triads we must determine the limits of each spectral band. The upper and the lower limits of the ith band will be denoted as $k_i + x_i^{up}$ and $k_i - x_i^{low}$ respectively, with

$$x_i^{up} + x_i^{low} = \Delta k_i \,, \tag{3.4}$$

of course. The intermediate limits of the bands of two successive wave numbers should coincide; that is

$$k_i + x_i^{up} = k_{i+1} - x_{i+1}^{low} \,. \tag{3.5}$$

As a result of equations (3.4-5) we may conclude as a general rule, that $x_{i+1}^{low} = x_i^{low} + k_{i+1} - k_i - \Delta k_i$. This can also be written through equations (3.1-2) as $x_{i+1}^{low} = x_i^{low} + k_{1+1}\left(1 - 1/\lambda - \ln\lambda/\lambda\right) = x_i^{low} + k_1\lambda^i\left(1 - 1/\lambda - \ln\lambda/\lambda\right)$, and we finally deduce, for the lower intercepts, the equivalent more elegant formula

$$x_{i\geq 2}^{low} = x_1^{low} + k_1\left(1 - 1/\lambda - \ln\lambda/\lambda\right)\frac{\lambda^i - \lambda}{\lambda - 1} \,, \tag{3.6}$$

with x_1^{low} a free parameter. At the same time, the upper intercepts are given by (3.4). Obviously, we are now ready to proceed with the final calculation of the upper and the lower limits of each wave number band, $k_i^{up} = k_i + x_i^{up}$ and $k_i^{low} = k_i - x_i^{low}$ respectively (after some algebra) as

$$k_i^{low} = \frac{k_1\lambda^{i-1}\ln\lambda}{\lambda - 1} + \frac{k_1}{\lambda - 1}\left(\lambda - 1 - \ln\lambda\right) - x_1^{low}, \tag{3.7a}$$

$$k_i^{up} = \frac{k_1\lambda^{i}\ln\lambda}{\lambda - 1} + \frac{k_1}{\lambda - 1}\left(\lambda - 1 - \ln\lambda\right) - x_1^{low}. \tag{3.7b}$$

Without any serious loss of the generality we may set for convenience (this will help markedly the next section's calculations) $x_1^{low} = k_1(\lambda - 1 - \ln\lambda)/(\lambda - 1)$, and then the wave number band limits simplify to

$$k_i^{low} = \frac{k_1 \lambda^{i-1} \ln\lambda}{\lambda - 1} = \frac{\Delta k_i}{\lambda - 1}, \tag{3.8a}$$

$$k_i^{up} = \frac{k_1 \lambda^{i} \ln\lambda}{\lambda - 1} = \frac{\Delta k_{i+1}}{\lambda - 1}. \tag{3.8b}$$

In this context, it is quite simple to distribute the energy from the formation of the possible triads to each specific band as it will be shown in the next section.

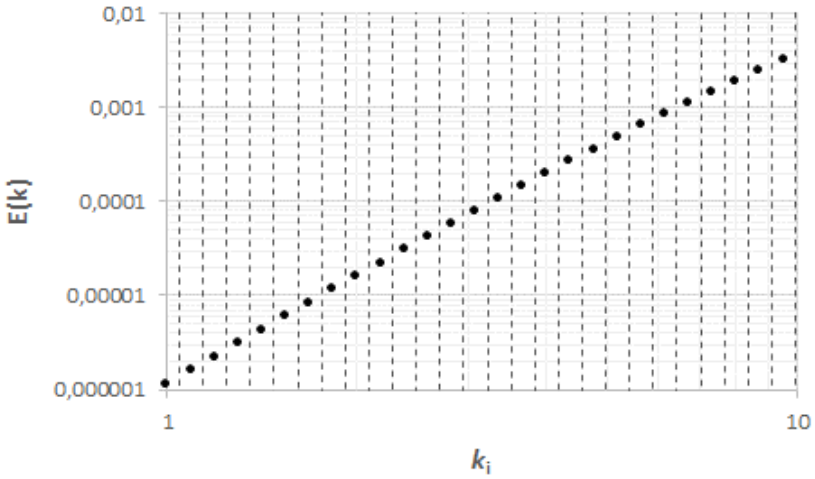

Figure 11. The discretized wave numbers and their bands.

In figure 11, we present an example of the discretized wave numbers by (3.1) and their bands of influence (3.8a,b) for a part of the energy spectrum $E(k) = k^4 e^{-k/8} / 786432$, and for the choices $k_1 = 2^{-9}$ and $\lambda = 2^{1/8}$. It is evident that the wave numbers by (3.1) are equally distant when plotted in a log-scale and they lay almost (but not exactly) in the geometric center of their bands. Actually, one can easily calculate that they lay somewhere between the geometric and the arithmetic center of their bands.

3.3 Formation of the triad interactions and allocation to the wave number zones

After the construction of the discrete domain we have a suitable framework in order to proceed with the core mechanism of the EDQNM formulation, the formation of the triad interactions among wave numbers k_x, k_y and k_z that form a triangle (*x*, *y*, *z* are integers). In our discrete realization the continuous formulation found in Lesieur and Ossia (2000), for the spectral evolution takes the following form

$$\left(\frac{\partial}{\partial t}+2\nu k_x^2\right)E(k_x,t)=\sum_{y=1}^{i\max}A\left(k_x,k_y,t\right)\Delta k_y, \tag{3.9}$$

where Δk_y is given by (3.2), *imax* is the highest available index of the discrete wave numbers in our application. The $A\left(k_x,k_y,t\right)$ term corresponds to the appropriate integration of the following expression

$$\theta_{xyz}(t)b_{xyz}E(k_z,t)\frac{k_x^3E(k_y,t)-k_xk_y^2E(k_x,t)}{k_yk_z}\ , \tag{3.10}$$

in terms of k_z over an interval from $k_z^{\min}$ to $k_z^{\max}$, with

$$k_z^{\min}=\left|k_x-k_y\right|, \tag{3.11a}$$

$$k_z^{\max}=Maximum\left(k_x+k_y,k_{i\max}\right), \tag{3.11b}$$

in order of k_x, k_y and k_z to be able to form triangles (that is the triadic concept). The appearing in (3.10) b_{xyz} and θ_{xyz} terms are given respectively by

$$b_{xyz}=\frac{k_y}{k_x}\left(\frac{k_y^2+k_z^2-k_x^2}{2k_yk_z}\right)\left(\frac{k_x^2+k_z^2-k_y^2}{2k_xk_z}\right)+\frac{k_y}{k_x}\left(\frac{k_x^2+k_y^2-k_z^2}{2k_xk_y}\right)^3, \tag{3.12}$$

and

$$\theta_{xyz} = \frac{1 - e^{-\left[\mu_x + \mu_y + \mu_z + \nu\left(k_x^2 + k_y^2 + k_z^2\right)\right]t}}{\mu_x + \mu_y + \mu_z + \nu\left(k_x^2 + k_y^2 + k_z^2\right)}, \tag{3.13}$$

where

$$\mu_x = a_1 \left(\sum_{i=1}^{x} k_i^2 E(k_i) \Delta k_i \right)^{1/2}. \tag{3.14}$$

The first problem in the integration of (3.10) is the correct allocation of each triad to its respective wave number band. More specifically, the continuous integration of (3.10) for any specific pair of the wave numbers (k_x, k_y), allows for k_z to take all the values in the interval from $|k_x - k_y|$ to $k_x + k_y$, creating an infinite number of successive triangles. In the discrete case there is only a limited number of triangles that can be formed, depending on the available discrete values of k_z that exist in the previous interval, as shown schematically in figure 12.

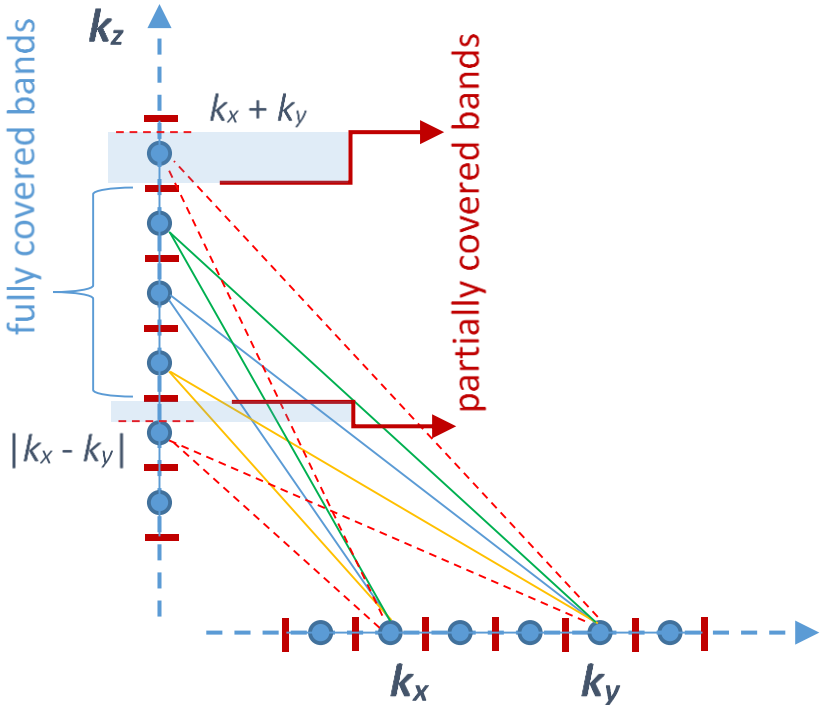

Figure 12. Schematic presentation of possible combinations of a pair of k_x and k_x with k_z wave numbers (●) to form triadic interactions. The triads in the interior of the k_x range (continuous lines) correspond to fully covered wave number bands (| |). Towards the upper and lower limits, however, the formed triads (dashed lines) cover only partially their bands.

In most of the cases, each triad formed, represents in terms of k_z, its whole representative band. This means that the integration, in the continuous case, of (3.10) for a specific (k_x, k_y) choice over a full band of k_z (3.8a,b) $\int_{k_z^{low}}^{k_z^{up}} \theta_{xyz}(t) b_{xyz} E(k_z,t) \frac{k_x^3 E(k_y,t) - k_x k_y^2 E(k_x,t)}{k_y k_z} dk_z$, can be approximated by the product of (10) for the specific triad (k_x, k_y, k_z) times the band width Δk_z, $\theta_{xyz}(t) b_{xyz} E(k_z,t) \frac{k_x^3 E(k_y,t) - k_x k_y^2 E(k_x,t)}{k_y k_z} \Delta k_z$. In some limiting cases however, that width should be modified due to the conditions 3.11.

3.4 The forcing term

The two most frequently studied types of isotropic turbulence are freely decaying, and forced statistically stationary turbulence. For studies in which one wishes stationarity for statistical sampling, forced turbulence is preferable over decaying turbulence. In this study we adopt a band-limited forcing scheme, by injecting the dissipative energy back to the system. Specifically, we add a forcing term *F*(*k*) to the kinetic-energy transfer in equation (3.9), in order to obtain a stationary energy spectrum. We share and distribute linearly the energy loss to the wave numbers in the interval between 1< k <2.5, in a way that resembles the DNS forcing that was adopted by Isihara et al. (2009).

3.5 The code

In the following lines we present the routine that was created using Visual Basic, as a macro in Microsoft Excel in order to perform forced EDQNM at high Reynolds number. The characteristics of each run are controlled externally through the Microsoft Excel sheet.

```
Sub EDQNM()
'
' EDQNM
' Macro recorded 10/8/2016 by Cyprus University of Technology
'
' Keyboard Shortcut: Ctrl+a
```

```
    Range("B1").Select
    maxtimesteps = Selection
    Range("B2").Select
    Dt = Selection
    Range("B3").Select
    kmax = Selection
    Range("B5").Select
    viscosity = Selection
    Range("B4").Select
    alpha1 = Selection
    Range("E1").Select
    alpha2 = Selection
    Range("B6").Select
    A = Selection
    Range("B7").Select
    every = Selection
    Range("B8").Select
    k1 = Selection
    Range("B9").Select
    F = Selection
    λ = 2 ^ (1 / F)
    ReDim DWN(256) As Double
    ReDim WN(256) As Double
    ReDim E(256) As Double
    ReDim KE(256)
    ReDim dis(256) As Double
    ReDim N(256) As Double
    ReDim Tk(256) As Double

'..... Initialization .....
    For m = 1 To kmax
    WN(m) = k1 * λ ^ (m - 1)
    DWN(m) = WN(m) * Log(λ)
    Worksheets("Sheet1").Cells(m + 11, 4).Select
    E(m) = Selection
    N(m) = viscosity * WN(m) ^ 2#
    Next
'.....Main Code.....
    For timestep1 = 1 To maxtimesteps / every
    Range("AB7").Select
    Selection = timestep1 - 1
    For timestep2 = 1 To every
    T = ((timestep1 - 1) * every + timestep2) * Dt + 1000000
    diss = E(1) * WN(1) ^ 2 * DWN(1)
    For m = 2 To kmax
    KE(1) = E(1) * DWN(1)
    diss = diss + E(m) * WN(m) ^ 2 * DWN(m)
    dis(1) = E(1) * WN(1) ^ 2 * DWN(1)
    KE(m) = KE(m - 1) + E(m) * DWN(m)
    dis(m) = dis(m - 1) + E(m) * WN(m) ^ 2 * DWN(m)
```

```
    Next
    eta = (viscosity ^ 2 / (diss * 2)) ^ (1 / 4)
    Le = (2 * KE(kmax)) ^ (3 / 2) / (2 * viscosity * diss)

    For k = 1 To kmax
    Tk(k) = 0
    For p = 1 To kmax
    qmax1 = Int(Log(ë ^ k + ë ^ p) / Log(ë))

    If (qmax1 <= kmax) Then qmax = qmax1 Else qmax = kmax
    qmin1 = Int(Log(Abs(ë ^ k - ë ^ p + 1E-20)) / Log(ë))

    If (qmin1 <= 0) Then qmin = 1 Else If (Log(Abs(ë ^ k - ë ^ p)) / Log(ë)) =
Int(Log(Abs(ë ^ k - ë ^ p)) / Log(ë)) Then qmin = Int(Log(Abs(ë ^ k - ë ^ p)) /
Log(ë)) Else qmin = Int(Log(Abs(ë ^ k - ë ^ p)) / Log(ë)) + 1
    If (qmax1 > kmax) Then DWNqmax = DWN(kmax) Else DWNqmax = WN(k) +
WN(p) - ((k1 * Log(ë) * ë ^ (qmax - 1)) / (ë - 1))
    If (qmin1 <= 0) Then DWNqmin = DWN(1) Else DWNqmin = ((k1 * Log(ë) * ë ^
(qmin)) / (ë - 1)) - Abs(WN(k) - WN(p))

    For q = qmin To qmax
    If (qmin = qmax) Then DWNq = WN(k) + WN(p) - Abs(WN(p) - WN(k)) Else If
(q = qmin) Then DWNq = DWNqmin Else If (q = qmax) Then DWNq = DWNqmax
Else DWNq = DWN(q)
    Theta = 1 / (alpha1 * ((2 * WN(k) * eta) ^ (-0 * WN(k) * eta) * dis(k) ^ (1 / 2) *
Exp(-((WN(k) * eta / 1.1) ^ 2)) + (2 * WN(p) * eta) ^ (-0 * WN(p) * eta) * dis(p) ^ (1
/ 2) * Exp(-((WN(p) * eta / 1.1) ^ 2)) + (2 * WN(q) * eta) ^ (-0 * WN(q) * eta) *
dis(q) ^ (1 / 2) * Exp(-((WN(q) * eta / 1.1) ^ 2))) + 1# * N(k) * Exp(-((WN(k) * eta /
0.65) ^ 2)) + 1# * N(p) * Exp(-((WN(p) * eta / 0.65) ^ 2)) + 1# * N(q) * Exp(-
((WN(q) * eta / 0.65) ^ 2)))

    bkpq = (WN(p) / WN(k)) * (((WN(p) ^ 2 + WN(q) ^ 2 - WN(k) ^ 2) / (2 * WN(p) *
WN(q))) * ((WN(k) ^ 2 + WN(q) ^ 2 - WN(p) ^ 2) / (2 * WN(k) * WN(q))) +
((WN(k) ^ 2 + WN(p) ^ 2 - WN(q) ^ 2) / (2 * WN(p) * WN(k))) ^ 3)
    Tk(k) = Tk(k) + (Theta * WN(k) / (WN(p) * WN(q))) * ((bkpq) * E(q) * (WN(k) ^
2 * E(p) - WN(p) ^ 2 * E(k))) * DWNq * DWN(p)
      Next

   Next
   Next


    S = 0#
    For k = 1 To kmax
    S = S + Tk(k) * WN(k) ^ 2 * DWN(k)
    Next
     S = S * ((1 / (2# * diss)) ^ (3 / 2)) * ((6# * 15# ^ (1 / 2)) / 7#)
       Range("C1").Select
    Selection = S
     A = diss * viscosity / (KE(18) - KE(13))
```

```
    For i = 1 To kmax
    If (i < 14) Then Aj = 0 Else If (i > 18) Then Aj = 0 Else Aj = A
    E(i) = E(i) + ((-E(i) * 2 * viscosity * WN(i) ^ 2 + E(i) * 2 * Aj + Tk(i)) * Dt)
    Next
 Next
 For m = 1 To kmax
    Worksheets("Sheet1").Cells(m + 11, 32 + timestep1).Select
   Selection = E(m)
   Worksheets("Sheet1").Cells(m + 11, 4).Select
   Selection = E(m)

   Worksheets("Sheet1").Cells(m + 16 + kmax, 32 + timestep1).Select
   Selection = Tk(m)
     Next
   Worksheets("Sheet1").Cells(kmax + 17 + kmax, 32 + timestep1).Select
   Selection = S
   Range("D11").Select
   Selection = "OK"
Next
End Sub
```

3.6 Forced EDQNM and comparison with respective DNS

In the following, we perform forced EDQNM with the presented code, in order to produce stationary spectra of isotropic turbulence. The specific application resembles the band limited forced DNS data by Isihara et al. (2009). Specifically we initialize the runs with a spectrum with total KE equal to 0.5 and keep this value constant by applying the forcing scheme presented earlier. The viscosity of the case is set to 0.00044, equal to the respective DNS choice. We used a k_{max} = 4096, resulting to a $k_{max}\,\eta$ value around 3 at the stationary state. The parameter λ is set equal to $2^{1/4}$ and the k_{min} equal to 0.125. With these choices, a total of 61 wave number bands were resolved. The a_1 parameter was set to 0.54 resulting to a Kolmogorov constant around 1.6, and the time dependence in 3.13 was omitted, since we deal with stationary cases. We run for sufficient time till a stationary state is achieved and we compare the spectrum with the respective DNS at Reynolds number around 700.

In figure 13 the compensated spectra show a remarkable coincidence with a profound bottleneck at the same position. The peak inside the forcing area is also comparable. However, any anomalous scaling effects do not appear in the EDQNM spectrum as in the case of the DNS spectra. The Reynolds number of the stationary case is 680 almost the same as in the respective DNS case (675) reflecting a good

performance in the macroscopic characteristics and the macro-scales of the produced turbulence. Also the dissipation spectra show the same qualitative behavior up to the peak at $k\,\eta$ around 0.2 (Fig. 14).

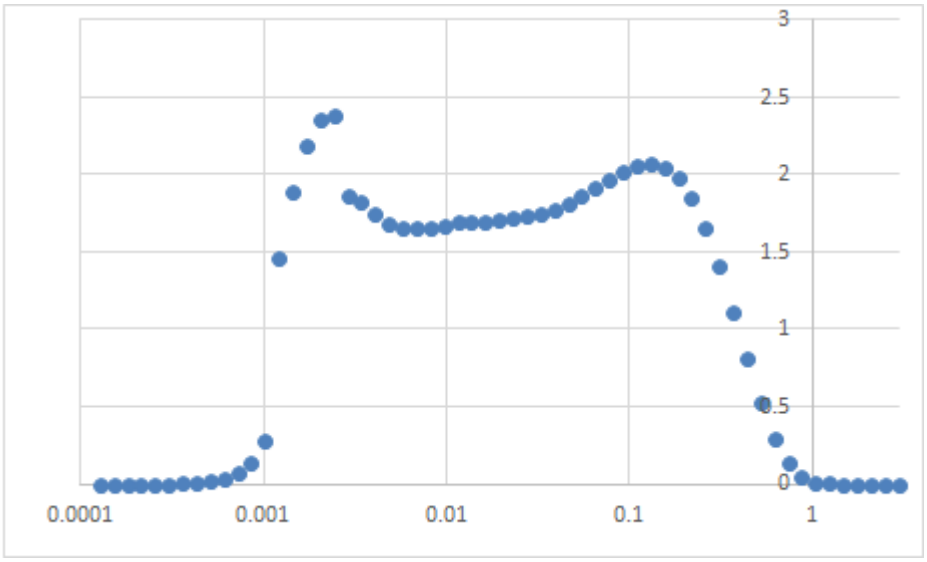

Figure 13. Comparison of the dissipation spectra from forced DNS (Isihara et al. (2009) and our EDQNM study (blue line).

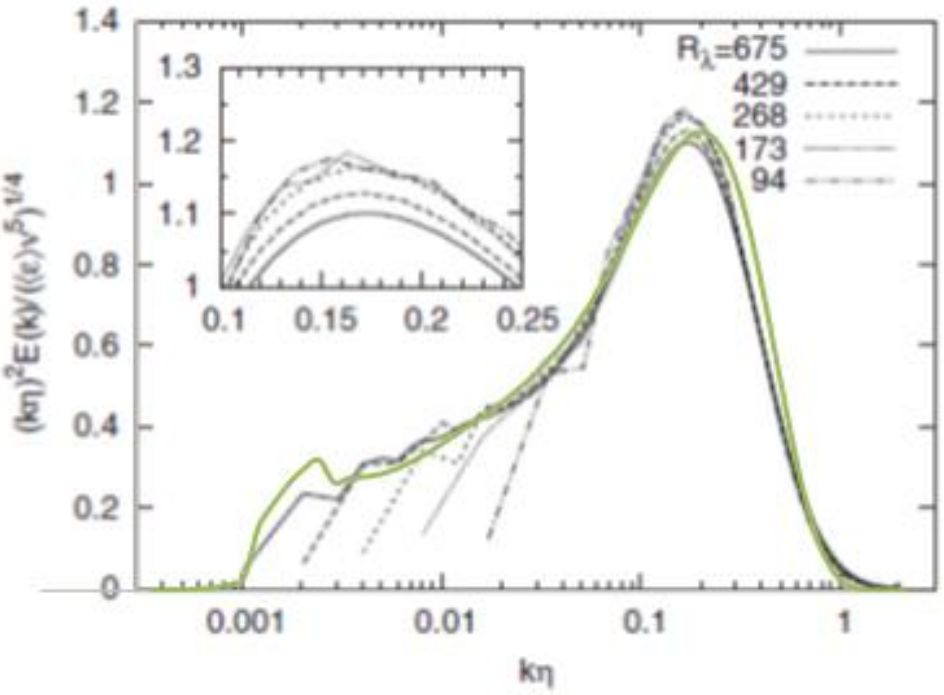

Figure 14. Comparison of the dissipation spectra from forced DNS (Isihara et al. (2009) and our EDQNM study (green line).

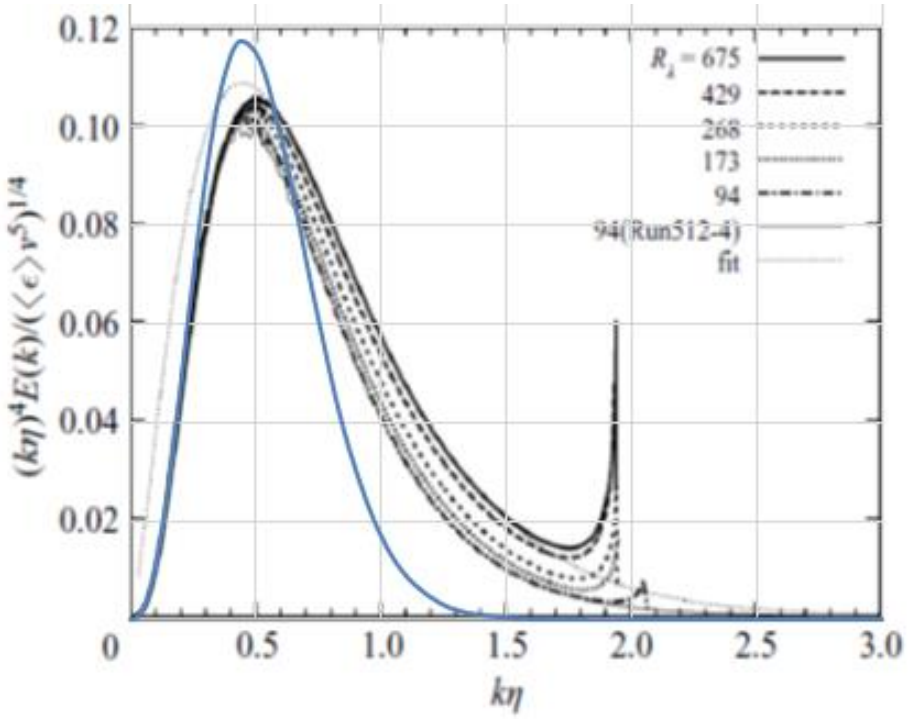

Figure 15. Comparison of the Palinstrophy spectra from forced DNS (Isihara et al. (2009) and our EDQNM study (blue line).

A different behavior follow the tails of the EDQNM spectra that appear steeper at the far dissipation range in comparison to the DNS data. This picture becomes clear by the comparison of the palinstrophy spectra in Figure 15. Although the maximum locates at the same position (arround $k\ \eta$ = 0.4), in the case of the EDQNM it is slightly higher and then the slopes of the spectra become very steep resulting in lower skewness values. In fact the skewness of the EDQNM stationary spectrum is 0.44, while the respective DNS corresponds to 0.68. This feature should be treated with different modelling of the θ term in 3.13 but this is left for future work.

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4. Conclusions

In the present work we gave a rough description of the present theory about isotropic turbulence. We presented the principal equations Navier-Stokes and discussed the Direct Numerical Simulations in order to solve them numerically. Finally we presented and analyzed in detail an EDQNM code for isotropic turbulence and we solved numerically a forced stationary case.

Isotropic turbulence is an ideal state where the motion properties, in the statistical sense, do not show any directional dependence. More strictly, they satisfy rotational and translational invariance. Isotropic turbulence is the simplest form of turbulent flow still maintaining all its fundamental characteristics. It has been the playground for theoretical work through the last century. Isotropy as a statistical notion is not manifest at the level of the velocity field dynamics but at the level of the correlation functions, where the averaging has removed all the irrelevant direction dependencies. Isotropic turbulence lives strictly in an infinite physical space. Via this energy cascade, turbulent flow can be realized as a superposition of a spectrum of flow velocity fluctuations and eddies upon a mean flow. The eddies are loosely defined as coherent patterns of flow velocity, vorticity and pressure. However, the isotropic flow, requiring first of all an infinite physical space to live in, is an idealization that cannot be strictly realized even in the clinical environment of numerical simulations. In the direct numerical simulations (DNS) of isotropic turbulence one usually solves the Navier-Stokes equation imposing periodicity. The idea is to introduce finiteness in space in a smooth manner. Isotropic flows are replaced by another kind of ideal flows that can be handled numerically.

In the infinite space the infinite sequence of equations has to be closed at some finite order, this is being done semi-empirically. In other words, we must truncate this set of equations by a model, each reasonable model is called a closure model. The Eddy Damped Quasi-Normal Markovian (EDQNM) is a sub-filter closure model applied in spectral wavenumber space rather than physical space which considers interactions between resolved and sub-filter wavenumbers by considering the statistics of their possible interactions. The EDQNM achieves closure by modeling the 4th spectral moments.

An EDQNM code for resolving forced isotropic turbulence was created in this work. The work may be considered as continuation of previous work done by

Michalis Pieris (2016) who wrote an EDQNM code for equally distant wave-numbers, however permitting the possibility of resolving large Reynolds numbers. The new code was applied in the case of for forced turbulence at Reynolds number around 1000, resembling high Reynolds DNS from the past. The comparison shows significant similarities in the macroscopic characteristics of the stationary state, however, it reveals a different behavior at the dissipative range with sharper tails and lower palinstrophy values at the EDQNM spectra.

References

1. Akylas E. (2015). Isotropy and Periodicity in Turbulence. Presentation at CUT, Limassol, Cyprus.
2. Akylas E. (2016). Personal Communication.
3. Batchelor, G. K. (1967), "An Introduction to Fluid Dynamics", Cambridge University Press, ISBN 0-521-66396-2
4. P. A. Davidson. Turbulence - An Introduction for Scientists and Engineers. Oxford University Press, 2004.
5. G Falkovich and K.R. Sreenivasan (2006). Lessons from hydrodynamic turbulence, Physics Today, vol. 59, no. 4, pages 43–49.
6. U. Frisch. Turbulence: The Legacy of A. N. Kolmogorov. Cambridge University Press, 1995.
7. Gravanis E. and E. Akylas (2016). Isotropic Turbulence in a compact space, submitted to the Journal of Fluid Mechanics.
8. T. Ishihara, T. Gotoh, Y. Kaneda, Study of High–Reynolds Number Isotropic Turbulence by Direct Numerical Simulation, Annu. Rev. Fluid Mech. **41**, 165 (2009)
9. T. von Karman and L. Howarth, On the statistical theory of isotropic turbulence, Proc. Roy. Soc. Lond. **A 164**, 192-215 (1938)
10. T. von K´arm´an and L. Howarth, On the statistical theory of isotropic turbulence, Proc. R. Soc. Lond. A 164 (1938) 192-215.
11. Kolmogorov, A. N. (1941), *The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers*, Dokl. Akad. Nauk. SSSR 30, 299-303. Reprinted in Proc. Roy. Soc. London A 434, 9-13 (1991).
12. KRAICHNAN, R. H. 1967 Phys. Fluids, 10, 1417.
13. Lumley, J.L., 1997. Some comments on turbulence, *Phys. Fluids A* **4**, 203-211.
14. Lesieur M. &S. Ossia (2000), 3D isotropic turbulence at very high Reynolds numbers: EDQNM study, Journal of Turbulence (1), pp. 1-24
15. Orszag, Steven A. (1970). "Analytical Theories of Turbulence". Journal of Fluid Mechanics. 41 (1970): 363–386.
16. Michael Pieris (2016). Linearly forced turbulence using EDQNM closure. BSc Thesis, CUT.
17. Richardson, L. F. (1922). Weather Prediction by Numerical Process. Cambridge University Press. 2nd Edn. with Foreward by Peter Lynch (2007).
18. J Stroh. Large-Eddy Simulation and Turbulent Energy Cascade: A brief overview of ideas and concepts, 2013.