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Comparison of analytical models for Ground Heat Exchangers

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ABSTRACT

GSHP systems, a form of shallow geothermal energy system, are used for heating and cooling usuing Ground Heat Exchangers (GHEs) that extract/reject heat from/to the ground. In such systems, the thermal response is represented by the temperature change within the GHE and the surrounding earth as a function of heat extraction or injection. There exist several mathematical models, both analytical and numerical, in the literature that govern the thermal response of GHEs. All such models are based on Fourier's law, which allows the determination of heat flow due to conduction. Regarding numerical models, they are obviously more accurate than analytical models and can represent the GHE in greater detail. However, numerical models may be too time-consuming for building energy simulations and consequently of limited use in practical applications. In such case analytical models can play a crucial role. Analytical models can be used as alternatives to one another, depending on the case and the accuracy required to solve a problem. Some of these will be applied to an existing case study of a vertical GHE in the Mediterranean island of Cyprus and be compared to the experimental data and conclusions will be derived regarding the accuracy and suitability of each.

Key Words: G*round Heat Exchanger; GHE analytical models; GHE analytical models comparison;*

1. INTRODUCTION

Ground Source Heat Pumps (GSHPs) are one of the most well-known types of geothermal energy systems that achieve a higher performance than the conventional Air Source Heat Pumps (ASHPs). GSHP systems extract/reject heat from/to the ground through coupling a HP with Ground Heat Exchangers (GHEs), and are used for heating and cooling. GHEs essentially consist of a network of underground tubes with a circulating refrigerant fluid.

Vertical or borehole (typical configuration) GHE, used for the case study here in a U-tube configuration, exchange heat between a circulating fluid and the ground. The ground is therefore acting as an energy source, or for hear rejection. The heat exchange acts primarily as a means of conducting heat to the ground. The thermal response of a vertical GHE is represented by the temperature change within the borehole and the surrounding ground (grout and soil) as a function of heat extraction or injection. There are models that address the heat transfer problem between the fluid (circulating) and the perimeter of the borehole, including the interference between the two parts of the pipe inside the borehole and the heat transfer between the borehole and the ground [1]. This study attempts to compare available analytical models in the literature with experimental data, to determine the performance of the analytical models and whether these models can widely be applied.

2. METHODOLOGY

There are quite a few mathematical models that have been created over the years to quickly and reliably predict GHEs operation. Some of them are analytical and numerical models, all of which are based on Fourier's law, which allows the determination of heat flow due to conduction. Depending on the situation and the accuracy required, different cases require different approaches for analytical models. As for numerical models, they are generally developed to represent the borehole geometry in greater detail than analytical models [2], [3], [4]. The finite difference, finite element and finite volume methods (FDM, FEM, FVM) are usually applied in order to solve an energy balance equation. Although numerical models are more accurate, they are more expensive and require more computational time and memory. Numerical models are used to retrieve an exact solution or parametric analysis. However, most numerical models have limited flexibility in GHE models and take considerable time to calculate them. Thus, they cannot be directly integrated into building energy simulation and are therefore of limited use in practical applications.

In this study, only analytical methods are presented. Such models include (i) instantaneous line source (ILS), (ii) continuous infinite line source (CLS), (iii) continuous cylindrical source (CCS), (iv) continuous Finite line source (FLS). A comparison among different models is provided as follows [5]:

a. Instantaneous line source (ILS)

$$\Delta T = \frac{Q}{4\pi\lambda L} e^{-r^2/(4at)} \tag{1}$$

where T is temperature (K), t is time (s), r is the radial distance of a point from a reference point (the center) (m), λ is thermal conductivity (W m^{-1}K^{-1}) independent from temperature, $\alpha = \frac{\lambda}{\rho c_p}$ is thermal diffusivity (m^{2}s^{-1}) of a homogeneous isotropic solid, and Q is the amount of heat per time (W) released at time $t = 0$, and L is the length of the line / borehole.

b. Continuous infinite line source (CLS)

$$\Delta T = -\frac{Q}{4\pi\lambda L} \mathrm{Ei}\left\{-\frac{r^2}{4at}\right\} \tag{4}$$

Where the *Ei* is the exponential integral, and Euler's constant $\gamma \approx 0.57722$, where

$$\mathrm{Ei}\{-n\} = \gamma + \ln(n) - n + \frac{1}{2}n^2 + O(n^3) \tag{5}$$

or

$$\Delta T \approx \frac{Q}{4\pi\lambda L}\left[\ln\left(\frac{4at}{r^2}\right) - \gamma + 4\pi\lambda R_b + \frac{r^2}{2at}\left(\ln\left(\frac{4at}{r^2}\right) - \gamma + 1 - \frac{amc_p}{\pi r^2 \lambda L}\left(ln\left(\frac{4at}{r^2}\right) - \gamma + 4\pi\lambda R_b\right)\right)\right] \tag{6}$$

where m is the mass of the borehole (kg), L the length of the borehole.

c. Continuous cylindrical source (CCS)

$$\Delta T = \frac{Q}{\pi^2 \lambda r}\int_0^\infty \left(e^{-\alpha x^2 t} - 1\right)\frac{[J_0(rx)Y_1(rx) - J_1(rx)Y_0(rx)]}{x^2[{J_1}^2(rx) + {Y_1}^2(rx)]}dx \tag{7}$$

where J_0, J_1, Y_0, Y_1 are the Bessel functions of the first and second kind, given (for $i = 1,2$) as

$$J_i(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\,\Gamma(k+i+1)} \left(\frac{x}{2}\right)^{2k+i}$$
$$Y_i(x) = \frac{J_i(x)\cos(i\pi) - J_{-i}(x)}{\sin(i\pi)} \tag{8}$$
$$\Gamma(n) = (n-1)!$$

d. Continuous Finite line source (FLS)

$$\Delta T = \frac{Q}{4\pi\lambda L} \int_0^L \left(\frac{\mathrm{erfc}\left(\sqrt{\frac{r^2 + (L/2 - x)^2}{4at}}\right)}{\sqrt{r^2 + (L/2 - x)^2}} - \frac{\mathrm{erfc}\left(\sqrt{\frac{r^2 + (L/2 + x)^2}{4at}}\right)}{\sqrt{r^2 + (L/2 + x)^2}} \right) dx \tag{9}$$

3. RESULTS

Regarding the case study, determining the thermal properties of the ground of the borehole in Limassol (real case study) of 127m depth and 152mm diameter for a U-tube of 40mm external diameter and bentonite thermal conductivity of $\lambda_b = 1.2$ W m^{-1} K^{-1}. The ground thermal conductivity λ varies between 0.6–1.5 W m^{-1}K^{-1}, while the diffusivity α varies between 0.4–0.68×10^{-6} m^2 s^{-1}. The power used for the thermal response test (TRT) was at 2900 W.

Corresponding results for the models presented above, namely the ILS, CLS, FLS, CCS, are presented in FIGURE 1, where the change in temperature with respect to time is presented. It is clearly observed that the most accurate models compared to the experimental results are CLS1 (equations 4 and 5) and CLS2 (equation 6) with an average error value of 0.0156 and 0.0194 respectively. The results of ILS, FLS and CCS are not as accurate and show an average error value of 3.309, 1.883, 3.317 respectively. It should be mentioned again that the suitability of each method depends on the physical application and most notably on the hypotheses and parameter values chosen

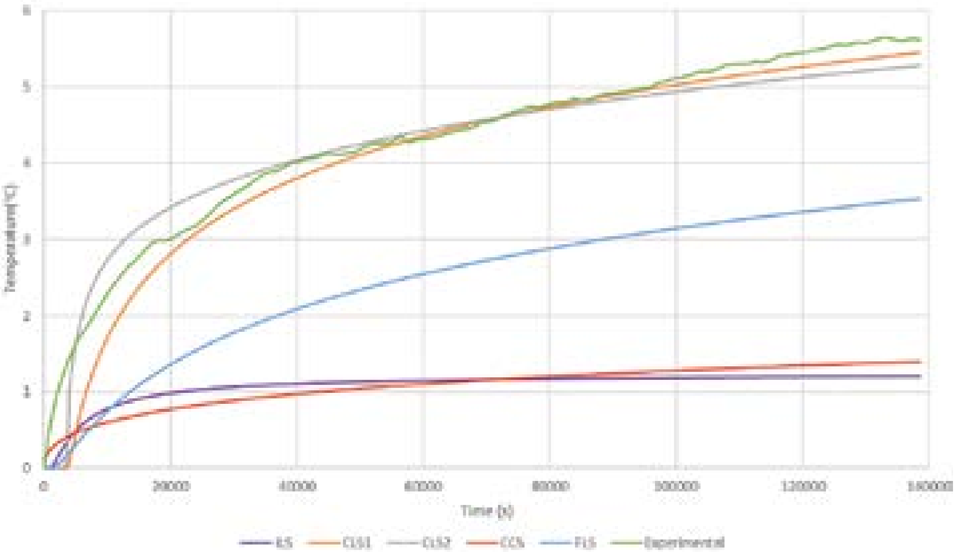

FIGURE 1. Analytical models comparison with experimental data

4. CONCLUSIONS

In this short study, different analytical models were compared against experimental geothermal data of a single U-tube borehole recorded in Cyprus. The analytical models investigated were the: Instantaneous line source (ILS), Continuous infinite line source (CLS), Continuous Finite line source (FLS), and Continuous cylindrical source (CCS).

The analytical models of GHE, observed with the highest accuracy were the CLS1 and CLS2, with very low error value (< 2%) respectively. It can be concluded that analytical models can be considered suitable in the simulation of geothermal models since they benefit much more than numerical models due to less computing time required (by computers) and are much cheaper than numerical models, however very close attention has to be placed on the selection of the parameters and the model.

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REFERENCES

[1] L. Lamarche and B. Beauchamp, "A new contribution to the finite line-source model for geothermal boreholes," *Energy Build*, vol. 39, no. 2, pp. 188–198, Feb. 2007, doi: 10.1016/j.enbuild.2006.06.003.

[2] L. Aresti, P. Christodoulides, and G. Florides, "A review of the design aspects of ground heat exchangers," *Renewable and Sustainable Energy Reviews*, vol. 92, pp. 757–773, 2018, doi: 10.1016/j.rser.2018.04.053.

[3] P. Christodoulides *et al.*, "Reviewing the Modeling Aspects and Practices of Shallow Geothermal Energy Systems," *Energies (Basel)*, vol. 13, no. 16, p. 4273, Aug. 2020, doi: 10.3390/en13164273.

[4] S. Javed, P. Fahlén, and J. Claesson, "Vertical Ground Heat Exchangers: A review of heat flow models," in *Proceedings of Effstock 2009: the 11th international Conference on thermal energy storages*, Stockholm, Sweden, 2009, pp. 53–78. doi: 10.1017/CBO9780511974885.006.

[5] P. Christodoulides, G. Florides, and P. Pouloupatis, "A practical method for computing the thermal properties of a Ground Heat Exchanger," *Renew Energy*, vol. 94, pp. 81–89, Aug. 2016, doi: 10.1016/j.renene.2016.03.035.