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3D nonlinear constitutive modeling for dynamic analysis of reinforced concrete structural members

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Abstract

Many constitutive models have been proposed in order to capture the realistic behavior of reinforced concrete structures under static loading conditions. Few of these numerical models manage to extend to dynamic problems. This is due to the fact that these models present increased computational demand and inability to simulate realistically the different types of mechanical behavior of reinforced concrete members. The purpose of this paper is to propose a computationally efficient constitutive method in order to simulate accurately the behavior of a wide range of reinforced concrete structural members under dynamic loading conditions.
The proposed material model is based on the Markou and Papadrakakis [1] model which was an extension of the Kotsovos and Pavlovic [2] work. A solution strategy which describes the behavior of concrete during the dynamic loading is presented. The proposed algorithm describes the cyclic behavior of concrete which is dominated by the development of microcracking, macrocracking and brittle failure. It uses the implicit integration method of Newmark in order to solve the equation of motion. The concrete domain is simulated by 8- and 20-noded hexahedral elements, which treat cracking with the smeared crack approach. The steel reinforcement is embedded inside the hexahedral meshing and modeled by truss and beam elements.
Accurate nonlinear dynamic analysis of reinforced concrete structures is very helpful in estimating the behavior of a concrete structures during an earthquake. Many concrete buildings have been designed according to the old seismic codes. Thus, an accurate and realistic modeling to assess their strength and their ability to carry the expected seismic forces is very important. The validity of the proposed method is demonstrated by comparing the numerical response with the corresponding experimental results of reinforced concrete members.

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1. Introduction

Many constitutive models have been proposed for the simulation of the reinforced concrete structures. Most of these approaches describe the material of concrete using uniaxial constitutive laws with strain softening and tension stiffening characteristics. The analysis of reinforced concrete structural members is characterized by excessive cracking of the material of concrete. Therefore, large concentrations of stresses are appeared in the cracked areas which may lead to numerical instabilities.. The complex behavior of concrete under cyclic and dynamic loading conditions makes the numerical procedure even more difficult to converge when nonlinearities are accounted for during the analysis. Therefore, a realistic 3D approach which is characterized by simplicity and computational efficiency is needed to be developed.

Most researchers use elastoplastic uniaxial constitutive laws in order to describe the mechanical behavior of concrete. Others use the "equivalent uniaxial strain" concept to combine the uniaxial laws with biaxial or triaxial behavior of concrete. Other approaches propose constitutive models based on biaxial or triaxial failure surfaces. In addition to that, many researchers use the compression field theory to treat the behavior of cracked RC elements subjected to shear. Another group of models combine the elastoplastic isotropic uncracked behavior of concrete with fracture energy based smeared crack approaches. Furthermore, many models which are based on damage mechanics are proposed for concrete behavior under monotonic and cyclic loading conditions.

The models based on the pre-mentioned concepts indicate the necessity of a 3D constitutive law without adding parameters which are not associated with the physical behavior of concrete at a material level. Most of these models are restricted to 2D analysis in order to capture the biaxial behavior of concrete structures thus do not provide the numerical tools to study the realistic 3D mechanical behavior of RC structures or even members that develop complicated forms of deformation. In the literature, few models [2-4] use 3D numerical simulations for dynamic analysis of RC structures.

The proposed model in this research work describes the triaxial behavior of concrete by using realistic assumptions without the need of introducing numerous concrete material parameters. The uniaxial compressive and tensile strengths, the Young Modulus of elasticity and the Poisson's ratio are the only material parameters which are needed to be defined for the analysis of concrete. The accuracy, numerical simplicity and the computational efficiency are the most important features in order to show the practical use of the model that can be easily implemented in predicting the dynamic behavior of any structural concrete member. Therefore, the model adopts the numerical approach which was proposed in [5] for cyclic loading conditions thus integrated herein for simulating the dynamic response of RC structures. Furthermore, the simulation of concrete behavior is based on the proposed model by Markou and Papadrakakis [1] which was an extension of the model presented by Kotsovos and Pavlovic [2].

2. Material constitutive law of reinforced concrete

The constitutive model of concrete describes the realistic 3D behavior of concrete by taking into account the out of plane stresses, which cause the triaxial stress-strain phenomena. Each state of stress-strain is mathematically described by hydrostatic and deviatoric components, where the proposed model uses two moduli of elasticity (bulk K and shear G) and an equivalent external stress ($\sigma_{id.}$) in order to describe the constitutive relations as presented by the combined approach of Kotsovos and Pavlovic [2]. The bulk modulus K and the shear modulus G describe the non-linear σ_0-$\varepsilon_{0(h)}$ and τ_0-$\gamma_{0(d)}$ behavior combined with the use of σ_{id} in order to take into account the coupling effect of τ_0-$\varepsilon_{0(d)}$ (h and d stand for hydrostatic and deviatoric components, respectively). The constitutive relations take the following form:

$$\varepsilon_0 = \varepsilon_{0(h)} + \varepsilon_{0(d)} = (\sigma_0 + \sigma_{id})/(3K_s) \tag{1}$$

$$\gamma_0 = \gamma_{0(d)} = \tau_0/(2G_s) \tag{2}$$

where *Ks, Gs* is the secant forms of bulk and shear moduli, respectively. The secant forms of bulk, shear modulus and σ_{id} are expressed as functions of the current state of stress which derived by regression analysis of the experimental data found in [2].

It is evident that at the first stages of loading, when the deviatoric stress is less than the 50% of the ultimate strength [1], the concrete material will practically behave in an elastic manner thus the elastic constitutive model is used. The constitutive material matrix of the uncracked concrete is presented in eq. 3.

$$D = \begin{bmatrix} 2G_t + \mu & \mu & \mu & 0 & 0 & 0 \\ \mu & 2G_t + \mu & \mu & 0 & 0 & 0 \\ \mu & \mu & 2G_t + \mu & 0 & 0 & 0 \\ 0 & 0 & 0 & G_t & 0 & 0 \\ 0 & 0 & 0 & 0 & G_t & 0 \\ 0 & 0 & 0 & 0 & 0 & G_t \end{bmatrix} \tag{3}$$

where $\mu = K_t - 2\,G_t/3$. When the deviatoric stress exceeds 50% of the ultimate strength, the parameters $K_t = K_t(\sigma_0, \tau_0, f_c)$ and $G_t = G_t(\sigma_0, \tau_0, f_c)$ are updated according to the current state of stress (f_c is the uniaxial compressive strength).

The ultimate strength is expressed by the value of the ultimate deviatoric stress by using the expressions of Willam and Warkne [6].

$$\tau_{0u} = \frac{2\tau_{0c}(\tau_{0c}^2 - \tau_{0e}^2)\cos\theta + \tau_{0c}(2\tau_{0e} - \tau_{0c})\sqrt{4(\tau_{0c}^2 - \tau_{0e}^2)\cos^2\theta + 5\tau_{0e}^2 - 4\tau_{0c}^2\tau_{0e}^2}}{4(\tau_{0c}^2 - \tau_{0e}^2)\cos^2\theta + (2\tau_{0e} - \tau_{0c})^2} \tag{4}$$

where the rotational variable θ defines the deviatoric stress orientation on the octahedral plane. The τ_{0e} $(\theta=0°)$ and τ_{0c} $(\theta=60°)$ correspond to the ultimate limit states of $\sigma_1=\sigma_2>\sigma_3$ (triaxial extension) and $\sigma_1>\sigma_2=\sigma_3$ (triaxial compression), respectively that derived through experimental data. The numerical model is based on the brittle nature of concrete, therefore, when the criterion of concrete is satisfied the material point loses its capacity abruptly. The combination of this theory with the smeared crack approach treats cracking by redistributing the released stresses to the surrounding uncracked concrete. The constitutive matrix of cracked concrete is modified by setting to zero the stiffness that corresponds to the direction of the maximum principle tensile stress. Similarly, the constitutive matrix is updated in the case of two cracks, which leaves only stiffness along the intersection of the two planes [5]. The stresses are calculated through the following expression:

$$\sigma_{Cr} = \mathbf{T}^{-1} \begin{bmatrix} \sigma_1 = 0 \\ \sigma_2 \\ \sigma_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

where $\mathbf{T}^{-1}$ is the inverse transformation matrix that is used to transform the principal stress axes to the initial x, y and z axes.

Furthermore, a flexible crack closing criterion is used herein which is crucial for the stability of the nonlinear analysis under cyclic loading conditions. The criterion which was introduced in [5], is based on the numerical phenomenon that relates to the strains that caused the initial formation of the cracks thus when unloading takes place the criterion accounts for limits of the values of strains that define the closure of cracks. The criterion of crack closure, takes the following form:

$$\varepsilon_i \leq a\varepsilon_{cr} \tag{6}$$

where ε_i is the current strain in the i-direction which is normal to the crack plane and ε_{cr} is the strain that caused the crack formation. Parameter a is a reduction factor which takes the following form:

$$a = 1 - \frac{\varepsilon_{cr}}{\varepsilon_{\max}} \tag{7}$$

Finally, the crack closure leads to the rebirth of the cracked Gauss point and the stresses are calculated through the use of a modified constitutive matrix. In the case of the closure of a crack in a material point which had only one crack which lied perpendicular to z' axis, the following constitutive matrix is used:

$$D = \begin{bmatrix} 2G_t + \mu & \mu & (1-a)\mu & 0 & 0 & 0 \\ \mu & 2G_t + \mu & (1-a)\mu & 0 & 0 & 0 \\ (1-a)\mu & (1-a)\mu & (1-a)(2G_t + \mu) & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta G_t & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta G_t & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta G_t \end{bmatrix} \tag{8}$$

where β is a shear retention factor which is β = 0.05 in this study. The constitutive matrix has to be transformed to global axes using the standard coordinate system transformation laws. The proposed model presents a numerical procedure which is based on the brittle nature of the material of concrete. Therefore, the model prevents the excessive concentrations of stresses in the cracked areas which may lead to numerical instabilities. The combination of this procedure with the smeared crack approach treats cracking by redistributing the cracked stresses to the uncracked areas. Additionally, the use of a flexible crack closing criterion which is crucial in dynamic loading conditions provides stability in the nonlinear analysis even when the loading is close to load carrying-capacity of the structures.

3. Numerical implementation

A numerical verification of the accuracy and efficiency of the proposed numerical method will be presented in this section by numerically investigating a two level RC frame under dynamic loading. The Newton-Raphson iterative method is used combine with an energy convergence tolerance criterion ($e_{tol.} = 10^{-4}$). The nonlinear Newmark integration method is used for the dynamic analysis. The steel bars are simulated as embedded natural beam-column flexibility based elements within the hexahedral concrete elements as described in [7]. The material of steel-reinforcement is modeled through the use of the Menegotto-Pinto [8] model that takes into account the Bauschinger effect. The steel reinforcement is considered to be fully bonded with the surrounding concrete.

The numerical implementation that was chosen, is a two storey RC frame denoted as L30, which was investigated by performing a dynamic analysis based on a research program concerned with the verification of the validity of the European code provisions for the design of earthquake-resistant structures (Carydis [9]). The geometric and reinforcement details are shown in Fig. 1a. The uniaxial cylinder compressive strength of concrete (f_c) was 50 MPa. The yielding stress (f_y) of steel reinforcement were 500 MPa. The masses of the frame were 2.87 and 2.62 tons at the lower and upper girder respectively. The frame was subjected to horizontal motion as shown by the green vectors in Fig.1b. The experimental test produces stable solutions although during the analysis, the structure presents considerable non-linear phenomenon such as excessive cracking (as illustrated by the numerical crack pattern in Fig. 1c) and yielding of the reinforcement bars.

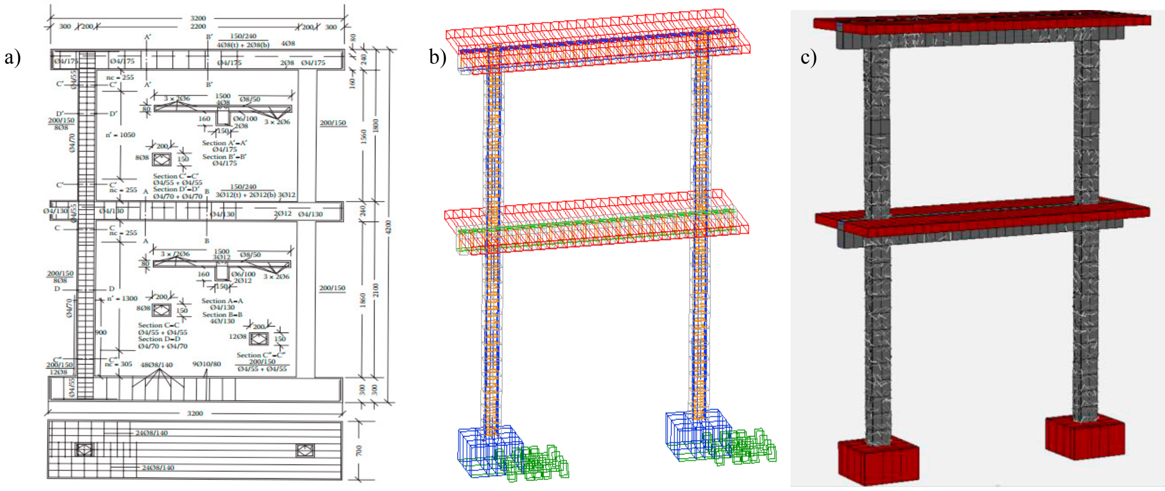

Fig. 1. a) Geometric and reinforcement details, b) 3D view of the FE mesh of 8-noded hexahedral and embedded rebar elements and c) the numerical crack pattern at the end of the test.

The concrete domains are modeled by 8-noded hexahedral finite elements and the steel reinforcement is modeled by natural beam-column flexibility based elements. As illustrated in Fig. 1b the embedded steel elements (transverse and longitudinal) have the exact location and direction of the actual reinforcement. Additionally, 224 concrete finite elements (10cm x 15cm x 15.5cm) and 1,891 steel reinforcement elements are used to model the RC domain. For the RC slabs, 128 8-noded hexahedral elements are used (red elements). The mass density of the RC slab-elements has been chosen appropriately in order to take into account the mass of the structure. Furthermore, 40 steel elements are used for the base of the RC frame in order to represent the rigid girder foundation (blue elements that are located at the base of the columns). The period of the model was found to be T = 0.63 s, by using the elastic properties of concrete. Therefore, the time step was set to ΔT = 0,007746 s for the nonlinear dynamic analysis.

The numerical curves are compared with the experimental ones in Figs. 2 and 3. The numerical model manages to capture adequately the dynamic response of the structure in terms of storey displacement time histories. The required NR iterations during the solution procedure are limited to an average of 4 to 5, underlining the numerical stability of the proposed method. The total required time for solving the nonlinear dynamic analysis was 16 minutes and the total number of dynamic increments solved was 2141.

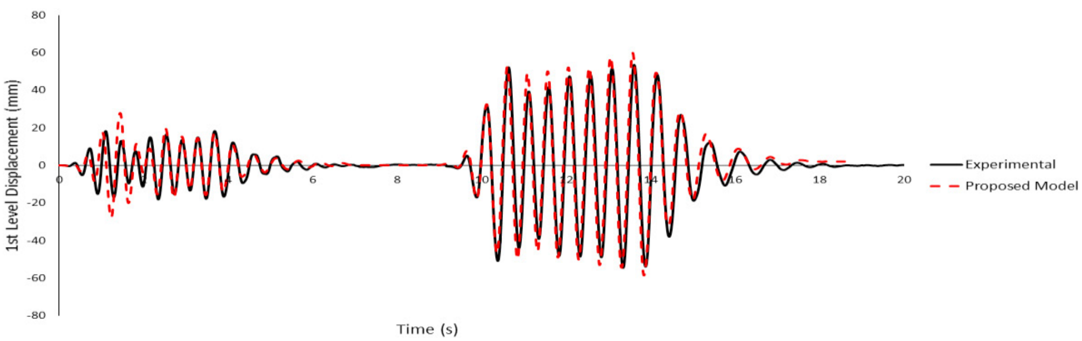

Fig. 2. Comparison between the numerical and experimental results of the 1st level displacement response.

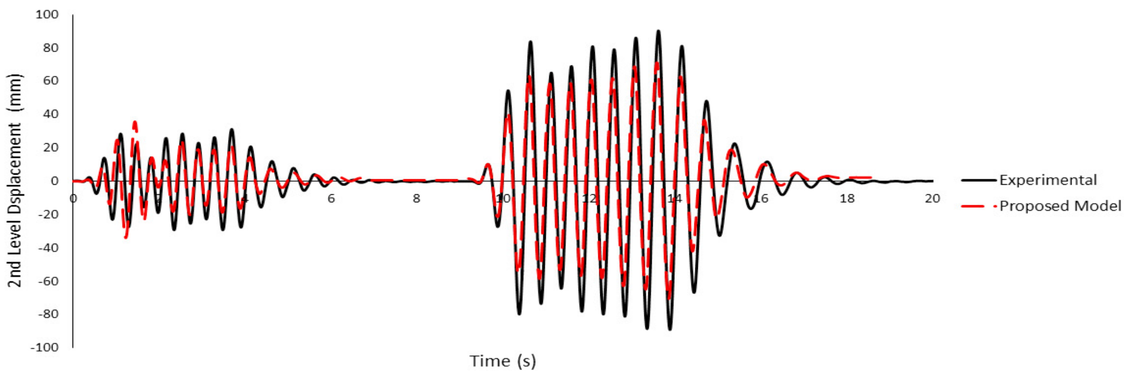

Fig. 3. Comparison between the numerical and experimental results of the 2nd level displacement response.

4. Conclusions

In this paper, a 3D solid finite-element model is proposed for the nonlinear dynamic analysis of structural members. The constitutive model combines a realistic assumption of the brittle behavior of concrete which fits perfectly with the smeared crack approach. The algorithmic implementation of crack opening and crack closure introduced in [5] provides the analysis with accuracy and numerical efficiency. The proposed model is implemented here for the solution of a two storey RC frame in order to capture its dynamic mechanical response. The numerical results of the RC frame that were presented in this work, demonstrate the ability of the proposed algorithm to capture experimental results, while the simulation of the 3D behavior of the RC members was performed with a minimum computational demand that makes the model appealing for the analysis of large-scale structures.

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