

DEVELOPMENT OF A NEW FUNDAMENTAL PERIOD FORMULA BY CONSIDERING SOIL-STRUCTURE INTERACTION WITH THE USE OF MACHINE LEARNING ALGORITHMS

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Abstract

The fundamental period of a structure is one of the key parameters utilized in the design phase to compute the seismic-resistant forces. Although the importance of seismic-resistant buildings is well understood it has been found that the current design code formulae, which are used to predict the fundamental period of reinforced concrete (RC) buildings are quite simplistic, failing to accurately predict the natural frequency, raising many concerns with regards to their reliability. The primary objective of this research project was to develop a formula that has the ability to compute the fundamental period of an RC structure, while taking into account the soil-structure interaction phenomenon. This was achieved by using a computationally efficient and robust 3D detailed modelling approach for modal analysis obtaining the numerically predicted fundamental period of 475 models, producing a dataset with numerical results. This dataset was then used to train a machine learning algorithm to formulate three fundamental period formulae using a higher-order, nonlinear regression modelling framework. The three newly proposed formulae were evaluated during the validation phase to investigate their performance using 60 new out-of-sample modal results, where, in this work, additional validation models are created and used to test the predictive abilities of the proposed fundamental period formulae. The findings of this research report suggest that the proposed fundamental period formulae exhibit exceptional predictive capabilities for the under-study RC multi-storey buildings, where they outperform all existing design code fundamental period formulae currently in effect.

Keywords: Fundamental Period Formula, Soil-Structure Interaction, Machine Learning Algorithms, Modal Analysis, Finite Element Method, Reinforced Concrete.

1 INTRODUCTION

The importance of seismic resistant design of structures and the ability thereof to capture the dynamic response of structures is crucial, especially in areas prone to seismic activity. One of the most important dynamic characteristics is the fundamental period, as it has a significant influence on the calculation of the seismic loads. During a seismic excitation, the interaction between the superstructure (building) and the substructure (soil) can become important as it starts to affect the stress-strain distribution within the superstructure, altering the initial expected results [1-3]. In general, it has been found that soil-structure interaction (SSI) can increase the fundamental period and the overall damping of the system, thus it is important to consider it in order to eliminate unsafe designs and unexpected damage development [4-5] during an earthquake excitation.

Numerous codes worldwide foresee their own methodology of computing the fundamental period of reinforced concrete (RC) structures. However, it is well documented that the current design codes all fail to consider the effect that SSI has on the fundamental period of a structure [3]. There are also some shortfalls with respect to the stiffness distribution of the structure, as the effect of the shear walls is not properly taken into considerations especially in the current Eurocode 8 [6], amongst others as presented in [1]. Thus, establishing design tools that would be able to predict the dynamic properties of RC buildings is of significant importance.

The methodology foresees the use of a computationally efficient and robust 3D modelling technique known as the HYMOD approach [7-9] in order to perform modal analysis to investigate the effect that SSI has on the fundamental period of RC structures. The overall approach foresaw the creation of a dataset comprising out of all the modal analysis results obtained from the various numerical models [1].

Thereafter this research work foresaw [1] the development of fundamental period formulae that were validated through an out-of-plane dataset and were found to have high accuracy. A number of machine learning (ML) and artificial intelligence (AI) algorithms were used to develop predictive models (including closed-form formulae), where it was found that the ability to predict the fundamental period of RC structures through a validation out-of-sample dataset was significantly high. This research work aims to further validate the proposed fundamental period formulae [1] through the use of additional fundamental period results that were developed for the needs of this manuscript.

2 MACHINE LEARNING ALGORITHM

The fundamental principles of the ML model are based on the formation of nonlinear terms consisting of various combinations of independent variables, up to the third degree [10]. The algorithm is able to automatically select nonlinear features, corresponding to the minimum prediction error. The data is normalized by subtracting the mean value from it as to eliminate any irregular outliers.

The algorithm was programmed to use 85% of the data to analyse the trends, by identifying relative relationships used to train the algorithm. The remaining 15% of the data is then used to evaluate the proposed fundamental period formulae. The algorithm presented in the next page represents the procedure for the generation of the formula which is developed by the various authors of Julia programming language [11]. Based on the numerical investigation performed for the needs of this work, it was confirmed that the proposed algorithm is efficient providing with the necessary tools in developing the predictive formulae.

Algorithm: Higher Order Regression

Input: XX^1 , YY^2 , nlf^3 **Output:** Prediction Formulae

1. Create all nonlinear features⁴ ($anlf$)
2. For $i=1:nlf$ do
3. For $j=1:anlf$ do
4. Add j to the model
5. Calculate Prediction Error
6. End
7. Keep in the model the j^{th} feature which yields the minimum prediction error
5. End

Return: Prediction Formula

¹Independent Variables, ²Dependent Variable, ³desired number of nonlinear features, ⁴with all combinations up to the 3rd degree.

3 NUMERICAL MODELS AND DATASET DEVELOPMENT

To develop a design formula through training, requires a relatively large number of models that will be analyzed under a modal analysis thereof. The HYMOD approach was used to generate models of RC structures founded on soft, medium and hard soil. This was achieved by extending the work performed by Gravett et al. [7] to add more models to the existing database that was used for the investigation of the SSI effect on RC buildings [1].

The models all originated from the 4-storey RC building studied by Markou et al. [12] with two parallel frames connected by four out-of-plane central beams and a continuous slab of 150mm resulting in a plan area consisting of 3 bays (see Fig. 1). The total span of each parallel frame was 8.9m and had a perpendicular distance of 6.25m between the two frames. Each floor is vertically spaced 3m center-to-center with the central bay infilled with an RC shear wall of 2.9 x 0.25 m section as shown in Fig. 2. It should be noted that the frame of the RC building was designed based on the old Cyprus code [13] and then retrofitted with RC infill shear walls. The reinforcement details for the in-plane beams, columns and shear walls are shown in Fig. 2.

The 475 modal results were generated from models that were modified geometrically, where different soil domains were also assumed. Table 1 shows the minimum and maximum dimensions of the models based on the research work presented in [1]. Fig. 3 shows the case of a 2-storey RC frame with multiple openings, which was then modified to 4, 6, 8 and 10-storey buildings. For more information related to the models' geometries, one may refer to [1].

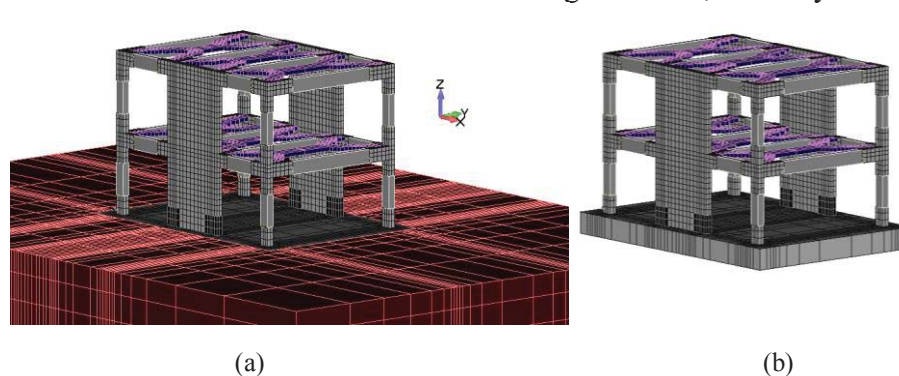


Figure 1. Single span RC mesh of a 2-storey structure with shear walls (a) flexible-base SSI mesh (b) fixed-base with raft foundation mesh.

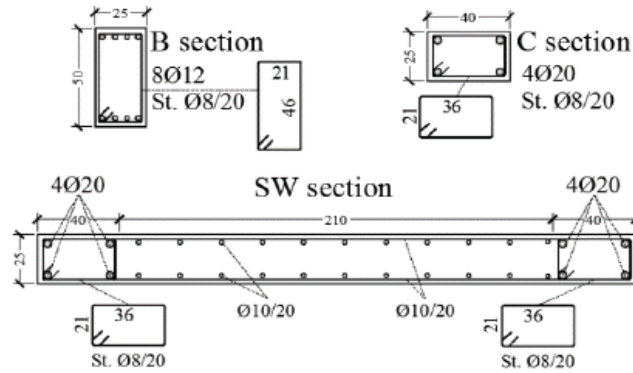


Figure 2. Reinforcement details of the (B)eam section, (C)olumn section and (SW) Shear Wall section respectively [9].

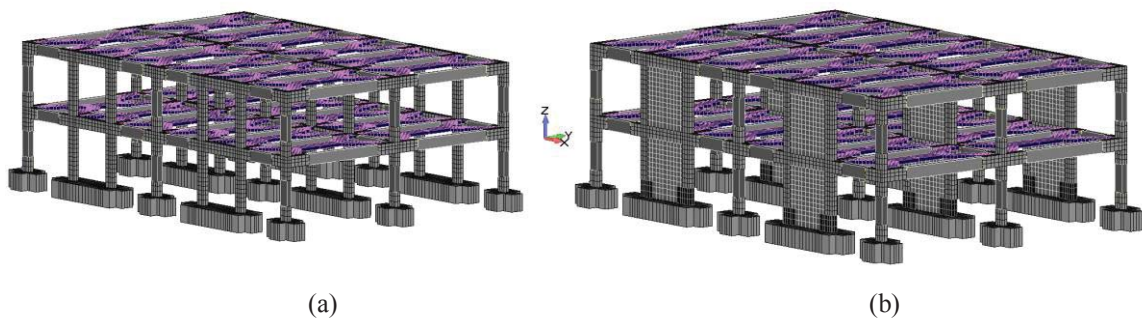


Figure 3. Double span mesh of a 2-storey RC structure without SSI (a) with shear walls (b) without shear walls.

Variables	Minimum	Maximum
Soil Depth (m)	3	60
Soil E (kPa)	65 000	700 000
H (m)	6	30
L (m)	6.25	18.25
B (m)	6.25	18.25
ρ (%)	0	82.9

Table 1: Minimum and maximum values of the HYMOD meshes.

In order to further investigate the performance of the proposed formulae, a new set of additional validation models were created for the needs of this paper. A diverse set of 15 random models were created which yielded a total of 30 numerically derived fundamental periods. The aim of this validation set was to create random models that have geometries that significantly deviate from the typical geometries used to generate the dataset which was used to extract the proposed formulae.

These models were analyzed by assuming a fixed or flexible base (with and without considering the effect of SSI), which then yielded a total of 15 new models. This was done in order to investigate the proposed formulae to predict the fundamental mode of out-of-sample models since these additional models differ substantially from those in the original validation set [1]. It is important to note that all of the additional validation models fall within the range of values as set out in Table 1. Fig. 4 shows the layout of the five additional validation models, where the model in Figs 4d and 4e foresee the analysis of asymmetric frames in-terms of the shear wall positioning. This type of geometries was not assumed within the dataset that was used to train the models and derive the proposed fundamental period formulae. It is also important to note

here that, the newly developed models foresaw the discretization of a 5m deep soil when the SSI was accounted for, where the soil Young modulus was equal to 300 and 700 MPa. Therefore, 5 fixed models, 5 models with 300 MPa soil and another 5 with 700 MPa soil.

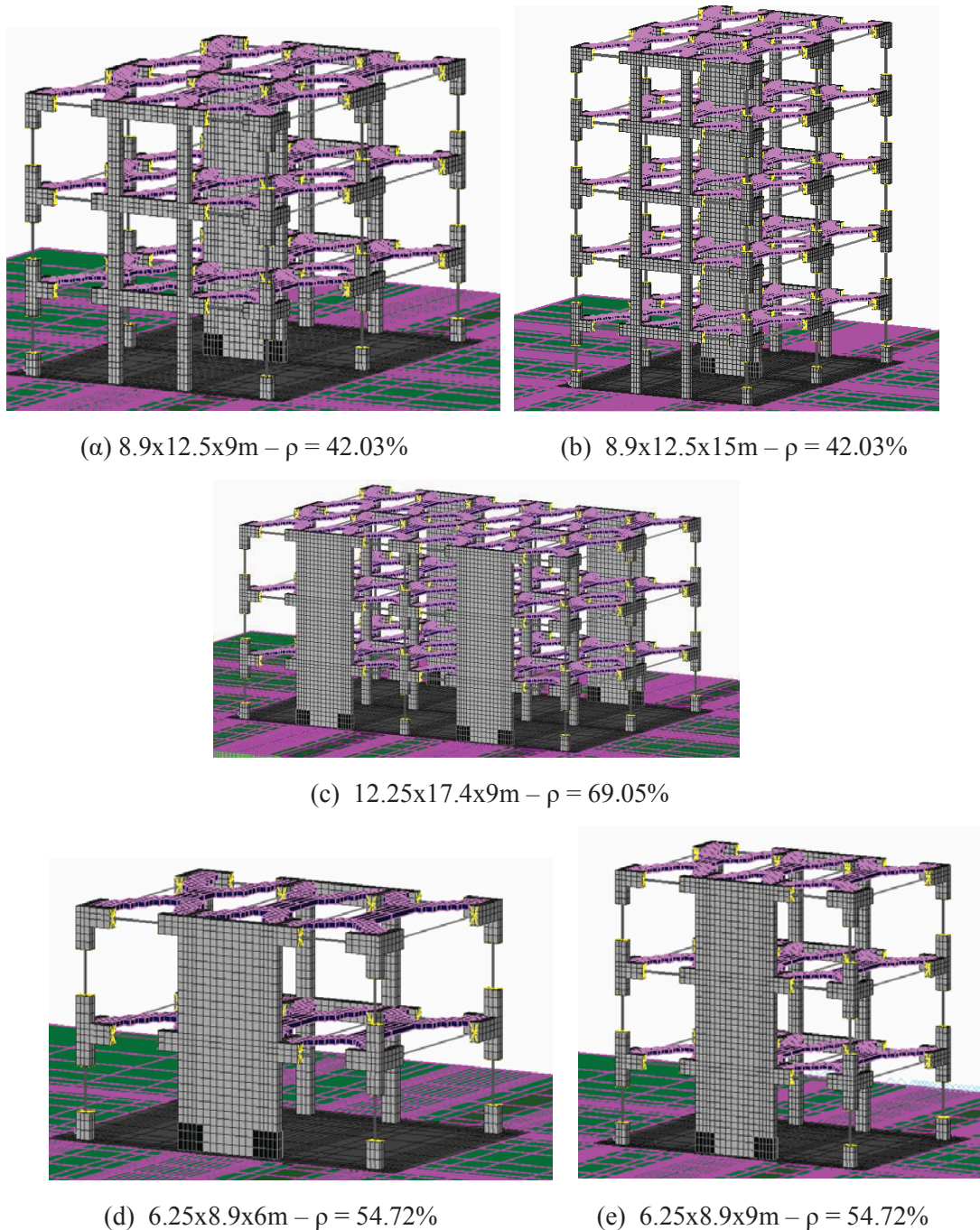


Figure 4. RC building models with soil. (a) 3-storey and (b) 5-storey with a shear wall in the middle, (c) 3-storey shear walls at the perimeter and (d) 2-storey and (e) 3-storey with a single shear wall (asymmetric cases).

4 PROPOSED FUNDAMENTAL PERIOD FOMULAE

Upon successful completion of the ML implementation, three design formulae were generated. The proposed formulae are currently only valid for the range of values shown in Table 1,

as previously discussed. The accuracy of the design formulae is directly related to the correlation between the numerically predicted period and those of the formulae. This in return depends on the number of features used within the design formulae.

Therefore, three individual features were considered (3, 5, and 20) to develop and parametrically investigate the numerical response of the developed formulae. It is important to note herein that the building features that were accounted for, during the training process are the following:

H the building's height (m)

ρ the percentage of shear walls (%)

L the length of building parallel to the oscillating direction (m)

B the width of the building perpendicular to the oscillating direction (m)

E_s the soils' modulus of elasticity (kPa)

D the soil depth (m)

The three different assumptions in terms of the number of features within each formula yielded three different formulae as shown below:

3-Features Formula:

$$T = (0.0332205 \cdot H) - (0.000123101 \cdot \rho \cdot H) + (16.726x10^{-6} \cdot L \cdot B^2) - 0.0275279 \quad (1)$$

5-Features Formula:

$$T = (0.0324913 \cdot H) - (86.048x10^{-6} \cdot \rho \cdot H) + (20.105x10^{-6} \cdot L \cdot B^2) - (28.11x10^{-14} \cdot \rho \cdot L \cdot E_s) - (53.766x10^{-6} \cdot \rho \cdot L) - 0.0173241 \quad (2)$$

20-Features Formula:

$$\begin{aligned} T = & (0.0328677 * H) - (143.475x10^{-6} \cdot \rho \cdot H) - (79.921x10^{-8} \cdot L \cdot B^2) \\ & - (10.458x10^{-14} \cdot \rho \cdot L \cdot E_s) - (55.061x10^{-6} \cdot \rho \cdot L) \\ & - (71.214x10^{-12} \cdot E_s \cdot D^2) + (5.016x10^{-6} \cdot L \cdot H \cdot D) \\ & - (2.93x10^{-18} \cdot B \cdot E_s^2) + (7.121x10^{-10} \cdot B \cdot E_s) + (8.702x10^{-6} \cdot L \cdot H^2) \\ & + (93.621x10^{-8} \cdot L \cdot \rho^2) - (6.093x10^{-6} \cdot L \cdot H \cdot \rho) + (3.351x10^{-6} \rho \cdot H^2) \\ & + (60.549x10^{-12} \cdot B \cdot E_s \cdot D) + (16.982x10^{-5} \cdot B^2) - (64.89x10^{-6} \cdot H^2) \\ & - (9.776x10^{-12} \cdot H \cdot E_s \cdot D) + (1.477x10^{-17} \cdot H \cdot E_s^2) \\ & - (44.111x10^{-10} \cdot H \cdot E_s) - (2.891x10^{-6} \cdot H \cdot D \cdot B) + 3.8476x10^{-4} \quad (3) \end{aligned}$$

Each formula was compared against the numerical results from the modal analysis in order to evaluate the average absolute error which resulted in 9.04%, 7.86% and 5.35% for the 3-, 5- and 20-feature period formulae, respectively [1]. An additional set of 60 out-of-sample modal results were developed [1], where the proposed formulae managed to predict the results with a high accuracy and outperform any existing fundamental period formula that can be found in the international literature. However, the ability of these formulae to predict out-of-sample cases should be further investigated through the use of more unconventional in-terms of geometry buildings as it is going to be presented in the next section.

5 ADDITIONAL VALIDATION RESULTS

Numerically computed periods derived from the additional validation models that were developed herein (30 additional modal results) were compared against those obtained from the formulae for each case. The numerically predicted periods were plotted against those obtained from the formulae for each case along with their correlation as can be seen in Figs 5 – 7.

By evaluating the correlation between the relationships of the numerically calculated periods against those obtained from the formulae for each case, it is easy to observe that a high correlation was achieved for each formula. The relationship obtained from the 20-feature correlation

indicates that the formula yields a very good prediction deriving a prediction error of 8.5% (see Table 2). The 5- and 3-feature formulae have less accuracy given that their complexity and ability to account for the contribution of each feature of the building and its soil is not as high as that of the 20-feature formula. It is also interesting to note at this point that even though the newly developed validation models vary substantially from the original models used to train the formulae [1], the proposed formulae were still able to accurately predict the fundamental periods (Figs 5 – 7) demonstrating their ability to predict out-of-sample results.

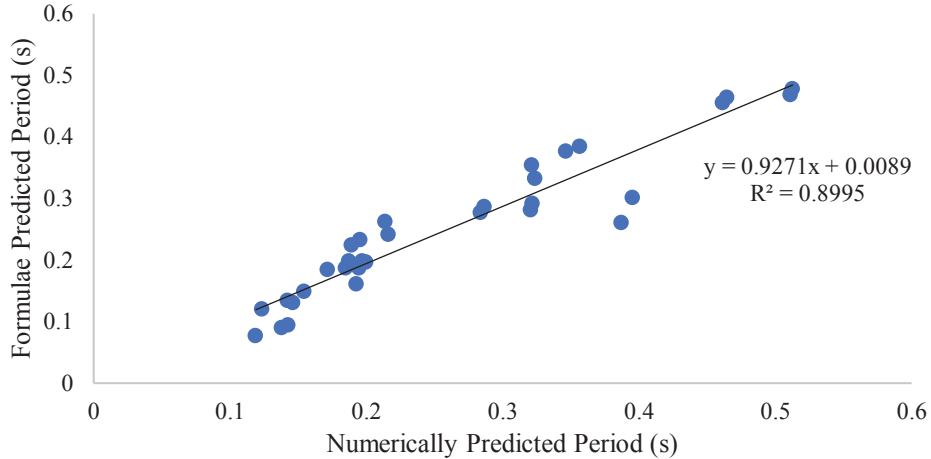


Figure 5. Extra validation data. Correlation of the 3-feature formula.

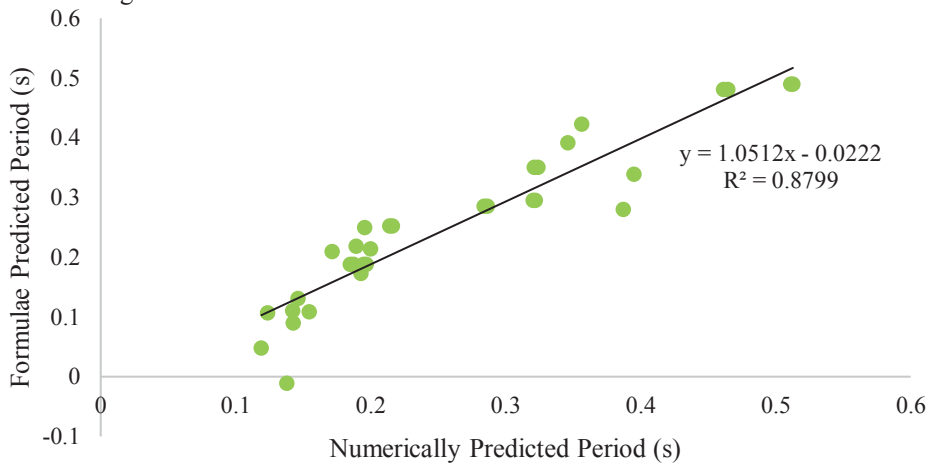


Figure 6. Extra validation data. Correlation of the 5-feature formula.

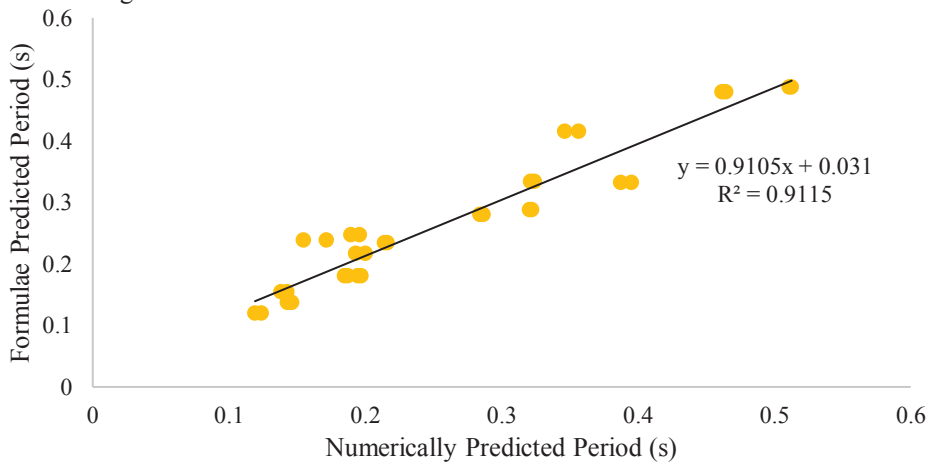


Figure 7. Extra validation data. Correlation of the 20-feature formula.

Table 2 shows the comparison between the proposed formulae and the design formulae of Eurocode [6] and NEAK [14]. It is easy to observe that the Eurocode 8 formula derives the highest mean absolute error, while the 20-feature formula the lowest, which is a finding further supporting the research findings of [1].

Description	Formula	Mean Absolute Error
3-Feature Formula	Eq. 1	13.0%
5-Feature Formula	Eq. 2	11.4%
20-Feature Formula	Eq. 3	8.5%
NEAK	$T_{NEAK} = 0.09 \frac{H}{\sqrt{L}} \sqrt{\frac{H}{H + \rho L}}$	17.7%
EC 8	$T_{EC} = C_T H^{0.75}$, with: $C_T = 0,075$	62.0%

Table 2: Fundamental Period Formulae predictions on the additional validation data.

6 CONCLUSIONS

The parametric investigation of three newly proposed fundamental period formulae was performed on 30 additional out-of-sample modal results and their ability to predict the fundamental period of RC buildings was performed. The 3-, 5- and 20-feature formulae displayed minimal errors when compared to the validation sets, concluding in very promising results thus verifying the findings reported in [1].

It was also found that the 20-feature formula exhibited a higher overall accuracy due to its high number of features, a finding that is in line with the work presented in [1]. The 3-feature formula had the highest error as it was not able to include the SSI related parameters, where the 5-feature formula was found to provide an improved predictive ability. It was also observed that, even though the out-of-sample models that were used to generate the additional validation modal results were dissimilar to the models used to generate the training dataset in-terms of geometry, the proposed formulae managed to derive good predictions with high accuracy. This highlights the extendibility of the proposed fundamental period formulae.

It is also interesting to note at this point that the 20-feature formula is not “elegant”, consisting of numerous features, but according to the authors’ foresight, this will be the future of Civil Engineering design, where ML and AI-generated models will be directly integrated within software tools that will be used to design our structures. Finally, future work foresees the development of additional models that will extend the spectrum in terms of the geometry of RC buildings, developing new formulae that will be able to predict the fundamental period of structures of various shapes. The case of steel structures will also be investigated in a similar project that is currently active.

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