A Multiresolution Model of Iterative Regularized Image Restoration

Ioannis Stephanakis Anastasios Doulamis Nikolaos Doulamis Nicolas Tsapatsoulis

Department of Electrical and Computer Engineering, National Technical University of Athens
9 Heroon Polytechniou str., 157 73 Zographou, Athens
Greece

stephan@image.ntua.gr adoulam@image.ntua.gr ndoulam@image.ntua.gr ntsap@softlab.ntua.gr

Abstract

A model of iterative regularized restoration of images based upon wavelet filter banks is proposed in this paper. Regularized restoration is a method of solving ill-posed image restoration problems. Wavelet filter-banks designed upon arbitrarily sampling lattices are proposed in order to replace the conventional regularization operator. A regularization parameter corresponds to each decomposition channel of the wavelet filter-bank. The proposed model estimates regularization parameters iteratively. The current estimate of the restored image is used in such an estimation. The model assumes that the degradation may be represented by a perfect reconstruction filter bank. Factorizations of unitary matrices using Givens rotations allow for efficient representations of a variety of degradations. Should both the degradation and the smoothing filter be decomposed by wavelet filter banks, the restoration problem may be split into independent restoration problems in each transformation channel. Regularization parameters are evaluated iteratively in each channel. Numerical results indicate better ISNR (Improvement in Signal-to-Noise-Ratio) figures than conventional iterative regularization methods.

1. Introduction

Wavelet functions exhibit very good localization properties in frequency and in spatial/time domain and provide an efficient way of representing 1-D and 2-D signals. Construction of wavelet and scale functions of finite support is possible by iterative filtering and subsampling [1]. Wavelet transformation of signals allows for processing in separate subbands [2]. Several methods based on multiresolution analysis via wavelets and orthogonal wavelet filter-banks have appeared recently in the literature [3] for such applications as image/speech coding and compression [2], motion estimation in video sequences, neural wavelet networks [4], speech recognition and image restoration [5].

Digital images are degraded due to motion blur, defocusing, atmospheric turbulence and long exposure times, [6]. The linear degradation model assumes that the degraded image, denoted as **y**, is related to the original image **s** through the degradation matrix **K** and additive white Gaussian noise **n**

$$\mathbf{y} = \mathbf{K}\mathbf{s} + \mathbf{n} \tag{1}$$

where vectors **y**, **s** and **n** are produced by raster scanning and stacking of the pixel values of the corresponding images. These vectors have N^2 elements for images having size (NxN) pixels. **K** is a block Toeplitz matrix which expresses the convolution of the original image with the degradation filter $K(m_1,m_2)$ and has dimensions N^2 by N^2,

$$\mathbf{K} = \begin{pmatrix} K(0,0) & K(0,-1) & \cdots & K(0,1) & K(-1,0) & K(-1,-1) & K(-1,-2) & \cdots \\ K(0,1) & K(0,0) & & K(0,2) & K(-1,1) & K(-1,0) & K(-1,-1) & \cdots \\ \vdots & & \ddots & & \vdots & & & \\ K(1,0) & K(1,-1) & & K(1,1) & K(0,0) & K(0,-1) & K(0,-2) & \\ \vdots & & & & \vdots & & & \ddots \end{pmatrix} \tag{2}$$

Regularized restoration techniques [7] tackle the ill-posed problem of image restoration by assuming a smoothing filter as a regularization operator and minimizing a Lagrangian functional. Such a functional includes the restoration error and the outcome of the filtering with the smoothing filter,

$$\min_{\hat{\mathbf{s}}} J_\lambda(\hat{\mathbf{s}}) = \min_{\hat{\mathbf{s}}} \left(\|\mathbf{y} - \mathbf{K}\hat{\mathbf{s}}\|^2 + \lambda \|\mathbf{C}\hat{\mathbf{s}}\|^2 \right) \tag{3}$$

where **C** is the filtering matrix with the smoothing filter and λ is the so-called regularization parameter. There are several methods of estimating the regularization parameter λ, like iterative methods [8] and statistical methods [9],[10]. This paper proposes and analyzes an iterative model for image restoration which establishes regularization parameters in each wavelet channel. The outline of the proposed model is presented in Figure 1.

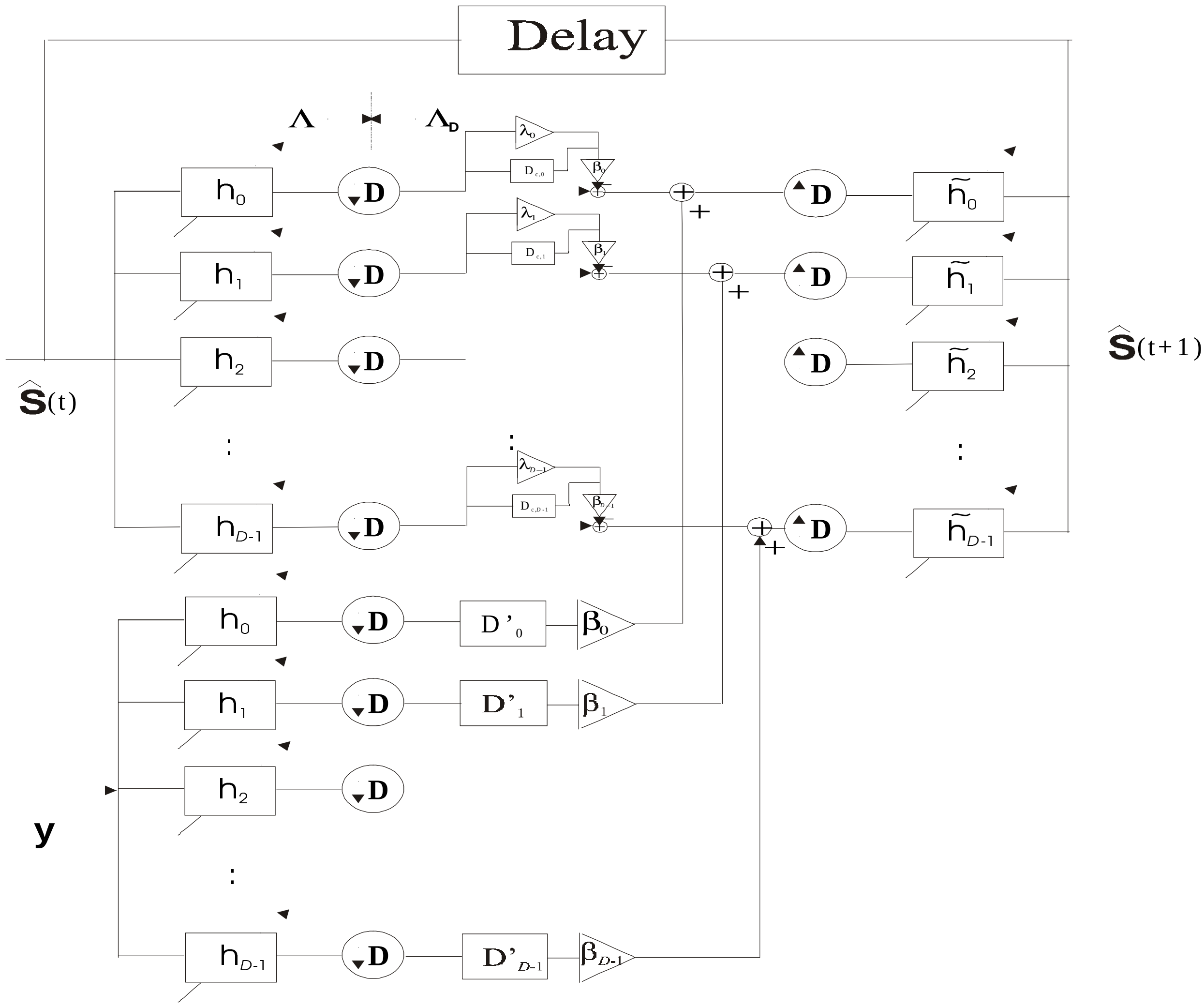


Figure 1: Block diagram of the proposed iterative model for solving the equation of regularized restoration

2. A model of regularized image restoration using filter-banks

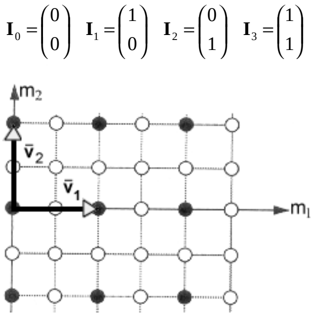
Regularized image restoration using wavelet filter banks as the smoothing operator is formulated in the sequel as an optimization problem. Wavelet factorizations via unitary matrices are suggested to decompose the degradation matrix **K** into independent frequency channels. The equation of the regularized restoration method is solved iteratively in each of these channels.

2.1 Formulation of regularized restoration as an optimization problem

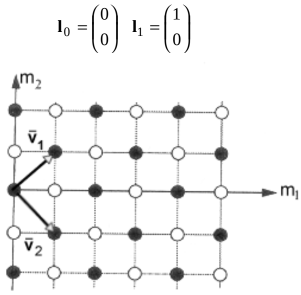
Generalization of regularized image restoration to multiresolution spaces has been presented in [11]. The restored image $\hat{\mathbf{s}}$ is found by minimizing the following Lagrangian cost,

$$\min_{\hat{\mathbf{s}}} J_{(\lambda_0,\lambda_1\ldots\lambda_{D-1})}(\hat{\mathbf{s}}) = \tag{4}$$
$$= \min_{\hat{\mathbf{s}}} \left(\|\mathbf{y}-\mathbf{K}\hat{\mathbf{s}}\|^2 + \lambda_0 \left\|\mathbf{W}_0^{\mathrm{T}}\hat{\mathbf{s}}\right\|^2 + \ldots + \lambda_{D-1}\left\|\mathbf{W}_{D-1}^{\mathrm{T}}\hat{\mathbf{s}}\right\|^2 \right)$$

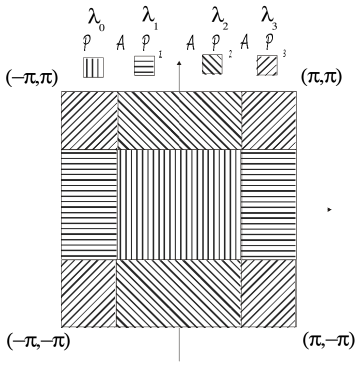
where λ_0, λ_1, … λ_{D-1} are the regularized restoration parameters and $\mathbf{W}_0^{\mathrm{T}}$, $\mathbf{W}_1^{\mathrm{T}}$ … $\mathbf{W}_{D-1}^{\mathrm{T}}$ are the matrices of spatial filtering with wavelets for the D distinct channels of image decomposition. They have dimensions $\frac{N^2}{D}$ rows by N^2 columns for N by N images. Should one denote the impulse responses of filters belonging to the filter bank as



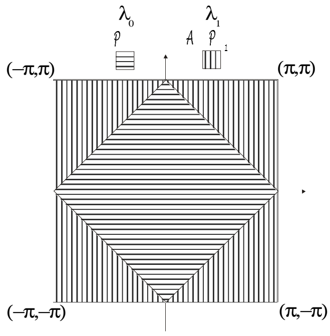
a: Sampling lattice $\Lambda_\mathbf{D}$ with $\mathbf{D}=\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}$.



b: Sampling lattice $\Lambda_\mathbf{D}$ with $\mathbf{D}=\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$ (quincunx sublattice)



c: Regularized restoration parameters and corresponding frequency regions for $\mathbf{D}=\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}$.



d: Regularized restoration parameters and corresponding frequency regions for $\mathbf{D}=\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$.

Figure 2: Sampling matrices and corresponding regularized restoration parameters

$h_0(m_1,m_2)$, $h_1(m_1,m_2)$ … $h_{D-1}(m_1,m_2)$, matrix $\mathbf{W}_j^\mathrm{T}$ may be expressed similar to $\mathbf{K}$, Eq. (2), as,

$$\mathbf{W}_j^\mathrm{T}=\begin{pmatrix} h_j(0,0) & h_j(0,-1) & \cdots & h_j(0,1) & h_j(-1,0) & h_j(-1,-1) & h_j(-1,-2) & \cdots \\ h_j(0,1) & h_j(0,0) & & h_j(0,2) & h_j(-1,1) & h_j(-1,0) & h_j(-1,-1) & \cdots \\ \vdots & & \ddots & & \vdots & & & \\ h_j(1,0) & h_j(1,-1) & & h_j(1,1) & h_j(0,0) & h_j(0,-1) & h_j(0,-2) & \\ \vdots & & & & \vdots & & & \ddots \end{pmatrix} \qquad (5)$$

Matrices $\mathbf{W}_0^\mathrm{T}$, $\mathbf{W}_1^\mathrm{T}$ … $\mathbf{W}_{D-1}^\mathrm{T}$ obey the orthonormality conditions by construction, i.e.

$$\begin{gathered}\mathbf{W}_0\mathbf{W}_0^\mathrm{T}+\mathbf{W}_1\mathbf{W}_1^\mathrm{T}+\ldots+\mathbf{W}_{D-1}\mathbf{W}_{D-1}^\mathrm{T}=\mathbf{I} \\ \mathbf{W}_p^\mathrm{T}\mathbf{W}_q=\mathbf{0} \text{ if } p\neq q \text{ or } \mathbf{W}_p^\mathrm{T}\mathbf{W}_q=\mathbf{I} \text{ if } p=q.\end{gathered} \qquad (6)$$

An alternative representation of the filter bank is through its polyphase matrix $\mathbf{H}_p$ which is comprised by the decimated components of the filters belonging to the filter bank,

$$\mathbf{H}_p(e^{j\omega})=\begin{pmatrix} h_{0,0}(c) & h_{0,1}(e^{j\omega}) & \cdots & h_{0,N-1}(e^{j\omega}) \\ h_{1,0}(e^{j\omega}) & h_{1,1}(e^{j\omega}) & & h_{1,N-1}(e^{j\omega}) \\ \vdots & & \ddots & \\ h_{N-1,0}(e^{j\omega}) & h_{N-1,1}(e^{j\omega}) & & h_{N-1,N-1}(e^{j\omega}) \end{pmatrix} \qquad (7)$$

The decimated components of filter h_j are denoted as $h_{j,0}$, $h_{j,1}$ … $h_{j,N-1}$. Decimation is carried out upon lattices $\Lambda_\mathbf{D}$ and their displacements by the corresponding cosset vectors $\mathbf{I}_0$, $\mathbf{I}_1$ … etc. Such lattices are presented in Figure **2**. Perfect reconstruction filter banks of orthonormal wavelet filters satisfy

$$\tilde{\mathbf{H}}_p(e^{j\omega})\mathbf{H}_p(e^{j\omega})=\mathbf{I}, \qquad (8)$$

where $\tilde{\mathbf{H}}_p(e^{j\omega})$ is the conjugent transpose matrix of $\mathbf{H}_p(e^{j\omega})$.

Proposition: The equation of the regularized restoration takes the form

$$\left(\mathbf{K}^\mathrm{T}\mathbf{K}+\lambda_0\mathbf{W}_0\mathbf{W}_0^\mathrm{T}+\ldots+\lambda_{D-1}\mathbf{W}_{D-1}\mathbf{W}_{D-1}^\mathrm{T}\right)\hat{\mathbf{s}}=\mathbf{K}^\mathrm{T}\mathbf{y}. \qquad (9)$$

Proof: We take the derivative $\dfrac{\partial J_{(\lambda_0,\lambda_1\ldots\lambda_{D-1})}}{\partial\hat{\mathbf{s}}}=\mathbf{0}$ or else

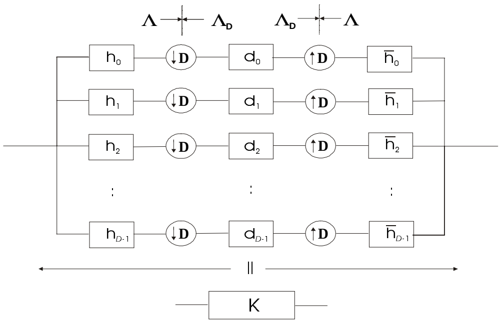


Figure 3: Decomposition of the degradation matrix by a perfect reconstruction wavelet filter bank

$$-2\mathbf{K}^{\mathrm{T}}(\mathbf{y}-\mathbf{K}\hat{\mathbf{s}})+2\lambda_0\mathbf{W}_0\mathbf{W}_0^{\mathrm{T}}\hat{\mathbf{s}}+\ldots+2\lambda_{D-1}\mathbf{W}_{D-1}\mathbf{W}_{D-1}^{\mathrm{T}}\hat{\mathbf{s}}=\mathbf{0} \quad (10)$$

where $\mathbf{K}^{\mathrm{T}}$ denotes the transpose degradation matrix. Eq. (10) results finally in Eq. (9). The solution of Eq. (9) yields the restored image □

Further simplification of this relationship is possible by making certain assumptions regarding the degradation matrix **K**. The iterative solution of Eq. (9) independently in each decomposition channel is proposed in the sequel should the degradation matrix be decomposed by a perfect reconstruction wavelet filter bank.

2.2 Representation of the degradation matrix by a wavelet filter-bank

Let matrix **K** be decomposed as in Figure 3,

$$\begin{aligned}\mathbf{K}=&\sum_{\mathbf{m}'\in\Lambda_{\mathbf{D}}}\mathbf{W}_0\, d_0(\mathbf{m}')\mathbf{P}_{\mathbf{m}'}\mathbf{W}_0^{\mathrm{T}}+\ldots\\ &\ldots+\sum_{\mathbf{m}'\in\Lambda_{\mathbf{D}}}\mathbf{W}_{D-1}d_{D-1}(\mathbf{m}')\mathbf{P}_{\mathbf{m}'}\mathbf{W}_{D-1}^{\mathrm{T}}\end{aligned} \quad (11)$$

where $\mathbf{P}_{\Delta\mathbf{m}}$ is a permutation matrix whose elements take the values of zero and one and shifts the 2-D channel degradation filters $d_0(\mathbf{m}')$, $d_1(\mathbf{m}')$ … $d_{D-1}(\mathbf{m}')$ in Eq. (11) by $\Delta\mathbf{m}$. Permutation matrices allow for expressing convolution as a sum of matrix products.

Proposition: Should the degradation matrix **K** be decomposed as in Eq. (11), the regularized restoration equation, Eq. (9), is split into D independent equations one for each decomposition channel,

$$\begin{aligned}(\mathbf{D}_{c,0}+\lambda_0\mathbf{I})\mathbf{W}_0^{\mathrm{T}}\hat{\mathbf{s}}&=\mathbf{D}_0^{'}\mathbf{W}_0^{\mathrm{T}}\mathbf{y}\\ (\mathbf{D}_{c,1}+\lambda_1\mathbf{I})\mathbf{W}_1^{\mathrm{T}}\hat{\mathbf{s}}&=\mathbf{D}_1^{'}\mathbf{W}_1^{\mathrm{T}}\mathbf{y}\\ &\vdots\\ (\mathbf{D}_{c,D-1}+\lambda_{D-1}\mathbf{I})\mathbf{W}_{D-1}^{\mathrm{T}}\hat{\mathbf{s}}&=\mathbf{D}_{D-1}^{'}\mathbf{W}_{D-1}^{\mathrm{T}}\mathbf{y}\end{aligned}\ . \quad (12)$$

Matrices $\mathbf{D}_{c,0}$, $\mathbf{D}_{c,1}$…$\mathbf{D}_{c,D-1}$ are used to convolve, upon lattice $\Lambda_{\mathbf{D}}$, the 2-D series $d_{c,0}(\mathbf{m}')$, $d_{c,1}(\mathbf{m}')$, … $d_{c,D-1}(\mathbf{m}')$, (which are defined as $d_{c,0}=d_0*\tilde{d}_0$, $d_{c,1}=d_1*\tilde{d}_1$, … $d_{c,D-1}=d_{D-1}*\tilde{d}_{D-1}$) with the wavelet

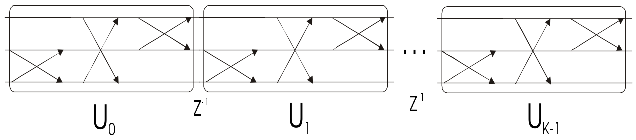


Figure 4: Factorization of the polyphase matrix of a filter bank with unitary matrices and Givens rotations

coefficients of the restored image, $\mathbf{W}_q^{\mathrm{T}}\hat{\mathbf{s}}$. Channel filters $\tilde{d}_0$, $\tilde{d}_1$ … $\tilde{d}_{D-1}$ are the reconstruction filters defined as $\tilde{d}_0(\mathbf{m}')=d_0(-\mathbf{m}')$, $\tilde{d}_1(\mathbf{m}')=d_1(-\mathbf{m}')$, … $\tilde{d}_{D-1}(\mathbf{m}')=d_{D-1}(-\mathbf{m}')$, $\mathbf{m}'\in\Lambda_D$. Convolution matrices $\mathbf{D}_0^{'}$, $\mathbf{D}_1^{'}$ … $\mathbf{D}_{D-1}^{'}$ convolve the channel reconstruction filters with the wavelet coefficients of the degraded image, $\mathbf{W}_q^{\mathrm{T}}\mathbf{y}$, upon $\Lambda_{\mathbf{D}}$.

Proof: One gets the following results starting from Eq. (11) and taking into account that $\mathbf{P}_{\Delta\mathbf{m}''}^{\mathrm{T}}\mathbf{P}_{\Delta\mathbf{m}'}=\mathbf{P}_{\Delta\mathbf{m}'-\Delta\mathbf{m}''}$ due to the construction of the permutation matrix $\mathbf{P}_{\Delta\mathbf{m}}$,

$$\begin{aligned}\mathbf{K}^{\mathrm{T}}&=\sum_{\mathbf{m}'\in\Lambda_{\mathbf{D}}}\mathbf{W}_0\, d_0(-\mathbf{m}')\mathbf{P}_{\mathbf{m}'}\mathbf{W}_0^{\mathrm{T}}+\ldots+\sum_{\mathbf{m}'\in\Lambda_{\mathbf{D}}}\mathbf{W}_{D-1}d_{D-1}(-\mathbf{m}')\mathbf{P}_{\mathbf{m}'}\mathbf{W}_{D-1}^{\mathrm{T}}\\ \mathbf{K}^{\mathrm{T}}\mathbf{K}&=\sum_{\mathbf{m}'\in\Lambda_{\mathbf{D}}}\mathbf{W}_0\sum_{\mathbf{m}''\in\Lambda_{\mathbf{D}}}\left(d_0(\mathbf{m}'')d_0(\mathbf{m}''-\mathbf{m}')\right)\mathbf{P}_{\mathbf{m}'}\mathbf{W}_0^{\mathrm{T}}+\ldots\\ &\quad+\sum_{\mathbf{m}'\in\Lambda_{\mathbf{D}}}\mathbf{W}_{D-1}\sum_{\mathbf{m}''\in\Lambda_{\mathbf{D}}}\left(d_{D-1}(\mathbf{m}'')d_{D-1}(\mathbf{m}''-\mathbf{m}')\right)\mathbf{P}_{\mathbf{m}'}\mathbf{W}_{D-1}^{\mathrm{T}}\end{aligned} \quad (13)$$

Eqs. (12) are obtained by substitution of Eqs. (13) into Eq. (9), multiplication on the left with wavelet matrices $\mathbf{W}_j^{\mathrm{T}}$ and using the orthonormality conditions in Eqs. (6). □

It is is not difficult to satisfy the assumption that the degradation matrix **K** is decomposed as in Eq. (11) since there are several factorizations of the polyphase matrix $\mathbf{H}_p(\boldsymbol{\omega})$ of a perfect reconstruction filter bank. It may be factorized by unitary matrices $\mathbf{U}_{n,m}$ as shown in Figure 4, [2],

$$\mathbf{H}_p(\boldsymbol{\omega})=\left(\prod_{n=1}^{N-1}\prod_{m=1}^{2}\mathbf{U}_{n,m}\mathbf{D}_m(e^{j\omega_m})\right)\mathbf{U}_0 \quad (14)$$

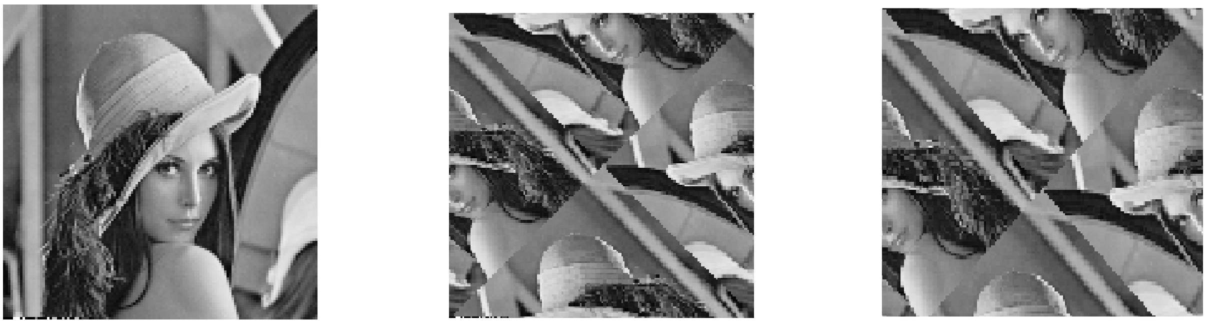
where $\mathbf{D}_m(e^{j\omega m})=\mathrm{diag}\{1\ 1\ \ldots\ 1\ e^{-j\omega m}\}$. Unitary matrices are expressed by Givens rotations (Ref. [12], p. 469) as

$$\mathbf{U}=\mathbf{G}_{1,1}\ldots\mathbf{G}_{1,D}\mathbf{G}_{2,3}\ldots\mathbf{G}_{D-1,D} \quad (15)$$

where

$$\mathbf{G}_{p,q}=\begin{pmatrix}1&&&&\\&\cos(\theta)&\cdots&-\sin(\theta)&\\&\vdots&1&\vdots&\\&\sin(\theta)&\cdots&\cos(\theta)&\\&&&&1\end{pmatrix} \quad (16)$$

and p, q denote the row and column of trigonometric

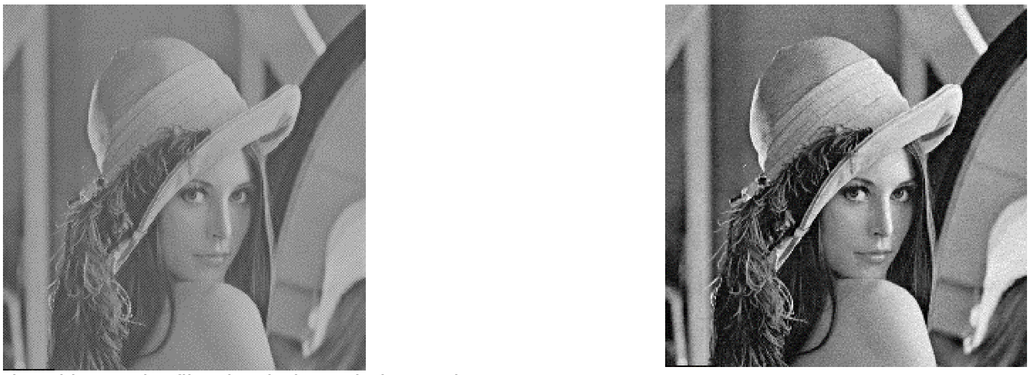


a: Original image

b: First polyphase component corresponding to coset vector $(0,0)^T$

c: Second polyphase component corresponding to coset vector $(1,0)^T$

Figure 5: Original and polyphase components of "Lena" for sampling upon the quincunx sublattice



a: Degradated image by filter bank degradation and additive noise

b: Iterative restored image with two regularization parameters (β_0=3 and β_1=2)

Figure 6: Degraded and restored image of "Lena"

elements. This allows for variable degradations to be decomposed by a perfect reconstruction filter bank like the one shown in Figure **2**.

3 Iterative solution of the regularization equation

The iterative solution of Eqs. (12) is proposed as

$$\begin{aligned}
\mathbf{W}_0^{\mathrm{T}}\hat{\mathbf{s}}(0) &= \beta_0\mathbf{D}_0^{'}\mathbf{W}_0^{\mathrm{T}}\mathbf{y} \\
\mathbf{W}_0^{\mathrm{T}}\hat{\mathbf{s}}(t+1) &= (\mathbf{I}-\beta_0\lambda_0)\mathbf{W}_0^{\mathrm{T}}\hat{\mathbf{s}}(t)+\beta_0\left(\mathbf{D}_0^{'}\mathbf{W}_0^{\mathrm{T}}\mathbf{y}-\mathbf{D}_{c,0}\mathbf{W}_0^{\mathrm{T}}\hat{\mathbf{s}}(t)\right) \\
&\vdots \\
\mathbf{W}_{D-1}^{\mathrm{T}}\hat{\mathbf{s}}(0) &= \beta_{D-1}\mathbf{D}_{D-1}^{'}\mathbf{W}_{D-1}^{\mathrm{T}}\mathbf{y} \\
\mathbf{W}_{D-1}^{\mathrm{T}}\hat{\mathbf{s}}(t+1) &= (\mathbf{I}-\beta_{D-1}\lambda_{D-1})\mathbf{W}_{D-1}^{\mathrm{T}}\hat{\mathbf{s}}(t)+ \\
&\quad +\beta_{D-1}\left(\mathbf{D}_{D-1}^{'}\mathbf{W}_{D-1}^{\mathrm{T}}\mathbf{y}-\mathbf{D}_{c,D-1}\mathbf{W}_{D-1}^{\mathrm{T}}\hat{\mathbf{s}}(t)\right)
\end{aligned} \tag{17}$$

This is an extension of the general iterative solution of the regularized restoration equation presented in [13],[14],[15]. Regularization parameters are evaluated iteratively according to the relationship,

$$\lambda_q(t) = \frac{\varepsilon^2}{\left\|\mathbf{W}_q^{\mathrm{T}}\hat{\mathbf{s}}(t)\right\|^2+\delta_q} \tag{18}$$

where ε^2 equals $\mathbf{n}^{\mathrm{T}}\mathbf{n}$ and δ_q is a small positive constant. It is necessary for convergence that the iteration parameter β_q, $q \in \{0,1 \ldots D-1\}$, should satisfy the following equation

$$0 < \beta_q < \frac{2}{\max_i\left(\lambda_q\mathbf{I}+\mathbf{D}_{c,q}\right)} \tag{19}$$

where $\max_i\left(\lambda_q\mathbf{I}+\mathbf{D}_{c,q}\right)$ stands for the maximum eigenvalue of matrix $\left(\lambda_q\mathbf{I}+\mathbf{D}_{c,q}\right)$. Thus iterative parameters β_q are channel dependent.

4 Experimental results

The degradation is assumed to be decomposed by a perfect reconstruction filter bank upon the quincunx sampling lattice with sampling matrix $\mathbf{D}=\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$. The following factorization holds,

$$\mathbf{K}_p(\boldsymbol{\omega}) = \tilde{\mathbf{H}}_p(\boldsymbol{\omega}) diag\{d_0, d_1\} \mathbf{H}_p(\boldsymbol{\omega}), \qquad \text{where}$$

$$\tilde{\mathbf{H}}_p(\boldsymbol{\omega}) = \begin{pmatrix} \cos(\theta_0) & \sin(\theta_0) \\ -\sin(\theta_0) & \cos(\theta_0) \end{pmatrix} \left(\prod_{n=1}^{N-2} \prod_{m=1}^{2} \begin{pmatrix} 1 & 0 \\ 0 & e^{j\omega_m} \end{pmatrix} \begin{pmatrix} \cos(\theta_{n,m}) & \sin(\theta_{n,m}) \\ -\sin(\theta_{n,m}) & \cos(\theta_{n,m}) \end{pmatrix} \right) \tag{20}$$

and $\mathbf{H}_p(\boldsymbol{\omega})$ is the polyphase matrix of the filter bank. The factorization parameters are presented in Table 1. This is a degradation which affects the high spatial frequencies of the image more than its low spatial frequencies. Usual linear blur may be approximated by a matrix product like Eq. (20). The original image of "Lena" along with its two polyphase components is presented in Figure 5.

Table 1: Filter bank factorizations according to Eq. (20) for the degradation

Degradation parameters							
d_0	θ_0	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	$\theta_{4,1}$	$\theta_{5,1}$	$\theta_{6,1}$
0.65	-60°	55.5°	42.5°	-20.5°	4.5°	4°	1°
d_1		$\theta_{1,2}$	$\theta_{2,2}$	$\theta_{3,2}$	$\theta_{4,2}$	$\theta_{5,2}$	$\theta_{6,2}$
0.30		-26.5°	-71°	162.5 °	6°	1°	4 °

The linear degradation model of Eq. (1), which assumes additive white noise, is adopted. The standard deviation of additive noise is σ=0.01. The degraded image **y** is shown in Figure 6a. An estimate of the s.t.d. of the noise is obtained from the flat regions of the corrupted image. This estimate is used in evaluating ε^2 in Eq. (18) which yields λ(t) vs iteration. Iterative regularized restoration is carried out as described by Eq. (8) (see Figure 1) for various iterative parameters β_0 and β_1. The algorith is terminated if $\frac{\|\hat{\mathbf{s}}(t+1)-\hat{\mathbf{s}}(t)\|^2}{\|\mathbf{s}(t)\|^2} \le 10^{-6}$. The results are compared against standard iterative solutions of the regularized restoration equation that appear in the literature. The Laplacian is used as the smoothing filter in conventional methods. Table 2 gives the values of the parameters of the iteration β as well as the final improvement in Signal-to-Noise-Ratio (ISNR) defined as $10\log(\frac{\|\mathbf{y}-\mathbf{s}\|^2}{\|\mathbf{y}-\hat{\mathbf{s}}\|^2})$. Iterative regularized image restoration in multiresolution channels yields the same final ISNR for two different pairs of iterative parameters β_0 and β_1. This value is better than the ISNR value obtained for conventional iterative regularized restoration. The restored image of "Lena" with β_0=3 and β_1=2 is shown in Figure 6b. Convergence rates differ in each case. The evolution of the values of the regularization parameters obtained from Eqs. (18) are presented in Figure 8 for both multiresolution channels, while for the Laplacian operator in Figure 7.

5 Conclusion

A novel approach of iterative regularized image restoration in multiresolution channels is proposed. A perfect reconstruction filter bank which is defined upon arbitrarily sampling lattices decomposes the degradation filter as well as the smoothing filter into independent wavelet channels. The corresponding systems of regularization equations are solved iteratively. The regularization parameters are evaluated at each iteration step. Restoration results obtained with the proposed method are better than results obtained with conventional regularization methods.

Table 2: Iteration parameters and final improvements in Signal-to-Noise-Ratio (ISNR) after convergence

Case	Laplacian	Filter bank	
Iteration parameters	β=0.5 δ=0.01	β_0=0.5 β_1=1.0 δ_0=0.1 δ_1=0.1	β_0=3.0 β_1=2.0 δ_0=0.1 δ_1=0.1
Number of iterations	33	45	25
ISNR [dB]	9.3	12.4	12.4

6 References

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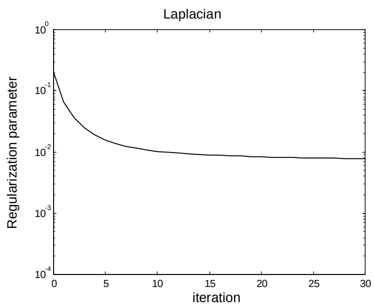
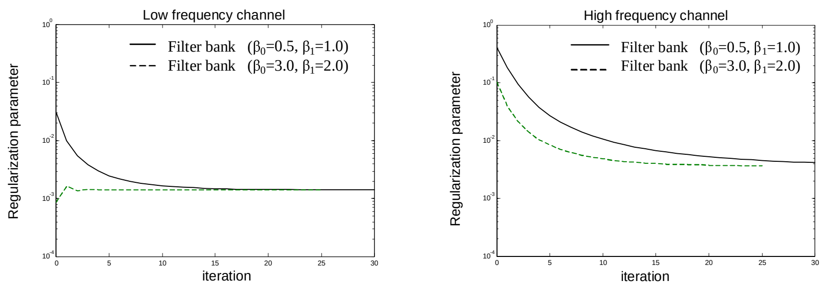


Figure 7: Evolution of regularization parameter for iterative restoration using the Laplacian



a: Low frequency channel

b: High frequency channel

Figure 8: Evolution of regularization parameters for iterative restoration using the wavelet filter bank