Cyprus University of Technology | Faculty of Management and Economics

Doctoral Dissertation

Essays on Behavioural Finance and Financial Econometrics

Polina Ellina

Limassol, April 2021

CYPRUS UNIVERSITY OF TECHNOLOGY

FACULTY MANAGEMENT AND ECONOMICS

DEPARTMENT COMMERCE, FINANCE AND SHIPPING

Doctoral Dissertation

Essays on Behavioural Finance and Financial Econometrics

Polina Ellina

Limassol, April 2021

Approval Form

Doctoral Dissertation

Essays on Behavioural Finance and Financial Econometrics

Presented by

Polina Ellina

Supervisor:

Signature ____

Member of the committee:

Signature ____

Member of the committee:

Signature ____

Cyprus University of Technology

April 2021

Copyrights

The approval of the dissertation by the Department of Commerce, Finance, and Shipping does not imply necessarily the approval by the Department of the views of the writer.

Acknowledgements

This PhD dissertation is the result of the effort and support of many people to whom I am very grateful.

First and foremost, I would like to express my sincere gratitude to my supervisor, Professor Panayiotis Theodossiou for the continuous support and his insightful feedback through this dissertation. His guidance, patience, and motivation helped me to accomplish this long-awaited goal and complete my dissertation. It was an honour for me having him as mentor since he has shared exceptional scientific knowledge through our discussions.

Besides my advisor, I would also like to thank Dr Christos Savva, Dr Panayiotis Andreou, and Dr Demetris Koursaros for their insightful knowledge, advice, and discussions that contributed to the accomplishment of this dissertation. I would like to deeply thank Professor Kyriaki Kosmidou and Professor Neophytos Lambertides for their helpful comments and remarks. Also, my sincere thanks go to the Faculty of the Department of Commerce, Finance, and Shipping at the Cyprus University of Technology for their ongoing support through these years. I am very grateful to all of you. I would also like to thank my friend Andreas Kouspos, Mrs Demetra Christodoulidou and Mrs Marianna Loizidou for their invaluable support.

Last, but not least, I would like to thank my family and my husband for their continuous encouragement. This dissertation is dedicated to you, to whom I owe my deep gratitude.

ABSTRACT

In the academic literature, financial data have been proved to violate the assumption of Normality. For this reason, skewness, and/or kurtosis distributional characteristics are presented in financial series. At the same time, this highlights the importance of using higher moment distributions that take into account these characteristics. Using mathematical and advanced statistical probability theory, this dissertation contributes to the literature by conducting three chapters in which the purpose is to explain and develop models to explore the following financial topics: 1. behavioural finance and more specifically the probabilistic behaviour of psychological biases, 2. the stochastic behaviour of Bitcoin using an asymmetric framework, and 3. the measurement of stock price crash risk using an outlier resistant technique.

The first chapter presents a probabilistic framework to define and analyse the well-known psychological biases of overconfidence, optimism, underconfidence, and pessimism on the perceptions of managers about the mean and risk (overall risk, downside risk, value-at-risk, and expected shortfall) of the economic variables under consideration. Furthermore, the anchoring and adjustment heuristic has been found in the literature to be one of the reasons that overconfidence bias exhibits. This first chapter further investigates the interrelationship between anchoring and overconfidence bias using an adaptation process. Using an analytical generalized two-piece framework showed that anchoring and adjustment and overconfidence bias share an interconnection. The results reveal that overconfident and optimistic managers overestimate their expected value and underestimate their downside risk, value-at-risk and expected shortfall (positively skewed distribution). Overconfident managers also underestimate their overall risk. Underconfident and pessimistic managers underestimate their expected value and overestimate their risk (negatively skewed distribution). The overestimation or underestimation differs depending on the psychological bias. The empirical findings depict that the distribution of professional forecasters is negatively skewed and consequently they are underconfident. Accordingly, they underestimate the nominal and real GDP.

The second chapter analyses the stochastic behaviour of Bitcoin using an asymmetric framework. The extraordinary behaviour of Bitcoin is what makes it unique

and different. This chapter examines the stochastic behaviour of Bitcoin and exchange rates, the mean and volatility spillovers in the presence of asymmetry under a flexible general framework that accounts for skewness/kurtosis price of risk (ST-GJR-GARCH-SGED model), and the forecasting ability of the asymmetric model compared to already existing GARCH models under different probability distributions. The main empirical findings show that the skewness/kurtosis price of risk has an important role in the model. The empirical distribution of Bitcoin's returns exhibits skewness and extreme leptokurtosis. The latter result explains the extraordinary volatility of Bitcoin that leads to a higher peaked probability distribution compared to the rest of the assets. The findings also point out that there is a weak inter-relationship between Bitcoin and exchange rates and that the ST-GJR-GARCH-SGED model outperforms other GARCH specifications.

The third chapter focuses on the investigation of the stock price crashes using an outlier resistant method. Stock price crash risk referred to as the conditional skewness of the distribution of returns. When a negative firm-specific shock becomes public, there is a negative outlier in the return distribution leading to a crash. The residual returns have been taken by regressed the expanded market model since this model screens out the market crashes and only firm-specific events are considered. A binary crash risk measure is used to define crashes. Using the logarithmic transformation of the residual returns, a firm is considered to crash under the binary measure if at least one firm-specific weekly return is falling a threshold point of standard deviation below the mean firm-specific weekly returns. The presence of influential observations in the series may lead to a misestimation of the percentage of crashes due to the standard deviation that is inflated. This chapter develops a crash risk framework based on an outlier resistant method. Also, it proposes that a robust measure without the logarithmic transformation of the residual returns detects accurately the stock price crashes. Monte – Carlo simulations and empirical findings suggest that the robust method detects a higher percentage of crashes compared to the standard OLS methodology. Also, the detection based on the residual un-transform OLS returns leads to a lower percentage of crashes.

Keywords: Behavioural Finance, Psychological Biases, SGED, Conditional Asymmetry, Conditional Kurtosis, Crash Risk, Outliers, Outlier-resistant method.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

INTRODUCTION

Theoretical and empirical research in Finance and Economics has shown that financial and economic data do not follow a Gaussian (Normal) distribution, e.g. McDonald and Newey (1988), McDonald et al. (1995), Hansen et al. (2010), and others. The distributional characteristics of skewness and/or kurtosis are presented in financial series. Up to this time, there has been extensive literature that investigates the skewed behaviour of data using asymmetric models. Therefore, the need to explain financial data using models that account for more than two moments of a probability distribution drives researchers to use asymmetric probability distributions. Furthermore, the presence of outliers in the data drives researchers to use outlier-resistant methods.

The family of SGT distribution developed by Theodossiou (1998) nested by several well-known distributions, such as Skewed T (ST), Skewed Generalized Error Distribution (SGED), Skewed Normal (SN), Cauchy (C), Laplace (LP), Uniform and Normal (N). Each distribution has different moment functions, moment generating function, cumulative distribution, and moments. The SGT is a fifth parameter distribution, where, using the log-likelihood maximum likelihood technique gives the estimated parameters. These parameters are k, n, λ, μ, σ. The parameters k and n are the two parameters that control the tails and the peakedness of the distribution. The asymmetry parameter, λ, controls the shape of the probability distribution. If the asymmetry parameter is positive, it generates a positively skewed distribution and if it is negative, it generates a negatively skewed distribution. The expected value and standard deviation are the well-known measures used in finance in many cases, e.g. portfolio analysis, etc. Setting $k = 2$, $n = \infty$, and $\lambda = 0$ gives the Normal distribution, $k = 1$ and $n = \infty$ and $\lambda = 0$, the Laplace, and so on. Figure 0.1 presents the SGT family of distributions.

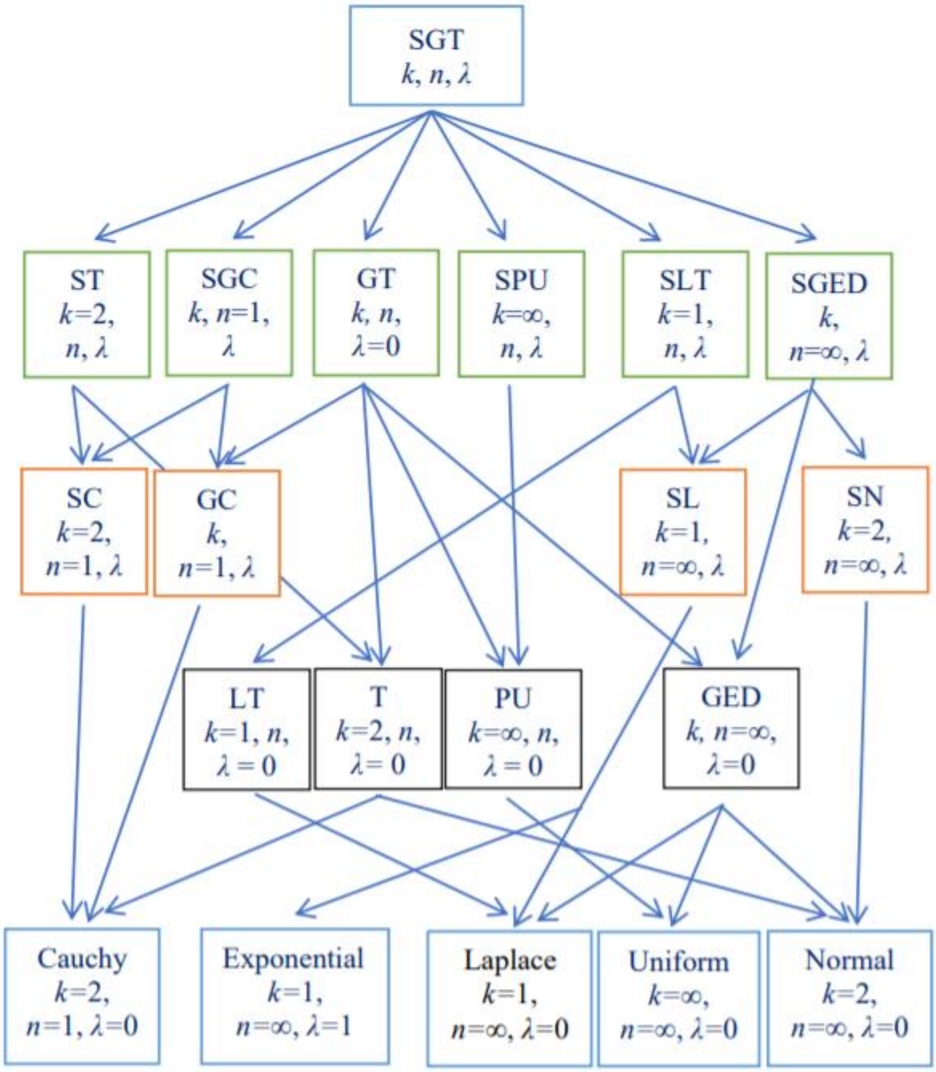


Figure 0.1. The SGT Family.[1]

Evidently, outliers are presented in financial data and in many cases, they include important information. For example, a negative outlier return of a firm may be the announcement of the CEO regarding an important decision of the firm. The standard ordinary least square technique (OLS) underperforms in the presence of outliers. This is the reason that there is a large body of literature about outlier-resistant methods. However, when there are no outliers in the return series, the outlier resistant method performs as well as the ordinary least square (OLS).

[1] This figure was taken from Theodossiou (2021).

In this dissertation, some of the skewed distributions and a robust resistant method are used to investigate three different financial issues: to explain the psychological biases using a probabilistic approach under the Skewed Normal (SN) and the Skewed Generalized Error (SGED) distributions, to explore the time-varying behaviour of Bitcoin and exchange rates using an asymmetric model under the SGED, and to identify the stock price crashes using a robust outlier technique.

The first chapter focuses on the development of a probabilistic framework to define and analyse the impact of the well-known psychological biases of overconfidence, optimism, underconfidence, and pessimism on the perceptions of managers about the expected value, overall risk, and the tail risks (downside risk, value-at-risk and expected shortfall) of economic variables. This framework has used the Skewed Normal distribution, which is an easily acceptable distribution in the literature, to explain the characteristics of each manager that suffer from psychological distortions. The rational view is assumed as followed by a Normal distribution. This framework further incorporates the anchoring and adjustment heuristic to explain the overconfidence bias. In other words, this study investigates the interrelationship between anchoring and overconfidence bias using an adaptation process. In this case, the framework relaxes the assumption of Normality and assumes the forecaster's errors followed by the Skewed Generalized Error Distribution (SGED).

The results reveal that overconfident (underconfident) managers overestimate (underestimate) their expected value and underestimate (overestimate) their risks. In this case, the probability distribution is positively (negatively) skewed. Similarly, optimistic (pessimistic) managers overestimate (underestimate) the performance of an economic variable, therefore, their probability distribution is positively (negatively) skewed. The overestimation of optimistic managers is larger compared to overconfident managers. The tail risk measures (downside risk, value-at-risk and expected shortfall) are underestimated by overconfident and optimistic managers and overestimated by their counterparts. It has also been proven that the subjective forecasting beliefs under a speed of adjustment parameter can explain an over/under confident behaviour. The forecasting errors yield to a tight (heavy) and skewed distribution for the overconfident (underconfident) experts. Empirically, the results show that professional forecasters underestimate the economic variable. In this case, the distribution of professional forecasters is negatively skewed.

The second chapter focuses on the investigation of the stochastic properties of Bitcoin and compares it with major exchange rates: Euro, Japanese Yen, Canadian Dollar, and British Pound. This investigation has been conducted using a flexible framework that accounts for the skewness and kurtosis price of risk. Using the ST-GJR-GARCH-M model under the SGED distribution, the risk-return relationship is investigated. The examination of this relationship comes since Bitcoin behaves uniquely and extraordinarily. Also, this chapter examines the presence of conditional heteroscedasticity, asymmetric volatility, and other dependencies in the return series. The link of the time-varying skewness and kurtosis price of risk to downside and upside volatility under ST-GJR-GARCH-SGED framework (Savva and Theodossiou, 2018) helps to investigate the mean and risk effects of Bitcoin and other exchange rates and accounts for the impact of the downside and upside risk effects, along with distributional effects due to the heavy tails' characteristics of such series. At the same time, an investigation regarding the shocks' correlation of Bitcoin and exchange rates showed the issue of the low correlation. Furthermore, by examining the behaviour of Bitcoin, this study sheds light on the trading and hedging capabilities helping investors to decide whether to incorporate it or not in their portfolios. Additionally, the model is extended to examine the bivariate behaviour of Bitcoin and exchange rates (spillover effects). Lastly, the forecasting ability of this model is compared to other existing GARCH specification models under different probability distributions.

The main findings illustrate that the empirical distribution of Bitcoin's returns exhibits skewness and extreme leptokurtosis. Skewness and kurtosis have also been found in all the series; however, the kurtosis of Bitcoin is about 2.5 times higher than the other assets. This result explains the extraordinary volatility of Bitcoin that leads to a higher peaked probability distribution compared to the rest of the assets. These findings suggest that skewness and kurtosis characteristics play an important role in the model. The implications are important for those who use Bitcoin as a financial asset.

Regarding the mean and volatility spillover effects, there is a negligible relationship between Bitcoin and exchange rates, and it is a useful asset to diversify the portfolio's risk since it behaves in a very different way relative to the other assets. Additionally, Bitcoin's behaviour is extremely leptokurtic when compared to other assets. The shape distributional characteristic of Bitcoin is not affected when spillover effects are presented. Interestingly, the findings show that the model ST-GJR-GARCH under the

Skewed Generalized Error Distribution (SGED) performs better than the rest models highlighting the importance of the Skewed Generalized Error distribution to forecast Bitcoin prices as this distribution captures data with leptokurtic characteristics well.

The third chapter focuses on the investigation of the measurement of crash risk as a negative outlier event using a robust-outlier technique. Crash risk is a new area in finance that academic literature has yet to investigate fully. Evidently, it captures asymmetry and it is the conditional skewness of the distribution of returns. When a negative firm-specific shock becomes public, there is a negative outlier in the return distribution leading to a crash. The presence of outliers in the series leads to a misestimation of the standard deviation that was used to construct the binary crash risk measure. This chapter develops a crash risk framework using a model developed by Theodossiou and Theodossiou (2019). The model presents the analytical equations of the mean, variance, and residual returns, shows the impact of outliers to identify a crash event and proposes a robust measure to estimate crashes.

The binary crash risk measure is constructed by taking the logarithmic transformation plus one of the residual returns from the expanded market index model (Dimson, 1979). The reason why the literature has used this model was to screen out the market shocks and consider only firm-specific events. By using the logarithmic transformation of the residual returns, a firm was found to crash under the binary measure if at least one firm-specific weekly return fell a threshold point of standard deviation below the mean firm-specific weekly returns.

Monte – Carlo simulations and empirical findings show that the standard methodology (OLS) detects a lower percentage of crashes in relation to the robust methodology. This is due to the outliers that affect the variance equation. Moreover, the un-transformed OLS measure detects a lower percentage of crashes compared to the log transformed measure. Importantly, 6,145 firm-year observations are detected in the robust measure and not in the standard logarithmic OLS method. Also, 10,934 firm-year observations are detected using the robust un-transform residual returns and not in the un-transform residual return. A case study of firms in different industries has shown the beneficial use of the robust method in comparison to the OLS.

The findings suggest that to avoid the misspecification of crashes it is better to use a robust technique that corrects the inflation of standard deviation driven by outliers. The pricing and forecasting of stock price crash risk are important for decision-makers and risk management.

To sum up, this dissertation develops three different mathematical and probabilistic frameworks to explore three financial puzzles related to behavioural finance and financial econometrics. Specifically, three frameworks are developed using skewed probability distributions, GARCH models, and robust statistics. All the models are also tested using Monte-Carlo simulations and empirical analysis. The conclusions provide insightful findings on the importance of asymmetric models to investigate different issues in the financial literature.

1 Psychological Biases and Skewness

1.1 Introduction

In the last years, there has been a growing interest in a subfield of finance, behavioural finance. Using psychological theories, many researchers try to understand stock market anomalies, the decisions of managers, and so on. The rejection of the rational expectation hypothesis claims that the decisions of people are based on psychological biases. Some well-known psychological biases are overconfidence and optimism while one of the three heuristics is anchoring and adjustment. These psychological distortions are under investigation in the academic literature since managers, investors, and others are not fully rational in their behaviour.

Statman (2017) in his book ‘Finance for Normal People: How Investors and Markets Behave’ describes extensively the behaviour of people using emotions. He suggests that many people do not react in a rational manner and he describes their behaviour using cognitive and emotional errors.

Moreover, the best seller book by Kahneman (2011) titled ‘Thinking, Fast and Slow’ describes in detail the reactions of people based on various distortions such as representativeness, overconfidence, etc. Interestingly, this book develops and explains theories in a simple way that researchers used to clarify several financial phenomena in their empirical studies. In part II, it analyses in an extensive way the heuristics and biases using psychological literature, while, in part III Kahneman explains the overconfidence and optimism biases and the consequences of having these characteristics in one’s behaviour.

Hubris and miscalibration theories are commonly used to explain the overconfidence bias. Hubris occurs when people overestimate the probability of desirable outcomes, e.g., Roll (1986). On the other hand, calibration measures the accuracy of predicted probabilities. Literature concluded that the perceived probability distribution of overconfident people has been characterized as too tight, e.g. Alpert and Raiffa (1982) and Kyle and Wang (1997). This means that overconfident managers tend to overestimate the probabilities of favourable events and underestimate their range while underconfident managers tend to overestimate the probability of unfavourable events and overestimate their range.

Gibran’s (1951) analogy highlights that optimists see the rose and pessimists see the thorns of the rose. A second analogy often used is that of a half-full glass representing optimism or a half-empty glass which stands for pessimism. Furthermore, Taylor and Brown (1988) mention that optimists believe that the future will be great for them. They also feel more capable, skilled, knowledgeable than their peers and they underestimate a negative event to happen to them.

As mentioned above, one of the three heuristics that Tversky and Kahneman (1974) presented in their paper was anchoring and adjustment. This heuristic concerns the way how people estimate their choices based on their beliefs and make decisions that deviate from the rational choice theory. It is a cognitive bias that distorts people’s decisions in a lot of ways. The literature has found that anchoring may reinforce other biases such as overconfidence (Russo and Schoemaker, 1992; Kahneman and Tversky, 1979).

Despite the existing theoretical and empirical literature on psychological biases and heuristics, this first chapter focuses on an alternative investigation of these psychological distortions. An advanced statistical and probabilistic framework that accounts for skewness as an important parameter explains the over/under confident and optimism/pessimism behaviours. The findings depict that overconfident managers overestimate the probability of desirable events, overestimate their expected values and underestimate their risk. Furthermore, optimistic managers overestimate to a greater extent the mean of an economic variable. Monte-Carlo simulations confirmed the conclusions, while an empirical application showed that professional forecasters underestimate the true value of an economic variable.

Accordingly, the next section presents the literature review regarding these biases. Section 1.3 presents the probabilistic framework including the skewed normal and the skewed generalized error distributions. Section 1.4 expresses the probability distributions of the psychological biases. Furthermore, it analyses the perceptions of managers about mean and various risks of a random economic variable. Also, it shows the inter-relationship between anchoring and overconfidence bias. Section 1.5 presents Monte Carlo simulations to represent the psychological biases (overconfidence, optimism, underconfidence, pessimism) while section 1.6 presents the empirical findings supporting the statistical framework. Summary and conclusions are presented in section 1.7.

1.2 Literature Review

Overconfidence and Optimism Biases

Moore and Healy (2008) describe the three characteristics of overconfident people. These are: 1. overestimation of the performance, 2. over-placement of performance compared to others, and 3. excessive precision.

The first characteristic relates to the overestimation of one's actual performance. This characteristic occurs due to the planning fallacy and illusion of control. Planning fallacy was proposed by Kahneman and Tversky (1979) and it describes the underestimation of the time needed to complete a task. Therefore, individuals that suffer from this phenomenon tend to overestimate their ability to control and underestimate the time needed to complete a task, e.g., Langer (1975). The second characteristic, over-placement, is the belief of overconfident people that they are better than others. Svenson (1981) concluded that 93% of US drivers believe that they drive better than others. These drivers believe that they are more skilful, and they are at a lesser risk compared to the others. The third characteristic is over-precision. According to Moore and Healy (2008, p.4), this characteristic was empirically investigated using questionnaires. Participants were asked to estimate the 90% confidence intervals of their answers. They found that the estimated intervals are too narrow. This means that people believe that they answer correctly and consequently, they are too sure about themselves. Kyle and Wang (1997) explained the overconfidence bias using conceptual probabilistic statements. They stated that "a trader is overconfident if his distribution is too tight and underconfident if his distribution is too loose". This chapter contributes to the literature in the sense that it explains the psychological biases using an analytical statistical and probabilistic theory. What is more, Moore and Healy (2008) summarize the characteristics of overconfidence bias which makes up the basis to build a model and explain the biases using probability and statistical theory.

Overconfidence bias has also been explained using the theories of hubris and miscalibration, e.g., Oberlechner and Osler (2012) and Ben-David et al. (2013). Hubris occurs when people overestimate the probability of desirable outcomes, e.g., Roll (1986). For example, Camerer and Lovallo (1999) investigated the optimistic behaviour of managers regarding business failure. Calibration measures the accuracy of predicted probabilities. It is a probabilistic tool in decision making, e.g., Lichtenstein et al. (1982).

In the literature, the probability distribution of overconfident people has been characterized as too tight, e.g., Alpert and Raiffa (1982) and Lichtenstein et al. (1982) and that of underconfident people as too loose, e.g., Kyle and Wang (1997). In other words, overconfident agents tend to overestimate the probabilities of desirable events, underestimate the probabilities of undesirable events as well as underestimate their range. Underconfident agents tend to overestimate the probability of undesirable events, underestimate the probabilities of desirable events, and overestimate their range.

The behavioural corporate finance literature regarding managers is exhausting. There is a growing body of research suggesting that managers are often irrational, e.g., Baker et al. (2007) and Shefrin (2001, 2005). Baker et al. (2007) divide the literature into two sides, investors, and managers. In this chapter, the focus is on the second group that is, managers. Thusly, a framework is developed to explain the beliefs of overconfidence, optimism, and the well-known heuristic from Kahneman and Tversky (1979) which is anchoring and adjustment.

Kahneman, 2011 (chapter III, p. 248) stated that "Optimism is normal, but some fortunate people are more optimistic than the rest of us. If you are generally endowed with an optimistic bias, you hardly need to be told that you are a lucky person-you already feel fortunate". Empirical evidence supports that optimism bias plays a dominant role in decision-making under the conditions of expected value and risk.

The unrealistic optimism bias characterizes people who believe that negative (unfavourable) events are less likely to happen to them than to others, e.g., Weinstein (1980), Weinstein and Lachendro (1982), and Weinstein and Klein (1996). More specifically, Weinstein (1980, p. 806) stated that "according to popular belief, people tend to think they are invulnerable. They expect others to be victims of misfortune, not themselves. Such ideas imply not merely a hopeful outlook on life, but an error in judgment that can be labelled unrealistic optimism". Consequently, these individuals tend to underestimate the probability of undesirable events and overestimate that of desirable events.

Optimistic managers expect good rather than bad things to occur in their life, e.g., Kunda (1987). This way of thinking leads managers to believe that they are invulnerable and have unrealistically positive expectations. They generate a theory that follows their predictions of favourable outcomes, and they do not accept any other unfavourable outcome. They, therefore, underestimate the probability of failure, e.g., March and

Shapira (1987). Pessimistic managers have the opposite expectations and beliefs, e.g., Scheier and Carver (1985).

Several empirical papers in finance investigated overconfidence and optimism biases. For instance, Malmendier et al. (2011) and Malmendier and Tate (2015) investigated the explanatory ability of the overconfidence bias to firm finance decisions. Galasso and Simcoe (2011) showed that overconfidence bias is closely related to innovating decisions. They concluded that overconfident CEOs are more likely to make decisions about a new technologically innovative plan. Heaton (2002) focused his investigation on the implications of optimism bias on corporate finance decisions.

In summary, the literature alludes to two psychological biases: the overconfident and the optimistic and their counterparts – underconfident and pessimistic. However, more evidence is needed to distinguish them. To understand and compare the different psychological biases, it is essential to use a rational view that serves as a baseline. Rational individuals are assumed to have an unbiased view of the true distribution of economic variables under consideration.

Anchoring and Adjustment Heuristic

One of the three heuristics that Tversky and Kahneman (1974) presented in their paper was anchoring and adjustment. Anchoring and adjustment is a cognitive bias that affects people's beliefs, e.g. Davis et al. (1986). It occurs when people start their valuations using a starting point (e.g., median, mode) and then adjust their next valuation based on the previous value. By using an experiment, Tversky and Kahneman (1974), showed that future expectations are influenced by point estimates. Using an initial point (anchor), there are more possibilities for the next valuation to be closer to this anchor. Tversky and Kahneman (1974, p. 1129) stated that "subjects state overly *narrow* confidence intervals which reflect more *certainty* than is justified by their knowledge. This effect is attributable, in part at least, to anchoring". This anchor may also be the information that one has as memory/immediately available (see e.g., Tversky and Kahneman, 1973).

A lot of studies investigate whether anchoring and adjustment can explain the overconfidence bias (see e.g., Block and Harper, 1991). The overconfidence bias is the result of the unrealistic estimation of their actual ability. Over the years, the anchoring and adjustment heuristic and the overconfidence bias were examined statistically. For example, Lovie (1985) proved that the bias judgment on the first two moments of a probability distribution, that is, expected value and variance, was caused by anchoring and adjustment. Lichtenstein et al. (1982) noted that “When asked about an uncertain quantity, one naturally thinks first of a point estimate such as the median. This value then serves as an anchor. To give the 25th or 75th percentile, one adjusts downward or upward from the anchor. But the anchor has such a dominating influence that the adjustment is insufficient; hence the fractiles are too close together, yielding overconfidence.” The examination of these two cognitive biases (overconfidence and anchoring and adjustment) in terms of statistical analysis (e.g., confidence intervals, expected value, risk, etc) lead to further investigation of the interrelationship between them using advanced probability theory and statistics.

As aforementioned in the previous section, an overconfident agent overestimates the performance of the economic variable under consideration, see for example Svenson (1981), Roll (1986), and Moore and Healy (2008). Furthermore, their probability distribution can be characterized with tighter tails, e.g. Kyle and Wang (1997).

Anchoring and Adjustment, Overconfidence, and Professional Forecasters

The rational expectation hypothesis is tested in the case of forecasters, e.g. Holden and Peel (1990), Ehrbeck and Waldman (1996), Lovell (1986). The statistical properties of unbiasedness and efficiency are associated with rationality, e.g. Holden and Peel (1990), Muth (1961), and Neftci and Theodossiou (1991). When the predictions of professionals are far away from the true value, they make predictable errors, e.g. Baghestani and Kianian (1993). This is attributed, in many cases, due to overconfidence bias (Arkes, 2001). Deaves et al. (2010) found that forecasters are overconfident, and the correct prediction of the value lead them to increase their confidence. Overconfident forecasters overestimate the forecast value of the economic variable relative to the actual (positive forecast error) while underconfident forecasters underestimate the forecast value of the economic variable (negative forecast error). Forecasts are important to investors, policy makers, businesses, and others. For example, businesses are reluctant to make decisions when they believe that the economy will change, e.g. hire employees. This chapter will also focus on the investigation of the anchoring and adjustment heuristic in the case that agents are professional forecasters.

Bayesian forecasts are updated using subjective beliefs and the economic variable under consideration is, in many cases, far away from the actual value. This behaviour is closely related to psychological biases such as overconfidence. A ‘correct’ estimation of the economic variable is expected to yield forecasts close to the actual value. In other words, the forecasting errors should have zero mean.

Conservatism bias is a fallacy which claims that professional forecasters may be too conservative to adapt their beliefs as new information is available, e.g. Batchelor and Dua (1992). This means that professional forecasters narrow the confidence intervals around their forecasts leading to a psychological bias.

Moving on, various papers examined whether anchoring and adjustment can be explained by the overconfidence bias, e.g., Block and Harper (1991), Kahneman and Tversky (1979), Lichtenstein et al. (1982). Russo and Schoemaker (1992) have also pointed out that the existence of overconfidence bias is due to anchoring and adjustment. An overconfident forecaster has the illusion of control of everything (Langer, 1975).

People learn using their abilities and achievements. For example, most people tend to overestimate the probability of success because they have experienced previous success (Miller and Ross, 1975; Langer and Roth, 1975). Even if they fail, they attribute their failure to external factors and not to their inability (self – serving attribution bias). Heidhues et al. (2018) examined overconfidence bias using a learning process. By using a model, Gervais and Odean (2001) showed that traders tend to be overconfident since they learn using their abilities. Their abilities are based on their experience of success or failure (Chiang et al., 2011). If they have successful experiences, they become overconfident, while, if they have experiences of failure, they become underconfident.

The motivation behind this chapter is multi-fold. Firstly, it defines overconfidence and optimism biases using a skewed probabilistic framework and explains the differences between one another. What is more, this kind of analysis explains their counterparts – underconfidence and pessimism. Secondly, it examines these biases and the interaction between them. More specifically, the psychological biases are expressed graphically to enhance understanding through visualization. Thirdly, it explores the mean and risk (overall risk, downside risk, value-at-risk and expected shortfall) on the perception of managers of an economic variable under consideration. Lastly, it examines the impact of over/under confidence as well as anchoring and adjustment biases on professional forecasters valuations.

1.3 Probabilistic Framework

This section develops a two-piece probabilistic framework based on the skewed normal distribution (SN) and the skewed generalized error distribution (SGED) to model and explores the impact of behavioural biases, such as overconfidence, unrealistic optimism and anchoring and adjustment, on professional agents' perceptions about the mean, the variance and other risk measures of important economic variables. Such variables may be the return of an investment, the future cash flows, and the nominal and real gross domestic product (GDP). This section presents a two-piece generalized distribution of the agent's expectations about a random variable accounting for downside and upside uncertainty and derives the first two moments to investigate the probabilistic beliefs about their mean and risk perceptions. The Skewed Normal (SN) distribution will use to model the managers' perceptions about the mean and risk measures while the Skewed Generalized Error Distribution (SGED) will apply in the empirical application on the case that the agents that predict the economic variable under consideration are professional forecasters.

The use of the Skewed Normal distribution (SN) is due to its simplicity since it's a continuous three-parameter probability distribution. Furthermore, the distributional parameters of the Skewed Normal distribution (SN) can be used to capture psychological biases with regards to future outcomes of economic variables. Therefore, these distributional parameters can help to understand various psychological biases: 1. Overconfident and Optimistic, 2. Underconfident and Pessimistic, 3. and Anchoring and over/under confident.

Two – Tail Generalized Distribution

A two-tail generalized distribution (unimodal) is used to model an agent's expected outcome (Savva and Theodossiou, 2018). That is,

$$dF_x = c\left[f\left(\frac{x-m}{\phi_1}\right) dx\, I\left(x \le m\right) + f\left(\frac{x-m}{\phi_2}\right) dx\, I\left(x > m\right)\right] , \tag{1.1}$$

where m is the mode of x, φ_1 and φ_2 are the two tail parameters that control the left and right tails of the distribution of the mode, $\varphi = (\varphi_1 + \varphi_2)/2$ is the mean value of the two tail parameters[2], f is a symmetric probability density function (unimodal), and $I(.)$ is an

[2] where $c = 1/\varphi$, therefore, $c = 2 / (\varphi_1 + \varphi_2)$.

indicator function and takes the value of one when the condition of the function is met and zero otherwise.

These two-tail parameters can explain the shape of the distribution. When $\varphi_1 > \varphi_2$, the distribution is negatively skewed. This means that the probability mass below the mode is higher compared to the probability mass above the mode. When $\varphi_2 > \varphi_1$, the distribution is positively skewed. When $\varphi_1 = \varphi_2 = \varphi$ the distribution is zero skewness, therefore, symmetric. The two distinguish tail parameters can capture the downside and upside adjustments about uncertainty of agents' valuations.

When $\varphi_1 = \varphi_2 = \varphi$,

$$dF_x = \frac{1}{\phi} f\left(\frac{x-m}{\phi}\right)dx \tag{1.2}$$

In this case, the probability distribution is symmetric (skewness = 0), and unimodal.

Assume the transformation $z = (x - m)/\varphi_p$ for $p = 1$ and 2. The distribution of the random variable x can be represented in terms of that of z in the following way

$$dF_x = c\left[\phi_1 I\left(z \le 0\right) + \phi_2 I\left(z > 0\right)\right]dF_z, \tag{1.3}$$

where $dF_z = f(x(z))dz$ is unimodal and symmetric. This is due to the substitution of $x = m + \varphi_p z$ and $dx = \varphi_p dz$ into equation (1.1). In the case of dF_z can be used any symmetric distribution.[3]

The above equation can be rewritten as

$$dF_x\left(x\right) = \left(1 + sgn\left(z\right)\lambda\right)dF_z \tag{1.4}$$

using the transformation $x(z) = m + (1 + sgn(x - m)\lambda)\varphi z$ where dF_z can be any distribution and $sgn(x - m) = sgn(z)$.[4] This decomposition plays an important role in the derivation of the results of this study.

Using equation (1.4) the probability below the mode ($x \le m$) is

$$p = P\left(x \le m\right) = \int_{-\infty}^{m} dF_x = \left(1-\lambda\right)\int_{-\infty}^{0} dF_z = \frac{1-\lambda}{2}, \tag{1.5}$$

---

[3] $\int_{-\infty}^{\infty} dF = 1 \Rightarrow c\phi_1 \int_{-\infty}^{0} f\left(z_1\right)dz + c\phi_2 \int_{0}^{\infty} f\left(z_2\right)dz = 1 \Rightarrow c\phi_1 \frac{1}{2} + c\phi_2 \frac{1}{2} = 1 \Rightarrow c = \frac{2}{\left(\phi_1 + \phi_2\right)} = \frac{1}{\phi}$

[4] For example, $dF_z\left(z\right) = f_z dz = \left(1/\sqrt{2\pi}\right)\exp\left(-z^2/2\right)dz$ is the standard normal distribution.

because $\int_{-\infty}^{0} dF_z = \frac{1}{2}$. The probability above the mode ($x > m$) is

$$q = P\left(x > m\right) = \int_{m}^{\infty} dF_x = \left(1+\lambda\right)\int_{0}^{\infty} dF_z = \frac{1+\lambda}{2} = 1 - p. \tag{1.6}$$

Equations (1.5) and (1.6) are important in explaining the psychological biases and the perceptions of managers about the expected value and various risks (variance, value-at-risk and expected shortfall) of economic variables. It follows from the above equations that the asymmetry parameter can be re-written as

$$\lambda = q - p \tag{1.7}$$

Using equations (1.5) and (1.6) the probabilities below and above the mode can be re-written as (see also Theodossiou and Savva, 2016)

$$p = P\left(x \le m\right) = \int_{-\infty}^{m} dF_x(x) = \frac{\phi_1}{2\phi} = \frac{1-\lambda}{2}. \tag{1.8}$$

and

$$q = P\left(x > m\right) = 1 - p = \frac{\phi_2}{2\phi} = \frac{1+\lambda}{2}. \tag{1.9}$$

Therefore,

$$\lambda = 1 - 2p = q - p = \frac{\phi_2 - \phi_1}{\phi_1 + \phi_2}. \tag{1.10}$$

The asymmetry parameter, λ, takes negative values when $\varphi_1 > \varphi_2$ (negatively skewed distribution) and positive values when $\varphi_1 < \varphi_2$ (positively skewed distribution). Also, $\varphi_1 = (1-\lambda)\varphi$ and $\varphi_2 = (1+\lambda)\varphi$.

A Skewed General Probability Distribution

The values of a decision-making economic variable, denoted by x, is assumed to follow a skewed distribution, defined by

$$dF_x\left(x\right) = f_x dx \tag{1.11}$$

Moments

The mean and variance of x derived in Appendix I are

$$\mu = E\left(x\right) = m + M_1 = m + 2\lambda G_1\phi \tag{1.12}$$

and

$$\sigma^2 = var(x) = M_2 - M_1^2. \tag{1.13}$$

where M_s for s = 1, 2 is the moment function derived based on the specific probability distribution.

Equations (1.12) and (1.13) are the general mean and variance equations using different distributions of the SGT family (such as Skewed Generalized Error Distribution, Skewed Laplace, Skewed Normal, etc.). These equations are capable to characterize the perceptions of managers on the first two moments of a probability distribution, mean and variance. The moments of the Skewed Normal distribution (expected mean and variance) are derived in Appendix I.

Downside Risk and Upside Uncertainty

The standard deviation of unfavourable and favourable outcomes, or downside and upside values of x, are respectively

$$\sigma^2_{x|x\le m} \equiv \mathrm{var}\left(x \middle| x \le m\right) = M_2^- - \left(M_1^-\right)^2 \tag{1.14}$$

$$\sigma^2_{x|x\ge m} \equiv \mathrm{var}\left(x \middle| x \ge m\right) = M_2^+ - \left(M_1^+\right)^2 \tag{1.15}$$

; see Appendix I for the derivation of these equations.

These are the general equations that using different distribution specifications will explain the view of managers including forecasters on the mean and risk measures. The specific equations for the Skewed Normal (SN) distribution are presented in Appendix I.

Value-at-Risk

Value-at-risk (VaR) is a statistical risk measure commonly used in finance. It measures the maximum amount that is expected to be lost, for a given period, using a pre-defined small probability q. The value of q is usually set to 1%. Furthermore, the value-at-risk is equal to negative of the q-quantile value of x ($VaR_q = -x_q$).[5] The quantile value of x is obtained from the solution of the following equation (Ellina et al., 2020)

$$\int_{-\infty}^{x_{q,\lambda}} dF_x(x) = (1-\lambda)\int_{-\infty}^{z_{q,\lambda}} dF_z(z) = q \text{ or } \int_{-\infty}^{z_{q,\lambda}} dF_z(z) = \frac{q}{1-\lambda}.$$

Thus

$$z_{q,\lambda} = F_z^{-1}\left(q/1-\lambda\right)$$

[5] The inverse of the cumulative density function (cdf) provides the quantile.

where $x_{q,\lambda} = m + (1-\lambda)\varphi z_{q,\lambda}$. Because $q < p = (1 - \lambda) / 2$, $x_{q,\lambda} < m$ and $z_{q,\lambda} < 0$. Let $z_q = z_{q,\lambda=0}$ be the quantile value that satisfies the equation $\int_{-\infty}^{z_q} dF_z(z) = q$.

It can be easily confirmed from the above equations that for

$$\lambda < 0, \int_{-\infty}^{z_{q,\lambda}} dF_z(z) < \int_{-\infty}^{z_q} dF_z(z) \text{ and } z_{q,\lambda} < z_q < 0 \text{ or } -z_{q,\lambda} > -z_q > 0 \tag{1.16}$$

and for

$$\lambda > 0, \int_{-\infty}^{z_{q,\lambda}} dF_z(z) > \int_{-\infty}^{z_q} dF_z(z) \text{ and } z_q < z_{q,\lambda} < 0 \text{ or } -z_q > -z_{q,\lambda} > 0 . \tag{1.17}$$

The value-at-risk is

$$VaR_q = -x_{q,\lambda} = -m - (1-\lambda)\varphi z_{q,\lambda} . \tag{1.18}$$

Equations (1.16) and (1.17) are important in explaining the impact of behavioural biases on the perceptions of managers regarding the value-at-risk measure.

Expected Shortfall

The expected shortfall (*ES*) is a risk measure; it is also used in the field of finance to measure the risk exposure. Expected shortfall (at a small probability *q*) is the expected value in the worst scenario. In other words, expected shortfall is the average of losses that exceeding the value-at-risk,

$$ES_{q,\lambda} = -E\left(x \middle| -x > VaR_{q,\lambda}\right) = -m - (1-\lambda)\varphi E\left(z \middle| -z > -z_{q,\lambda}\right). \tag{1.19}$$

If follows easily from equations (1.16) and (1.17) that for

$$\lambda < 0, -x_{q,\lambda} > -x_q \text{ and } ES_{q,\lambda} > ES_q \tag{1.20}$$

and

$$\lambda > 0, -x_{q,\lambda} < -x_q \text{ and } ES_{q,\lambda} < ES_q \tag{1.21}$$

Equations (1.20) and (1.21) are important in explaining the impact of behavioural biases on the perceptions of managers regarding the expected shortfall measure.

SGED Distribution

The values of a decision-making economic variable, denoted by *x*, are modelled as a non-centered SGED distribution (Theodossiou, 2015),

$$dF_x(x) = \frac{1}{2\phi} k^{1-\frac{1}{k}} \Gamma\left(\frac{1}{k}\right)^{-1} \exp\left(-\frac{1}{k}\frac{(x-m)^k}{(1+(sgn(x-m)\lambda))^k \phi^k}\right) dx \tag{1.22}$$

where k and n are kurtosis parameters and $\Gamma(\cdot)$ is the gamma function.

The substitution of $z = (x - m)/ (1 + sgn(x - m)\lambda)\varphi$ [6] and $sgn(z) = sgn(x - m)$ in equation (1.22) gives the pdf of z

$$dF_z(z) = \frac{1}{2}k^{1-\frac{1}{k}}\Gamma\left(\frac{1}{k_i}\right)^{-1}\exp\left(-\frac{1}{k}|z|^k\right)dz$$

SGED Moments

Under the GED, the moment function of z (absolute) is

$$G_s \equiv E|z|^s = 2\int_0^\infty z^s dF_z = k^{1-\frac{1}{k}}\Gamma\left(\frac{1}{k}\right)^{-1}\int_0^\infty z^s \exp\left(-\frac{1}{k}z^k\right)dz \; ^7$$

$$= k^{1-\frac{1}{k}}\Gamma\left(\frac{1}{k}\right)^{-1}\frac{1}{k}k^{\frac{s}{k}+\frac{1}{k}}\int_0^\infty t^{\frac{s}{k}}e^{-t}\,dt = k^{\frac{s}{k}}\Gamma\left(\frac{s+1}{k}\right)\Gamma\left(\frac{1}{k}\right)^{-1}. \qquad (1.23)$$

For $s = 1, 2,\ldots$ and $s < n$.

The excess of the mode moments of the probability distribution can be computed by

$$M_s = E(x-m)^s = \int_{-\infty}^{\infty}(x-m)^s dF_x$$

$$= \varphi^s(1-\lambda)^{s+1}\int_{-\infty}^{0}z^s dF_z + \varphi^s(1+\lambda)^{s+1}\int_0^\infty z^s dF_z$$

where $(-1)^s\int_{-\infty}^{0}z^s dF_z = \int_0^\infty z^s dF_z$ and dF_z is symmetric.

Thus,

$$M_s = \left[(-1)^s(1-\lambda)^{s+1} + (1+\lambda)^{s+1}\right]\varphi^s\int_0^\infty z^s dF_z$$

$$= \frac{1}{2}\left[(-1)^s(1-\lambda)^{s+1} + (1+\lambda)^{s+1}\right]G_s\varphi^s = A_s\varphi^s \qquad (1.24)$$

---

[6] $x(z) = m + (1 + sgn(x - m)\lambda)\varphi z$.

[7] Let $t = (1/k)|z|^k$. Therefore, $z = k^{1//k}t^{1//k}$ and $dz = k^{\frac{1}{k}-1}t^{\frac{1}{k}-1}dt$.

where $A_s = \frac{1}{2}\left[(-1)^s(1-\lambda)^{s+1} + (1+\lambda)^{s+1}\right]G_s$ [8]

The first two moments are,

$$\mu \equiv E(x) = m + M_1 = m + 2\lambda G_1\phi = m + \delta\sigma \tag{1.25}$$

and

$$\sigma^2 \equiv \text{var}(x) = M_2 - M_1^2 = \left(\left(3\lambda^2+1\right)G_2 - 4\lambda^2 G_1^2\right)\phi^2 \tag{1.26}$$

where for s = 1, $M_1 = 2\lambda G_1\varphi$ and for s = 2, $M_2 = \left(3\lambda^2+1\right)G_2\varphi^2$, [9]

$$\theta \equiv 1\Big/\sqrt{\left(3\lambda^2+1\right)G_2 - 4\lambda^2 G_1^2} \tag{1.27}$$

$$G_1 = k^{\frac{1}{k}}\Gamma\left(\frac{2}{k}\right)\Gamma\left(\frac{1}{k}\right)^{-1} \tag{1.28}$$

and

$$G_2 = k^{\frac{2}{k}}\Gamma\left(\frac{3}{k}\right)\Gamma\left(\frac{1}{k}\right)^{-1}. \tag{1.29}$$

The Pearson's mode of skewness is

$$\delta = \frac{\mu - m}{\sigma} = 2\lambda G_1\theta. \tag{1.30}$$

SN Distribution

The non-centered Skewed Normal distribution (SN) is also used to model the values of a decision-making economic variable, denoted by *x*. That is,

$$dF_x(x) = f_x dx = \frac{1}{\varphi\sqrt{2\pi}}\exp\left(-\frac{1}{2}\frac{(x-m)^2}{\left(1+sgn(x-m)\lambda\right)^2\varphi^2}\right)dx\ , \tag{1.31}$$

[8] The standardized skewness and standardized kurtosis are respectively

$SK = \left(A_3 - 3A_2A_1 + 2A_1^3\right)\Big/\left(A_2 - A_1^2\right)^{\frac{3}{2}}$ and $KU = \left(A_4 - 4A_3A_1 + 6A_2A_1^2 - 3A_1^4\right)\Big/\left(A_2 - A_1^2\right)^2$ where

$A_s = 0.5\left[(-1)^s(1-\lambda)^{s+1} + (1+\lambda)^{s+1}\right]G_s$

[9] Note that $-(1-\lambda)^2 + (1+\lambda)^2 = 4\lambda$ and $(1-\lambda)^3 + (1+\lambda)^3 = 2\left(3\lambda^2+1\right)$.

where m, φ and λ defined previously, *sgn* is the sign function taking the value of –1 for $x < m$ and the 1, for $x > m$. A positive value of λ yields a positively skewed distribution and a negative value of λ a negatively skewed distribution. Otherwise, the distribution is symmetric, $\lambda = 0$.

SN Moments

The mean and variance of x, derived in Appendix I, are respectively

$$\mu = E(x) = m + \sqrt{8/\pi}\lambda\varphi = m + 1.5958\lambda\varphi \tag{1.32}$$

and

$$\sigma^2 = \mathrm{var}(x) = \left(1 + (3 - 8/\pi)\lambda^2\right)\varphi^2 = \left(1 + 0.4535\lambda^2\right)\varphi^2 \tag{1.33}$$

Equations (1.32) and (1.33) are functions of the asymmetry (λ) and tail parameters (φ) Furthermore, the expected value depends on the mode of distribution.

Downside Risk

The equation for the downside (standard deviation) of x is (see Appendix I for the derivations):

$$\sigma_D = \sqrt{1 - 2/\pi}\,(1 - \lambda)\varphi \tag{1.34}$$

Downside risk depends on the asymmetry parameter λ as well as on the tail scaling parameter φ.

1.4 Probability Distribution of Psychological Biases

To contrast the differences between the different types of managerial biases, the case of a manager with unbiased beliefs of the true distribution of x is used as benchmark. This type of manager will be referred to as the rational expectation manager. The below analysis presents the perceptions of managers separately under the Skewed Normal (SN) distribution. Table 1.1 shows the notations of each psychological bias.

Table 1.1. Notation of Psychological Biases

Psychological Bias	Notation
Overconfidence	*o*
Underconfidence	*u*
Optimism	*op*
Pessimism	*pe*

Notes. This table presents the notation of each psychological bias; overconfidence, underconfidence, optimism, and pessimism.

Assumption of Equal Distribution Modes

Overconfident, underconfident, optimistic, pessimistic, and rational managers are assumed to have the same perception of the mode. Such an assumption is not unreasonable since the mode coincides with the maximum likelihood point. More specifically, the probability mass around the mode reaches its maximum value at $x = m$. Mathematically,

$$\max_{x=m} dF = f\left(x = m\right)dx$$

This assumption means that the different types of managers have a similar located view of the distribution of the economic variable. Rational managers perceive the distribution of the economic variable as symmetric while overconfident (underconfident) managers perceive the distribution of the economic variable under consideration to be thinner (fatter) and skewed to the right (left).

1.4.1 Rational Expectations Manager

Rational managers are assumed to have an unbiased view of the true distribution of economic variables under consideration. In this case, the true distribution of the economic variable x is assumed to be symmetric (zero skewness) and normal. The distributional parameters for the rational manager are set to $\lambda = 0$, $\varphi = 1$, thus $dF_x = dF_z$ and $z = (x - m) / \varphi$. It follows easily from equations (1.32) and (1.33) that the expected value and standard deviation of x associated with the rational manager are

$$\mu = m \text{ and } \sigma = \varphi$$

The probabilistic analysis and proofs of the various managerial psychological biases is presented below.

1.4.2 Overconfidence and Underconfidence

Overconfidence is a bias where an individual's judgement of a favourable (positive) event is greater. Furthermore, the distribution of overconfident agents has been characterized as too tight. Overconfident managers tend to overestimate the probability of favourable events and underestimate the probability of unfavourable events. Moreover, in the literature, overconfidence bias has been associated with the overestimation of the value, as well as with the underestimation of the risk.[10]

Underconfident managers, on the other hand, have the opposite behaviour since they underestimate the value of a variable. Moreover, they tend to underestimate the probability of favourable events. The distribution of underconfident agents has been characterized as too loose.

The perceptions of overconfident and underconfident managers can be explained using the tail and asymmetric parameters φ and λ of the Skewed Normal (SN) distribution. The distributional parameters for the overconfident manager are denoted by φ_o and λ_o and the underconfident by φ_u and λ_u.

Proposition 1 An overconfident manager overestimates the probability of favourable outcomes, underestimates the probability of unfavourable outcomes, and imposes tighter tails on the probability distributions of the economic variables. This miscalibration leads to (a) positively skewed subjective probability distributions for the variables under consideration, (b) overestimation of their expected values, (c) underestimation of their overall risk and downside risk and (d) underestimation of value-at-risk and expected shortfall.

Proof

Assume that the values of x larger than the mode m are favourable outcomes (smaller values are unfavourable outcomes). The assignment of larger probabilities on favourable events implies that the probability mass above the mode is higher relative to the probability mass below the mode ($q > p$, see equations 1.5 and 1.6). This means that overconfident managers assign smaller probabilities for unfavourable events. This miscalibration implies that, for $c > 0$,

[10] They also have a tendency to overvalue the stock of their companies' and consequently incentive options provided to them, e.g., Palmon et al. (2008), and Palmon and Venezia (2013, 2015).

$$dF_x\left(m-c\middle|m,\varphi_o,\lambda_o\right) < dF_x\left(m+c\middle|;m,\varphi_o,\lambda_o\right),$$

where dF_x is the probability mass function for x and φ_o and λ_o are respectively the values of the tail and asymmetry parameters perceived by an overconfident manager. The above inequality implies that for any symmetric pair of values around the mode m, the probability of occurrence of the value $x = m + c$ perceived by the overconfident manager will be larger than that of $x = m - c$.

The above inequality can be re-written as

$$\left(1-\lambda_o\right)dF_z\left(-z\right) < \left(1+\lambda_o\right)dF_z\left(z\right), \text{for } z > 0,$$

where $z = (x - m) / \varphi_o$ is a standardized random variable and dF_z is the standard normal distribution.

Because $dF_z(-z) = dF_z(z)$, the above inequality implies that the asymmetry parameter for the overconfident manager is positive ($\lambda_o > 0$). Therefore, dF_x is positively skewed. The miscalibration of subjective probabilities of overconfident managers leads to a positively skewed probability distribution for x; proof, part (a).

The expected value of x implied by the overconfident manager's probability distribution is

$$\mu_o = m + 1.5958\varphi_o\lambda_o > m = \mu;$$

see equation (1.32). The asymmetry parameter is positive ($\lambda_o > 0$), therefore, the overconfident manager will overestimate the expected value of x ($\mu_o > \mu$); proof, part (b).

The standard deviation of x associated with the overconfident manager, see equation (1.33), is

$$\sigma_o = \sqrt{1+0.4535\lambda_o^2}\,\varphi_o.$$

Psychological theory claims that the distribution of an overconfident manager has been characterized as too tight. That is, $\varphi_o < \varphi = \sigma$. Therefore, this implies that an overconfident manager will underestimate risk ($\sigma_o < \sigma$). That is, σ_o will be smaller than the true value of σ, provided that

$$\sigma_o = \sqrt{1+0.4535\lambda_o^2}\,\varphi_o < \sigma = \varphi$$

or

$$\varphi_o < \varphi\Big/\sqrt{1+0.4535\lambda_o^2}.$$

The downside risk perceived by an overconfident manager, given by equation (1.34), is

$$\sigma_{o,D} = \sqrt{1-2/\pi}\left(1-\lambda_o\right)\varphi_o .$$

Because $\varphi_o < \varphi$ and $\lambda_o > 0$,

$$\sigma_{o,D} = \sqrt{1-2/\pi}\ \left(1-\lambda_0\right)\varphi_o < \sqrt{1-2/\pi}\ \left(1-\lambda_0\right)\varphi < \sqrt{1-2/\pi}\varphi = \sigma_D ,$$

where $\sigma_D = \sqrt{1-2/\pi}\phi$ measures downside risk for the rational agent; proof, part (c).

Because $\lambda_o > 0$, it follows from equations (1.17) and (1.18) that $-z_q > -z_{q,o} > 0$, where $z_{q,o}$ and z_q are respectively the quantile values for the overconfident and rational managers.

Because $\varphi_o < \varphi$ and $\lambda_o > 0$, the overconfidence manager's VaR is

$$VaR_{q,o} = -x_{q,o} = -m - \left(1-\lambda_o\right)\varphi_o z_{q,o} < -m - \left(1-\lambda_o\right)\varphi_o z_q < -m - \varphi z_q = -x_q = VaR_q ,$$

where VaR_q is the objective VaR measure. Also, it follows from (1.21) that for

$$\lambda_o > 0, ES_{q,\lambda_o} < ES_q .$$

where ES_q is the expected shortfall for rational managers; proof, part (d).

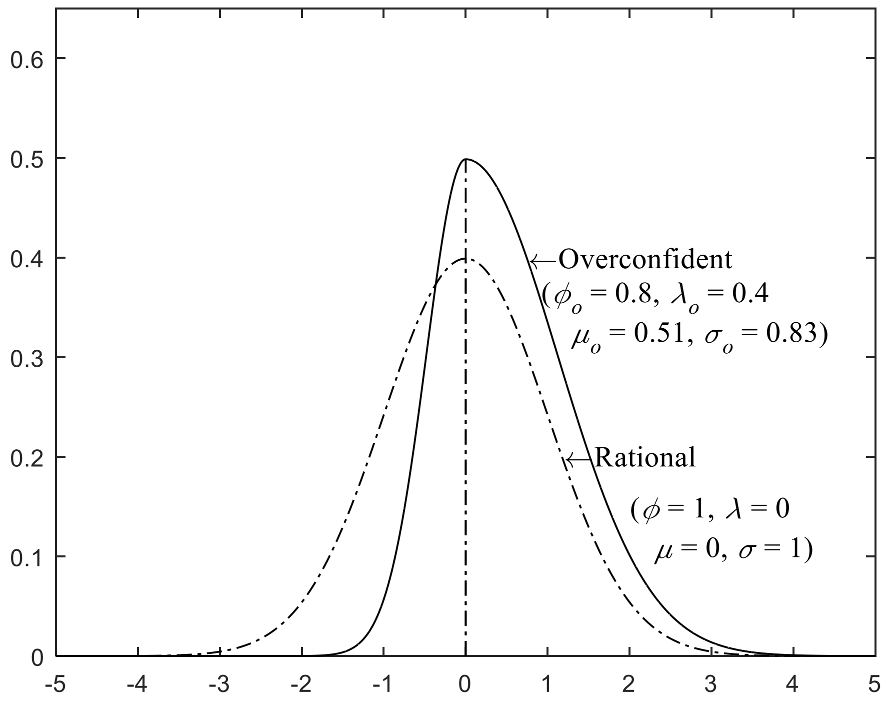


Figure 1.1. Overconfident and rational distributions

<u>Proposition 2</u> An underconfident manager underestimates the probability of favourable outcomes, overestimates the probability of unfavourable outcomes, and imposes wider tails on the probability distributions of the economic variables. This miscalibration leads to (a) negatively skewed subjective probability distributions for the variables, (b) underestimation of their expected values, (c) overestimation of their overall risk, downside risk, value-at-risk, and expected shortfall.

<u>Proof</u>

The assignment of larger probabilities on unfavourable outcomes implies smaller probabilities for favourable outcomes. This miscalibration implies that, for $c > 0$,

$$dF_x\left(m-c\middle|m,\varphi_u,\lambda_u\right) > dF_x\left(m+c\middle|;m,\varphi_u,\lambda_u\right),$$

where dF_x is the probability mass function for x and φ_u and λ_u are respectively the values of the tail and asymmetry parameters perceived by an underconfident manager. The above inequality implies that the probability of occurrence of the value $x = m + c$ perceived by the underconfident manager will be smaller than that of $x = m - c$.

The above inequality can be re-written as

$$(1-\lambda_u)dF_z(-z) > (1+\lambda_u)dF_z(z), \text{for } z > 0,$$

where $z = (x - m) / \varphi_u$ is a standardized random variable and dF_z is the standard normal distribution.

Because $dF_z(-z) = dF_z(z)$, the above inequality implies that the asymmetry parameter for the underconfident manager is negative ($\lambda_u < 0$). Therefore, dF_x is negatively skewed. This leads to a negatively skewed subjective probability distribution for x; proof, part (a).

The expected value of x implied by the underconfident manager's subjective probability distribution is

$$\mu_u = m + 1.5958\varphi_u\lambda_u < m = \mu;$$

see equation (1.32). Because $\lambda_u < 0$, the underconfident manager will underestimate the expected value of x ($\mu_u < \mu$); proof, part (b).

The overall risk of x for the underconfident manager is

$$\sigma_u = \sqrt{1+0.4535\lambda_u^2}\,\varphi_u > \sigma = \varphi,$$

because $\varphi_u > \varphi = \sigma$ (theory claims that the distribution for underconfident managers has been characterized as too loose).

Also, because $\lambda_u < 0$ and $\varphi_u > \varphi$, the downside risk for the underconfident manager is

$$\sigma_{u,D} = \sqrt{1-2/\pi}\left(1-\lambda_u\right)\varphi_u > \sqrt{1-2/\pi}\varphi_u > \sqrt{1-2/\pi}\ \varphi = \sigma_D,$$

where $\sigma_D = \sqrt{1-2/\pi}\phi$ is the downside risk for the rational agent.

Because $\lambda_u < 0$, it follows from equations (1.16) and (1.18) that $-z_{q,u} > -z_q > 0$, where $z_{q,u}$ and z_q are respectively the quantile values for the underconfident and rational managers.

Because $\varphi_u > \varphi$ and $\lambda_u < 0$, the underconfident manager's VaR is

$$VaR_{q,u} = -x_{q,u} = -m - \left(1-\lambda_u\right)\varphi_u z_{q,u} > -m - \left(1-\lambda_u\right)\varphi_u z_q > -m - \varphi z_q = -x_q = VaR_q,$$

It follows from equation (1.20) that for $\lambda_u < 0$,

$ES_{q,\lambda_u} > ES_q$.; proof, part (c).

Figure 1.1 presents the probability distributions of a rational and an overconfident manager. The three parameters of the skewed normal distribution for the rational manager are set to $m = 0$, $\varphi = 1$ and $\lambda = 0$. On the other hand, for the overconfident manager, the distributional parameters are set to $m = 0$, $\varphi_o = 0.8$ and $\lambda_o = 0.4$. The symmetric distribution for the rational manager is represented by the dotted curve. The distribution for the overconfident manager is skewed to the right. Therefore, the probability distribution is positively skewed. For these distributional parameters, the overconfident manager attaches a 0.7 (0.3) probability for the values on the right (left) of the mode. The expected value of the random variable x is $\mu_o = 0.51$ and the standard deviation is $\sigma_o = 0.83$. Therefore, the overconfident manager overestimates the expected value and underestimates the overall risk of the economic variable x.

Figure 1.2 shows the probability distributions of a rational and an underconfident manager. The range of values around the mode compared to reality tends to be larger for underconfident managers and smaller for overconfident managers. This implies that $\varphi_o < \varphi < \varphi_u$. The tail parameter φ controls the tails of a distribution around its mode. A value closer to zero means that the tails are more concentrated towards the mode. This means that for overconfident φ_o becomes smaller and λ_o larger and φ_u becomes larger and λ_u smaller for underconfident ($\varphi_o < \varphi_u$ and $\lambda_o > \lambda_u$).

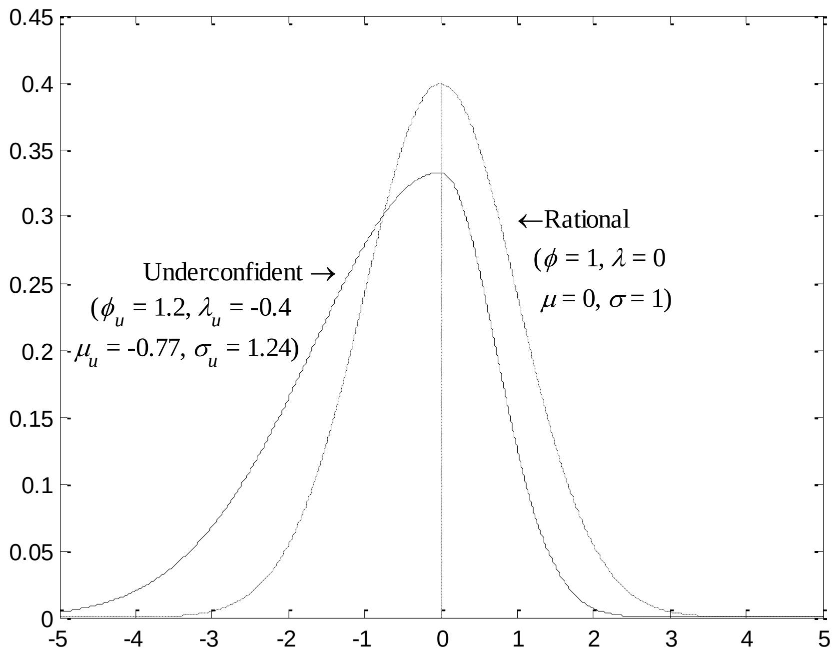


Figure 1.2. Underconfident and rational distributions

1.4.3 Optimism and Pessimism

Weinstein (1980) referred to the tendency of people to be optimistic about future events. He mentions that thoughts affect the amount of optimistic bias of various events. This means that optimistic people believe that unfavourable (favourable) events are less (more) likely to happen to them than to others. Therefore, optimistic individuals tend to overestimate the probability of favourable events and underestimate the probability of unfavourable events.

<u>Proposition 3</u> Unrealistic optimistic managers underestimate the probability of unfavourable events and overestimate the probability of favourable events. This miscalibration leads to (a) positively skewed subjective probability distributions for the economic variables under consideration, (b) overestimation of their expected values, (c) overestimation of overall risk, (d) underestimation of their downside risk, value at risk and expected shortfall.

<u>Proof</u>

The proof for parts (a) and (b) of proposition 3 is similar to that of Proposition 1, therefore, it is omitted. From the parts (a) and (b) of proposition 1, the asymmetry parameter is positive ($\lambda_{op} > 0$).

Because $\lambda_{op} > 0$ and $\varphi_{op} = \varphi$, the overall risk of x associated with the optimistic manager (psychological theory does not say anything about the tails) is

$$\sigma_{op} = \sqrt{1+0.4535\lambda_{op}^2}\,\varphi > \sigma = \varphi\,,$$

because $\sqrt{1+0.4535\lambda_{op}^2} > 1.$; proof, part (c).

Also, because $\lambda_{op} > 0$ and $\varphi_{op} = \varphi$, the downside risk for the optimistic manager is

$$\sigma_{op,D} = \sqrt{1-2/\pi}\,\left(1-\lambda_{op}\right)\varphi < \sqrt{1-2/\pi}\,\varphi = \sigma_D\,,$$

where $\sigma_D = \sqrt{1-2/\pi}\phi$ is the downside risk for the rational agent.

Because $\lambda_{op} > 0$, it follows from equations (1.17) and (1.18) that $-z_q > -z_{q,op} > 0$, where $z_{q,op}$ and z_q are respectively the quantile values for the optimistic and rational managers. Because $\lambda_{op} > 0$, the optimistic manager's VaR is

$$VaR_{q,op} = -x_{q,op} = -m - \left(1-\lambda_{op}\right)\varphi z_{q,op} < -m - \varphi z_q = -x_q = VaR_q\,,$$

Furthermore, it follows from equation (1.21) that for $\lambda_{op} > 0$,

$ES_{q,\lambda} < ES_q$. ; proof, part (d).

Unrealistically pessimistic managers will concentrate on the negative outcomes and therefore they will put a larger probability mass left to the mode than right. Their subjective probability distribution will be negatively skewed. In other words, pessimistic people tend to overestimate the probability of unfavourable events and underestimate the probability of favourable events.

Proposition 4 Unrealistic pessimistic managers overestimate the probability of unfavourable events and underestimate the probability of favourable events. This miscalibration leads to (a) negatively skewed subjective probability distributions for the variables under consideration, (b) underestimation of their expected values and (c) overestimation of their overall risk, downside risk, and expected shortfall.

Proof

The proof for parts (a) and (b) of proposition 4 is similar to that of proposition 2; therefore, it is omitted. Parts (a) and (b) of proposition 2 implies that the asymmetry parameter is negative ($\lambda_{pe} < 0$).

In this case, the overall risk of x associated with the pessimistic manager is

$$\sigma_{pe} = \sqrt{1+0.4535\lambda_{pe}^2}\,\varphi > \sigma = \varphi\,,$$

because $\sqrt{1+0.4535\lambda_{pe}^{2}} > 1$.

Also, because $\lambda_{pe} < 0$ and $\varphi_{pe} = \varphi$, the downside risk for the pessimistic manager is

$$\sigma_{pe,D} = \sqrt{1-2/\pi}\left(1-\lambda_{pe}\right)\varphi > \sqrt{1-2/\pi}\,\varphi = \sigma_D\,,$$

where $\sigma_D = \sqrt{1-2/\pi}\,\varphi$ is the downside risk for the rational agent.

Because $\lambda_{pe} < 0$, it follows from equations (1.16) and (1.18) that $-z_{q,pe} > -z_q > 0$, where $z_{q,pe}$ and z_q are respectively the quantile values for the pessimistic and rational managers. Because $\lambda_{pe} < 0$, the pessimist's VaR is

$$VaR_{q,pe} = -x_{q,pe} = -m-\left(1-\lambda_{pe}\right)\varphi z_{q,pe} > -m-\left(1-\lambda_{pe}\right)\varphi z_q > -m-\varphi z_q = -x_q = VaR_q\,.$$

and

$$ES_{q,\lambda} > ES_q \text{ .; proof, part (c).}$$

Overconfidence and Optimism

Figure 1.3 shows the psychological biases and the differences between the overconfident, the optimistic and the rational view manager. The overconfident manager involves overconfident managers who simplistically consider fewer options and have an optimistic view of the world. These managers underestimate negative outcomes and focusing on positive outcomes. This is obvious in figure 1.3 since a manager with overconfident model behaviour narrows the tails $\varphi_o = 0.8 < \varphi = 1$ and therefore, underestimates the uncertainty ($\sigma_o = 0.83 < \sigma = 1$), gives a larger discrete probability mass function and has an optimistic view of the expected mean ($\mu_o = 0.51 > \mu = 0$).[11]

Unrealistic optimistic managers underestimate the occurrence of unfavourable events and overestimate the occurrence of favourable events. At this point, remember the definition of an unrealistic optimistic manager; he/she believes that positive (desirable) events are more likely to happen than negative (undesirable). Specifically, in figure 1.3, a manager that exhibits these biases has an optimistic view of the expected value (μ_{op} = 0.64 > μ = 0) and overestimates the uncertainty ($\sigma_{op} = 1.04 > \sigma = 1$).[12] Therefore, the

[11] For the overconfident manager, the three Skewed Normal (SN) parameters are set to $m = 0$, $\varphi_o = 0.8$ and $\lambda_o = 0.4$.

[12] For the unrealistic optimistic manager, the three Skewed Normal (SN) parameters are set to $m = 0$, $\varphi_{op} = 1$ and $\lambda_{op} = 0.4$.

expected value for an unrealistic optimistic manager is higher than that of an overconfident manager (μ_{op} = 0.64 > μ_o = 0.51).

<u>Proposition 5</u> **Optimistic and overconfident managers overestimate the probability of favourable events; therefore, they face the same asymmetry parameter. At the same time, overconfident managers impose tighter tails on the distribution of the economic variable. This miscalibration leads optimistic managers to (a) overestimate to a greater extent the mean and overall risk and (b) underestimate to a lesser extent downside risk, value-at-risk and expected shortfall of economic variables under consideration.**

<u>Proof</u>

Optimistic and overconfident managers overestimate the probability of favourable events (higher probability mass function right the mode). This implies that they face the same asymmetry parameter ($\lambda_{op} = \lambda_o$). However, the tail parameter for overconfident managers is smaller than that of optimistic and rational managers ($\varphi_o < \varphi = \varphi_{op}$). Therefore, the following results for the mean and risk measures can be easily obtained.

Specifically, the mean and overall risk of x by the optimistic manager is higher compared to that of an overconfident manager. That is,

$$\mu_{op} = m + 1.5958\varphi\lambda_{op} > \mu_o = m + 1.5958\varphi_o\lambda_o \,,$$

$$\sigma_{op} = \sqrt{1+0.4535\lambda_{op}^2}\,\varphi > \sigma_o = \sqrt{1+0.4535\lambda_o^2}\,\varphi_o \text{; proof, part (a)}$$

Because $\lambda_{op} = \lambda_o$ and $\varphi_o < \varphi = \varphi_{op}$, the downside risk, value-at-risk and expected shortfall of x by optimistic compared to that of an overconfident manager's distribution are

$$\sigma_{op,D} = \sqrt{1-2/\pi}\left(1-\lambda_{op}\right)\varphi > \sigma_{o,D} = \sqrt{1-2/\pi}\left(1-\lambda_o\right)\varphi_o$$

and

$$VaR_{q,o} = -m - \left(1-\lambda_o\right)\varphi_o z_{q,o} < VaR_{q,op} = -m - \left(1-\lambda_{op}\right)\varphi z_{q,o} < VaR_q = -m - \varphi z_q \,.$$

$$ES_{q,o} < ES_{q,op}\,.\text{ ; proof, part (b)}$$

Note that because $\lambda_{op} = \lambda_o$, $z_{q,o} = z_{q,op}$.

The visualization of biases which was made possible through the representation of statistical distributions helps to understand the individual biases and their interplay.

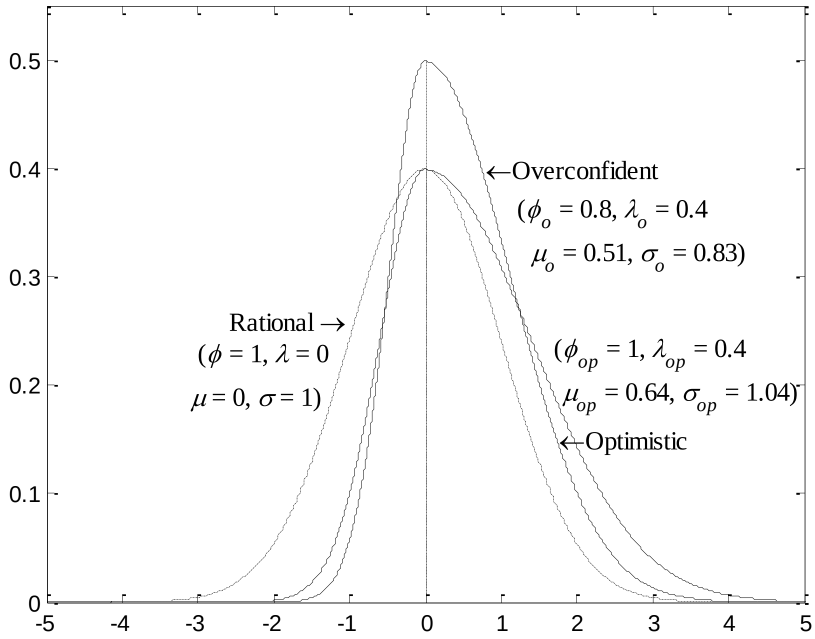


Figure 1.3. Overconfidence, Rational and Unrealistic Optimism biases

1.4.4 Anchoring and Adjustment: A Statistical Adaptation Model

This sub-section further develops the mathematical model to explain the probabilistic update of forecaster's beliefs using an adaptation process. This framework follows the above analysis to investigates the interrelationship between anchoring heuristic and over/under confidence bias. The following analysis will primarily focus on professional forecasters that predict the economic variable under consideration. The difference between the forecast value and the actual value of an economic variable it will be referred as the forecast error. A positive (negative) forecast error means that the forecast value is higher (lower) relative to the actual value. Therefore, forecasters overestimate (underestimate) the value of the economic variable. However, when the forecast error equals zero, forecasters estimate the correct value of the economic variable x.

This sub-section presents the agent's expectations of a random variable (e.g., forecast values) accounting for downside and upside uncertainty under the Skewed Generalized Error Distribution (SGED). In this case, the random variable x may represent any economic variable under consideration by the forecasters, such as the gross domestic product (GDP).

The two-piece probability distribution and the moments presented in section 1.3 (equations 1.1 and 1.22-1.30) are used to investigate the over/under confidence forecasting bias and the anchoring and adjustment heuristic.

Forecasts, due to the central limit theorem, must generally be asymptotically Normal (Gaussian). This implies a zero-asymmetry parameter ($\lambda = 0$) for forecasting errors. In this case, the forecast errors are expected to follow a Normal distribution. Namely, the two-piece distribution collapses to a zero mean ($\mu = m = 0$) Normal distribution with standard deviation $\sigma = \phi_1 = \phi_2$ and $\lambda = 0$. Therefore, the distribution of rational forecasters is Normal.

However, forecasts are updated using subjective beliefs and the economic variable under consideration is, in many cases, far away from the actual value. Because of psychological biases, the incorporation of prior beliefs (subjective probability beliefs) violates the normality assumption.[13] A 'correct' estimation of the economic variable is expected to yield forecasts close to the actual value. In other words, the forecasting errors should have zero mean.

The distribution of forecasting errors can be captured by a flexible probability density function that accounts for skewness and kurtosis characteristics. For this analysis, the SGED distribution will be used.

Note that, in the previous analysis, proposition 1 proved that overconfident agents overestimate the expected value of a random variable compared to a rational agent, $\mu_o > \mu = 0$ and $\lambda_o > 0$. In this case, $\varphi_{o,1} < \varphi_{o,2}$ and $(\varphi_{o,1} + \varphi_{o,2})/2 = \varphi_o < \varphi$. On the other hand, underconfident agents underestimate the expected value of an economic variable, $\mu_u < \mu = 0$ and $\lambda_u < 0$ (see equation 1.25, expected value of the SGED distribution). Therefore, $\varphi_{u,1} > \varphi_{u,2}$ and $(\varphi_{u,1} + \varphi_{u,2})/2 = \varphi_u > \varphi$. Consistent with the above analysis, positive forecast errors followed by overconfident forecasters, therefore, a positively skewed distribution. On the other hand, negative forecast errors are followed by underconfident forecasters, therefore, a negatively skewed distribution.

[13] Literature used Bayesian statistics to explain the beliefs of experts, e.g. Morris (1974, 1977) and Van den Steen (2001, 2004, 2011).

The summary propositions that are needed for the analysis of over and under confident professional forecasters are restated in propositions 6 and 7, respectively. The below propositions are directly derived from section 1.4.

Proposition 6 (Overconfident Forecaster). An overconfident forecaster

(i) assigns higher probability of favourable than unfavourable events,

(ii) the forecasting error distribution (forecast value minus actual value) for overconfident forecasters is positively skewed,

(iii) narrows the tails resulting a smaller tail parameter for the perceived forecasting error distribution,

(iv) overestimates the forecast economic variables relative to a rational forecaster.

Proposition 7 (Underconfident Forecaster). An underconfident forecaster

(i) assigns a lower probability of favourable than unfavourable events,

(ii) the forecasting error distribution (forecast value minus actual value) for underconfident forecasters is negatively skewed,

(iii) wider the tails resulting a larger tail parameter for the perceived forecasting error distribution relative to a rational forecaster,

(iv) underestimates the forecast economic variables relative to a rational forecaster.

The proofs for the above propositions are similar as in sub-section 1.4.2 (propositions 1 and 2); therefore, it is omitted.

Anchoring Using an Adaptation Expectation Process

Anchoring and adjustment is a heuristic where forecasters focus on their initial valuations (or the information provided to them) in calibrating predicted probabilities of economic variables. The anchors are reflected in the prior distributions and can be adjusted as new information arrives.

Anchoring affects the behaviour of overconfident and underconfident forecasters. As shown in the previous analysis, overconfidence and underconfidence biases can be represented using the two different tail parameters: the left and right tail parameters ϕ_1 and ϕ_2 of the distribution of forecasting errors. For overconfident forecasters, the asymmetry parameter is positive, $\lambda_o > 0$. Therefore,

$$\phi_{o,1} = (1-\lambda_o)\phi_o < \phi_{o,2} = (1+\lambda_o)\phi_o$$

The analysis of overconfidence bias in section 1.4 implies that

$$\phi_o = \left(\phi_{o,1} + \phi_{o,2}\right)/2 < \phi_c$$

where φ_c is the correct value of the tail parameter (see equation 1.8). For underconfident forecasters, the asymmetry parameter is negative, $\lambda_u < 0$. In this case,

$$\phi_{u,1} = \left(1-\lambda_u\right)\phi_u > \phi_{u,2} = \left(1+\lambda_u\right)\phi_u$$

and

$$\phi_u = \left(\phi_{u,1} + \phi_{u,2}\right)/2 > \phi_c \, .$$

Given an information set $\{I_t\}_1^s$, the overconfident forecaster's view is updated using the following adaptive expectation process that adjusts the left and right tail parameters of the forecasters

$$\phi_{o,i,s} = a\phi_c + \left(1-a\right)\phi_{o,i,s-1} \text{, for } s = 1, 2, \ldots, \tag{1.34}$$

where a is an adjustment coefficient measuring the speed of adjustment of the left and right tail parameters to the new information (i = 1 and 2) and $\varphi = \varphi_c$. The parameter a measures the persistence of the anchors. The speed of adaptation towards the true parameters depends on the parameter a. The lower the value of a the longer it takes for perceived values of the parameters to be closer to their true values. In cases of extreme overconfidence behaviour, the value of a = 0. In the setting of this chapter, rational individuals possess values of a = 1. However, when beliefs range on the interval [0, 1], it means that they adjust the values using real values and their valuations. Graphically, the subjective beliefs one year later are represented in figure 1.4.

	0	1
Left Tail:	$\varphi_{o,1,0}$	$\phi_{o,1,1} = a\varphi_c + \left(1-\alpha\right)\phi_{o,1,0}$
Right Tail:	$\varphi_{o,2,0}$	$\phi_{o,2,1} = a\varphi_c + \left(1-\alpha\right)\phi_{o,2,0}$

Figure 1.4. Graphical Illustration of Subjective Adjustment Beliefs Regarding Uncertainty one year later

The above equations demonstrate that the perceived values of the left and right tail parameters are weighted averages of past perceived and actual values. Using recursive substitutions

$$\phi_{o,i,s} = a\sum_{j=1}^{s}\left(1-a\right)^{j-1}\phi_c + \left(1-a\right)^s\phi_{o,i,0} \ , \tag{1.35}$$

for i = 1 and 2. The limit of the above equation is

$$\phi_{o,i,s} = \lim_{s \to \infty} \left(1 - a^{s}\right)\phi_c + \left(1 - a\right)^{s} \phi_{o,i,0} = \phi_c$$

The limit of the asymmetry parameter is

$$\lambda_{o,s} = \lim_{s \to \infty,\, a \neq 0} \frac{\phi_{2,s} - \phi_{1,s}}{\phi_{1,s} + \phi_{2,s}} = 0\,.$$

The subjective beliefs *s* years later represented in figure 1.5.

$$\begin{array}{cccc} 0 & t+1 & t+2 \quad \ldots & t+s \\ \phi_{o,i,0} & \phi_{o,i,1} & \phi_{o,i,2} & \phi_{o,i,s} = a\sum_{j=1}^{s}\left(1-a\right)^{j-1}\phi_c + \left(1-a\right)^{s}\phi_{o,i,0} \end{array}$$

Figure 1.5. Graphical Illustration of Subjective Adjustment Beliefs Regarding Uncertainty

The above analysis is illustrated via the following illustration.

Theoretical Illustration

Assume that the initial left and right tail parameters for an overconfident forecaster are $\varphi_{o,1,0} = 1$, $\varphi_{o,2,0} = 2$, respectively. The adjustment coefficient is $\alpha = 0.30$ and the actual value is $\varphi_c = 4$.

The first, second and final left adjustments in the next periods are

$$\phi_{o,1,1} = a\varphi_c + \left(1-\alpha\right)\phi_{o,1,0} = 4 * a + 1 * \left(1-\alpha\right) = 1.9$$

$$\phi_{o,1,2} = a\varphi_c + \left(1-\alpha\right)\varphi_{o,1,1} = 4 * a + 1.9 * \left(1-\alpha\right) = 2.53$$

$$\phi_{o,2,3} = a\varphi_c + \left(1-\alpha\right)\phi_{o,1,2} = 4 * a + 2.53 * \left(1-\alpha\right) = 2.971.$$

The first, second and final right adjustments in the next periods are

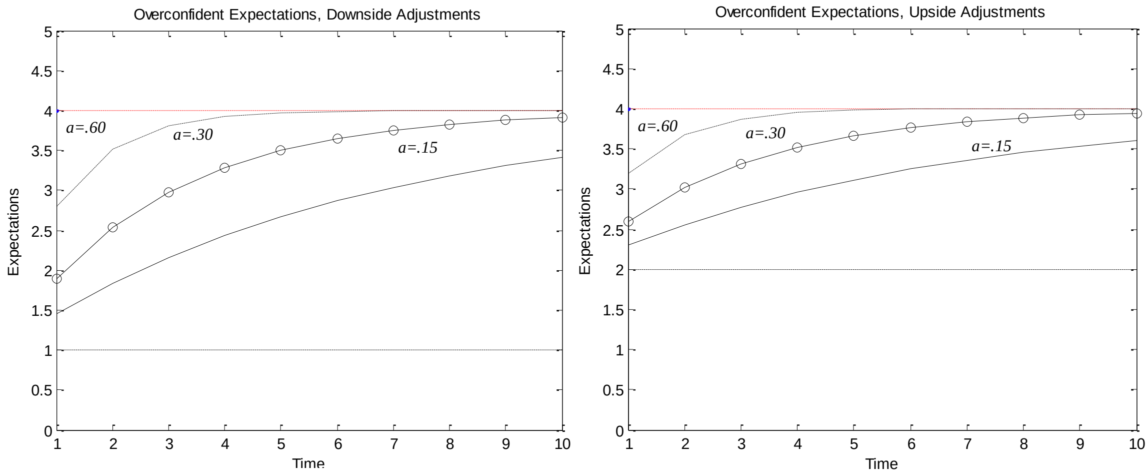
$$\phi_{o,2,1} = a\varphi_c + \left(1-\alpha\right)\phi_{o,2,0} = 4 * a + 2 * \left(1-\alpha\right) = 2.6$$

$$\phi_{o,2,2} = a\varphi_c + \left(1-\alpha\right)\phi_{o,2,1} = 4 * a + 2.6 * \left(1-\alpha\right) = 3.020$$

$$\phi_{o,2,3} = a\varphi_c + \left(1-\alpha\right)\phi_{o,2,2} = 4 * a + 3.020 * \left(1-\alpha\right) \approx 4$$

Both tail parameters converge towards their true value of $\varphi_c = 4$.

Figure 1.6. Overconfident Expectations



Notes. The left and right tail parameters for an overconfident forecaster are set to $\varphi_{o,1,0}$ = 1 and $\varphi_{o,2,0}$ = 2, respectively and the actual value of the tail parameter is φ_c = 4. Lines 1, 2, and 3 represent overconfident downside and upside forecaster valuations with the speed of adjustment parameter *a* = .15, .30 and .60, respectively.

For an underconfident forecaster, the left and right tail parameters set to $\varphi_{u,1,1}$ = 4, $\varphi_{u,2,1}$ = 3 and the actual value is φ_c = 2.

The first, second and final left adjustments in the next three periods

$$\phi_{u,1,1} = a\varphi_c + (1-\alpha)\phi_{u,1,0} = 2*a + 4*(1-\alpha) = 3.4$$

$$\phi_{u,1,2} = a\varphi_c + (1-\alpha)\phi_{u,1,1} = 2*a + 3.4*(1-\alpha) = 2.980$$

$$\phi_{u,1,3} = a\varphi_c + (1-\alpha)\phi_{u,1,2} = 2*a + 2.980*(1-\alpha) \approx 2.$$

The first, second and final right adjustments in the next three periods

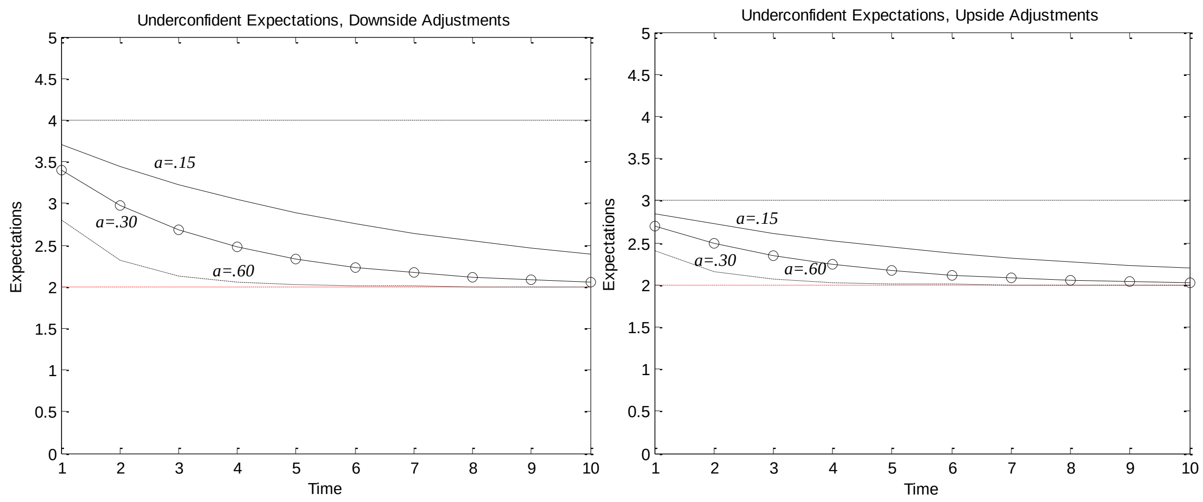
$$\phi_{u,2,1} = a\varphi_c + (1-\alpha)\phi_{u,2,0} = 2*a + 3*(1-\alpha) = 2.7$$

$$\phi_{u,2,2} = a\varphi_c + (1-\alpha)\phi_{u,2,1} = 2*a + 2.7*(1-\alpha) = 2.49$$

$$\phi_{u,2,3} = a\varphi_c + (1-\alpha)\phi_{u,2,2} = 2*a + 2.49*(1-\alpha) \approx 2.343.$$

Both tail parameters converge towards their true value of φ_c = 2.

Figure 1.7. Underconfident Expectations



Notes. The left and right tail parameters for an underconfident forecaster are set to $\varphi_{u,1,1} = 4$, and $\varphi_{u,2,1} = 3$, respectively and the actual value of the tail parameter is $\varphi_c = 2$. Lines 1, 2, and 3 represent underconfident downside and upside forecaster valuations with the speed of adjustment parameter $a = .15$, .30 and .60, respectively.

Figures 1.6 and 1.7 show the subjective beliefs of overconfident and underconfident forecasters, respectively. Both graphs showed that as the speed of adjustment parameter increases, the forecast valuations are faster close to the actual value. The speed of adjustment showed the behaviour of the forecasters for the economic variable x. For example, if there are two overconfident forecasters with speed of adjustment parameters 0.30 and 0.6, ($\alpha = 0.3$ and 0.6) this means that the first forecaster evaluates 0.70 based on his valuations and .30 on the actual values. On the other hand, the second forecaster evaluates 0.4 on his expectations and 0.6 on the actual values. The second forecaster's valuation will yield faster to the true value of the economic variable.

1.5 Monte Carlo Simulations

Two Monte-Carlo simulations are conducted to investigate the mean and risk perceptions of overconfident, underconfident, optimistic and pessimistic managers. The irrational managerial mean and risk perceptions are compared to those of a rational manager. A rational manager is characterized as having the true perception of the economic variable under consideration. The first simulation is illustrated by the means of a portfolio while the second simulation by the means of a capital budgeting example. The simulations are replicated as in the paper of Ellina et al. (2020).

The mode of the distribution of portfolio returns and cash flows is set to zero (m = 0). This assumption makes the comparison easier and does not distort the results. The distributional parameters for an overconfident manager are set to $\lambda = 0.3$ and $\varphi = 0.08$ and for an optimist manager to $\lambda = 0.3$ and $\varphi = 0.10$. For the underconfident manager, the distributional parameters are set to $\lambda = –0.3$ and $\varphi = 0.12$ and for the pessimistic manager to $\lambda = -0.3$ and $\varphi = 0.10$. The distributional parameters for the rational manager are set to $\lambda = 0$ and $\varphi = 0.10$.

1.5.1 Managerial Perceptions of a Portfolio Value Example

The value of the portfolio for the next year computed by $Value_1 = Value_0\ (1 + x) = 10(1 + x)$ where x is a normally generated return with zero-mode ($m = 0$), standard deviation 0.1 and $Value_0$ is the previous (initial) portfolio value. A Monte-Carlo simulation of a $T = 100{,}000$ portfolio returns is generated to estimate the mean and risk measures of each psychological bias: overconfident, optimism, underconfident and pessimism. Table 1.2 presents the mean and risk perceptions of each psychological bias.

The results reveal that overconfident and optimistic managers overestimate the next period expected value of their portfolios. The overestimation is larger in the case of optimistic (10.48) managers compared to the overconfident managers (10.38). On the other hand, underconfident managers underestimate the mean portfolio value of the next period. The underestimation is larger in the case of the underconfident manager (9.43) compared to the pessimist manager (9.52).

The findings also portray that overconfident managers underestimate the overall risk (0.82 < 1), downside risk (0.34 < 0.60) as well as the value-at-risk (1.23 < 2.32) and the expected shortfall (1.43 < 2.67) measures. Notably, optimistic managers also underestimate the tail risk measures (downside risk, value-at-risk and expected shortfall). On the other hand, underconfident and pessimistic managers overestimate the risk measures. The overestimation is more evident in the case of an underconfident manager.

Table 1.2. Managerial Perceptions of Portfolio Mean and Risk Measures

Bias	φ	λ	$E(V_1 \mid C)$	$\sigma(V_1 \mid C)$	$\sigma_D(V_1 \mid C)$	$VaR_{1\%}$	$ES_{1\%}$
Rational	0.1	0.0	10	1	0.60	2.32	2.67
Overconfident	0.08	0.3	10.38	0.82	0.34	1.23	1.43
Underconfident	0.12	–0.3	9.43	1.23	0.94	3.78	4.31
Optimist	0.1	0.3	10.48	1.02	0.42	1.53	1.79
Pessimist	0.1	–0.3	9.52	1.02	0.79	3.15	3.59

Notes. 100,000 skewed normal returns are generated using the distributional parameters of φ and λ as presented in the second and third columns. The next year investment values are computed by $V_1 = 10\ (1 + x)$ where x is a generated randomly return. The third and fourth columns are the mean and standard deviation of the portfolio values of each bias. The next column presents the downside risk computed as the standard deviation of the next year portfolio values that are below 10. The last two columns present the tail risk measures: value-at-risk (1%) and the expected shortfall (ES).

1.5.2 Managerial Perceptions of a Capital Budgeting Example

A second simulation is conducted using a capital budgeting example to investigate the managerial perceptions about the mean and risk of the future cash flows. The cash-flows of the project are CF_0 = -178.0777 and $CF_t = 40\ (1 + x)$, for t = 1, 2, …,5, where x is a generated randomly return using the distributional parameters of each bias and k = 0.04. The net present value of the project computed by

$$NPV = -CF_0 + \sum\nolimits_{t=1}^{N} CF_t\left(1+k\right)^{-t} = -178.0777 + \sum\nolimits_{t=1}^{5} 40\left(1+x\right)\left(1+0.04\right)^{-t},$$

is driven by the growth rate x of the cash flows. Table 1.3 presents the mean and risk measures of the project's net present value for each psychological bias using T = 100,000 generated values.

The results reveal that overconfident and optimistic managers overestimate the project's net present value compared to a rational manager. However, the overestimation is larger in the case of optimistic managers (8.53) compared to the overconfident manager (6.82). On the other hand, underconfident managers underestimate the project's net present value (-10.23). The findings also show that overconfident managers underestimate the overall risk (6.49 < 7.96), downside risk (2.48 < 4.81) as well as the value-at-risk (7.30 < 18.51) and the expected shortfall (9.08 < 21.25) measures. Optimistic managers also underestimate the tail risk measures (downside risk, value-at-risk and expected shortfall). On the other hand, underconfident and pessimistic managers overestimate the risk measures. The overestimation is more pronounced in the case of an underconfident manager.

Table 1.3. Managerial Perceptions of *NPV* Mean and Risk Measures

Bias	φ	λ	$E\left(V_1 \vert C\right)$	$\sigma\left(V_1 \vert C\right)$	$\sigma_D\left(V_1 \vert C\right)$	$VaR_{1\%}$	$ES_{1\%}$
Rational	0.10	0.0	0.00	7.96	4.81	18.51	21.25
Overconfident	0.08	0.3	6.82	6.49	2.48	7.30	9.08
Underconfident	0.12	-0.3	-10.23	9.75	8.10	34.27	38.11
Optimist	0.10	0.3	8.53	8.12	3.10	9.13	11.35
Pessimist	0.10	-0.3	-8.53	8.12	6.75	28.56	31.76

Notes. 100,000 skewed normal returns are generated using the distributional parameters of φ and λ as presented in the second and third columns, respectively. The project's net present values are computed using the following net present value equation: $NPV = -178.0777 + \sum_{t=1}^{5} 40(1+x)(1.04)^{-t}$ where x is a generated randomly cash flow growth rate. The third and fourth columns are the mean and standard deviation of the *NPV* values of each bias. The next column presents the downside risk computed as the standard deviation of the negative net present values of each bias. The last two columns present the tail risk measures: value-at-risk (1%) and the expected shortfall (ES).

1.6 Empirical Findings

1.6.1 Data Description and Summary Statistics

Data is collected by the Federal Reserve Bank of Philadelphia in its Bureau of Economic Analysis (BEA) which provides the estimates of macroeconomic indicators for the U.S. economy of nominal and real gross domestic product (GDP).[14] GDP data covers the period from 1965:Q3 to 2019:Q4. BEA publish three quarterly vintages estimations of

[14] https://www.philadelphiafed.org/surveys-and-data (accessed: April 2021).

GDP: First, second and third estimations; the preliminary estimates of each quarter of the final GDP growth rates are published every few days. In addition, annual updates are released in the late of July. The three first releases and the most recent are used as provided by Philadelphia FED on its website.

The difference between the forecast value and the actual value of an economic variable it will be referred as the forecast error (*fe*). Mathematically,

$$fe_{i,t} = f_{i,t} - a_t$$

where i = 1,2, 3, $f_{i,t}$ is the first, second and third estimate (respectively) and a_t is the final (actual) estimate.

Panel A Table 1.4 shows that the means are negative for all the forecast errors and increase across time. This means that all the preliminary estimates underestimate the final GDP growth rates. The standard deviations are close to two in all cases. Negative skewness and excess kurtosis are also presented in all cases. Also, the Bera–Jarque test of normality is rejected. In conclusion, there are skewness and kurtosis characteristics in the forecast errors. Panel B of Table 1.4 presents further statistics of the forecast errors including minimum, quantiles and maximum. The minimum distance between forecasts and actual price is approximately -8.00 while the maximum is almost 5 in the case of nominal GDP and 6 in real GDP (see also figures 1.8 and 1.9). Quantiles show that the deviations vary and therefore, the examination of the deviation between forecast and actual estimations needs consideration.

Overall, the statistics show that the forecast errors of the nominal and real GDP violate the assumption of Normality and present skewness and kurtosis characteristics. The findings show that the probabilistic characteristics of forecasters are not rational and may suffer from psychological biases. These findings are in line with the characteristics of an underconfident forecaster; see proposition 7.

1.6.2 Model Estimation

Maximum likelihood estimates (MLE) for the parameters of the mean, variance (standard deviation), and the two distributional parameters (k and λ) are obtained using the Berndt et al. (1974) procedure. The estimated parameters of the SGED distribution are obtained from the maximization of the log-likelihood specification of forecasting errors under the SGED distribution. That is,

$$L(b_i) = \sum_{t=1}^{T} \log f\left(b_i \,|\, fe_i\right) = \sum_{t=1}^{T} L_t\left(b_i\right) \text{ for } i = 1,2, \text{ and } 3.$$

where $f\left(b_i \,|\, fe_i\right)$ is the likelihood function of SGED, b is a column of the four estimated distributional parameters (expected value μ, standard deviation σ, skewness λ and kurtosis k). The distribution of each forecast error under SGED is

$$f\left(fe\right) = \frac{1}{2\theta\sigma} k^{1-\frac{1}{k}} \Gamma\left(\frac{1}{k}\right)^{-1} \exp\left(-\frac{1}{k}\frac{\left(fe - m + \delta\sigma\right)^k}{(1+\left(sgn(fe - m + \delta\sigma)\lambda\right)^k \theta\sigma^k}\right)$$

where all parameters are explained previously.

Table 1.4. Preliminary Statistics of the Nominal and Real GDP Forecast Error

	Nominal GDP			Real GDP		
	$fe_{1,t}$	$fe_{2,t}$	$fe_{3,t}$	$fe_{1,t}$	$fe_{2,t}$	$fe_{3,t}$
Part A. Preliminary Statistics						
Mean	-0.5534	-0.3804	-0.2983	-0.3998	-0.2977	-0.2419
St. Dev.	1.9433	1.8381	1.8579	2.0204	1.9939	2.0058
Skewness	-0.5022	-0.3914	-0.3336	-0.4101	-0.3855	-0.3150
Exc.Kurtosis	1.4359	1.5896	1.4882	1.8359	1.8716	1.4879
BJ	27.7641*	28.2540*	24.1612*	36.5591*	36.8768*	23.7150*
Part B. Quantiles						
Min	-7.3951	-7.9666	-8.1023	-8.9720	-8.3786	-7.7200
1%	-7.2291	-5.9112	-5.2154	-5.5906	-6.5491	-6.6400
5%	-4.0398	-3.2315	-3.2785	-3.6866	-3.5628	-3.5628
10%	-2.7911	-2.4729	-2.4643	-2.7345	-2.7031	-2.5827
25%	-1.5173	-1.4843	-1.4219	-1.5633	-1.6072	-1.5124
50%	-0.4278	-0.2818	-0.2963	-0.2918	-0.2178	-0.0408
75%	0.5538	0.6888	0.8720	0.9620	0.8818	1.0931
90%	1.7423	1.6906	1.6775	1.8874	1.9523	2.0228
95%	2.4875	2.6024	2.8183	2.5749	2.4534	2.6676
99%	3.9568	3.9747	4.3949	4.5745	4.9033	4.9033
Max	4.6992	4.8687	4.8687	5.6333	6.1962	6.3643
OBS	217	216	218	217	216	218

Notes. Nominal and real GDP forecast data is collected from Federal Reserve Bank of Philadelphia in its Bureau of Economic Analysis (BEA) and covers the period 1965:Q3 to 2019:Q4. The forecast errors are computed by $fe_{i,t} = f_{i,t} - a_t$ where i = 1,2, 3, $f_{i,t}$ are the first, second and third estimates and a_t is the final (actual) estimate. The first two rows present the first two moments, expected value and risk. The skewness and kurtosis are calculated by $SK = m_3 / m_2^{3/2}$ and $KU = m_4 / m_2^2$, where m_j is the j^{th} moment around the mean. The Normality test Bera–Jarque is computed using the equation $BJ = (T/24)(4SK^2 + KU^2)$. * denotes statistical significance at 1%.

Figure 1.8. Nominal GDP Forecast Errors

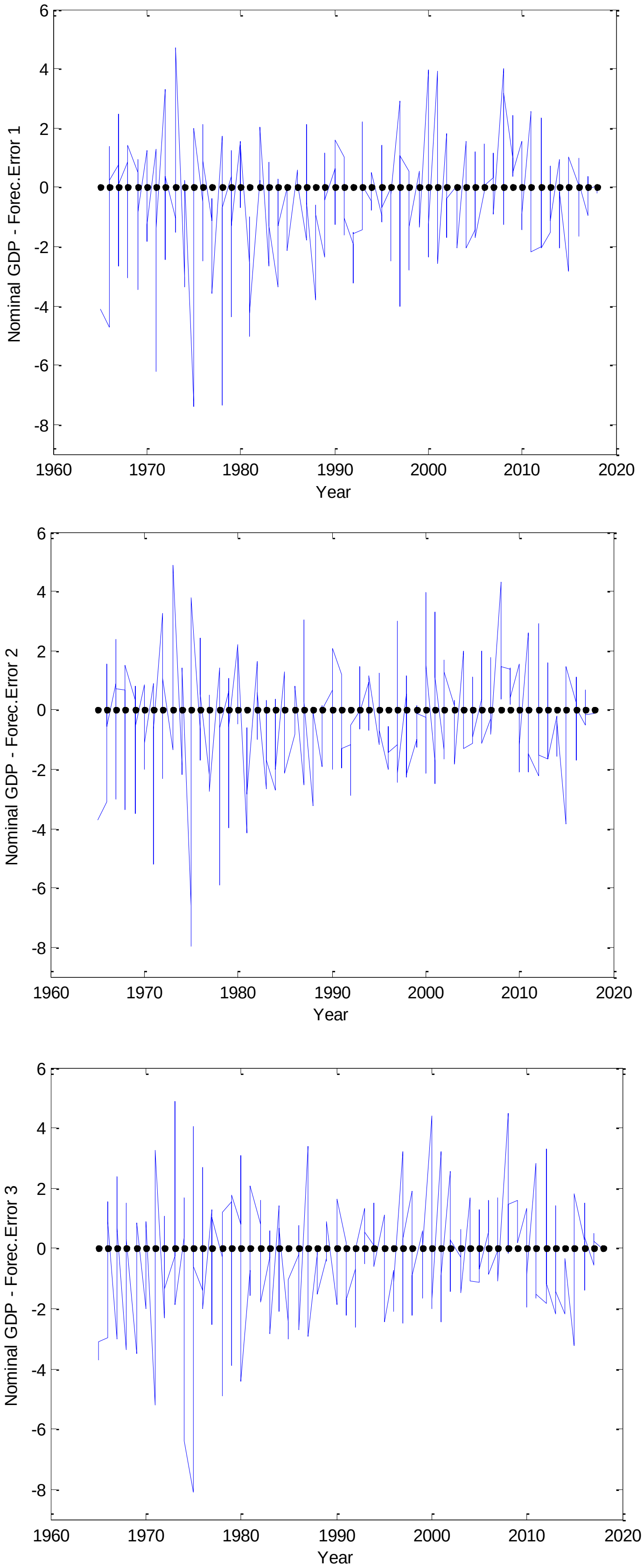
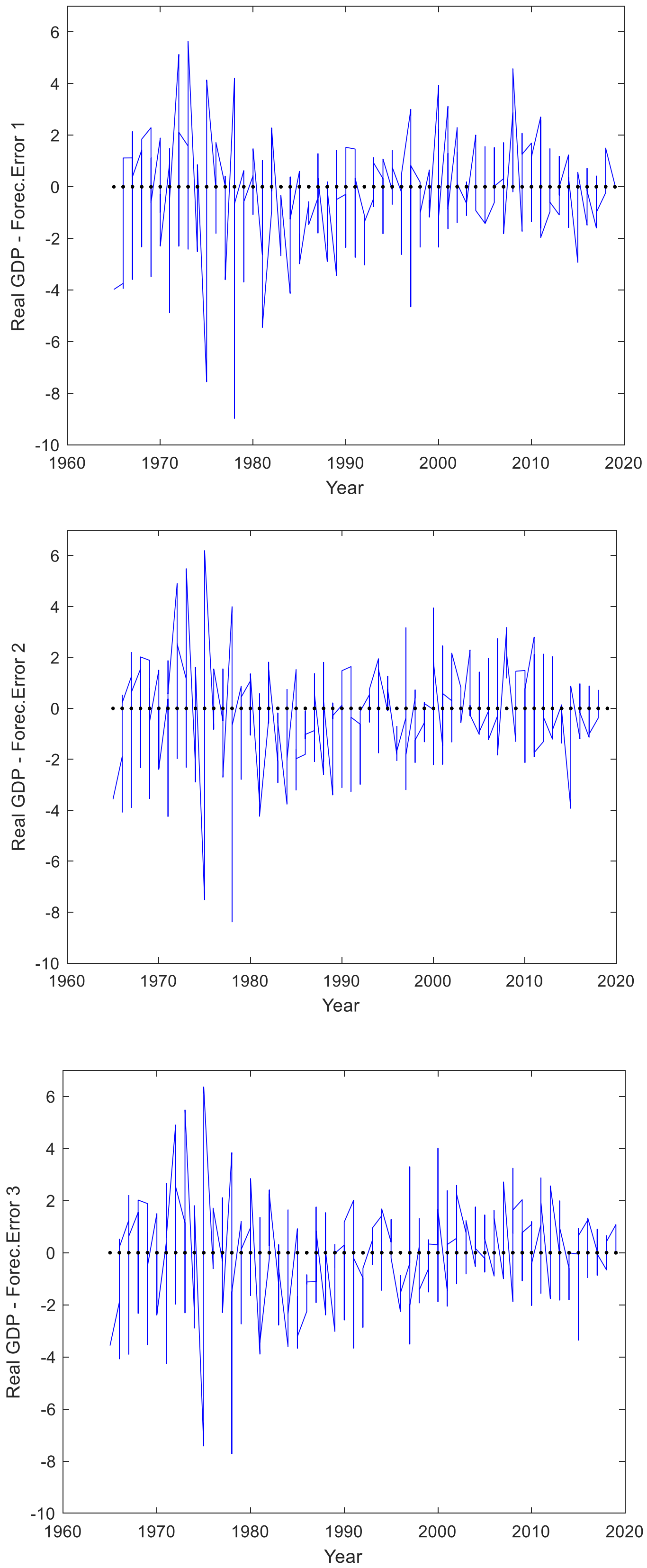


Figure 1.9. Real GDP Forecast Errors



1.6.3 Probability Distribution of Professional Forecasters

Table 1.5 presents the estimated distributional parameters of each forecast error for the nominal and real GDP respectively, under the SGED distribution.

The results based on the SGED distribution show that in both cases (nominal and real), the expected value is negative and statistically significant and increase across time. For example, the expected value of the first, second and third forecast errors of nominal GDP are -0.5541, -0.3793, and -0.3017, respectively. Professional forecasters are not rational since a rational forecaster would ideally have mean equals to zero. Any deviation from zero means that, on average, they are far away from the actual values. In this case, this is an underconfident behaviour of the forecasters (proposition 7). The negative sign of all the forecast errors and the fact that all the coefficients are close to each other show that professional forecasters are also conservative to the adaptation of a new information. This also confirms the previous findings of underestimation of the final GDP growth rates. The standard deviation, in all cases, is approximately two (and statistically significant). Specifically, the standard deviation of each forecast error for the Nominal GDP is 1.9306, 1.8269 and 1.8467, respectively.

The estimated parameter that controls the tails of the distribution, k, is in all cases, statistically significant. The parameter k is below two in all cases, indicating that the distribution is leptokurtic. The asymmetry parameter, which controls the shape of the distribution is negative in all cases. This means that the distribution of the forecasting errors is negatively skewed and has a higher peak relative to the normal distribution (tighter tails). This is an indicator of an underconfident behaviour (proposition 7).

Table 1.5 also shows that the left tail scaling parameter (φ_1) is larger than the right tail scaling parameter (φ_2) indicating that the predictions below the mode are higher compared to the predictions above the mode. If these parameters are equal, this indicates a rational behaviour. Since they are different, the Normality is violated and therefore, psychological distortions are capable to explain this behaviour. In this case, the distribution is negatively skewed. This means that professional forecasters are underconfident (proposition 7). Furthermore, the two different tail parameters are close to each forecast. In all cases, standardized skewness is negative while standardized kurtosis is around 4. Also, the log-likelihood ratio test for the SGED against the Normal distribution shows that Normality is rejected, therefore, SGED explains the behaviour of forecasting errors betters.

To sum up, the forecasting errors are not behaving rationally and therefore, they exhibit psychological distortions. This means that higher moment distributions (such as SGED) are capable to capture this behaviour. The conclusion is that forecasters are underconfident. Interestingly, they remain conservative across their forecasts.

Table 1.5. SGED Distributional Estimators

	Nominal GDP			Real GDP		
Parameter	$fe_{1,t}$	$fe_{2,t}$	$fe_{3,t}$	$fe_{1,t}$	$fe_{2,t}$	$fe_{3,t}$
Part A. Distributional Parameters						
μ	-0.5541	-0.3793	-0.3017	-0.4061	-0.3025	-0.2430
	(0.1826)*	(0.1667)*	(0.1634)*	(0.13824)*	(0.1846)*	(-0.188)
σ	1.9306	1.8269	1.8467	2.0055	1.9789	1.9971
	(0.1124)*	(0.0999)*	(0.1007)*	(0.1143)*	(0.1104)*	(0.1114)*
k	1.3814	1.4457	1.4547	1.3543	1.3788	1.4156
	(0.2388)*	(0.2323)*	(0.2299)*	(0.2017)*	(0.1879)*	(0.1922)*
λ	-0.1299	-0.095	-0.0634	-0.0783	-0.1039	-0.1129
	(0.1255)	(0.1242)	(0.1251)	(0.1273)	(0.1275)	(0.1306)
Part B. Additional Parameters						
φ	1.5700	1.5527	1.5972	1.6641	1.635	1.6627
φ_1	1.7740	1.7003	1.6986	1.7944	1.8050	1.8505
φ_2	1.3661	1.4052	1.4959	1.5338	1.4651	1.4749
mode	-0.1924	-0.1213	-0.1249	-0.1735	-0.0009	0.0875
LogL	-445.651	-433.227	-440.075	-454.26	-449.35	-455.95
LR-Normal	11.8557*	8.4867*	7.5733*	11.5316*	11.406*	9.2367*
SK	-0.3449	-0.2384	-0.1584	-0.2156	-0.2781	-0.2908
KU	4.1526	3.933	3.8882	4.1859	4.1318	4.0316
Obs.	217	216	218	217	216	218

Notes. The table presents the estimated parameters along with their standard errors of the first three forecasts (first, second, third) and the last revision used as the actual value obtained from the maximization of the SGED log-likelihood specification. The parameters μ, σ, λ and k represent the mean, standard deviation, the asymmetry and the shape parameters of the SGED distribution. Mode is computed by equation (1.25). Log-Likelihood is the maximum likelihood value and LR-Normal tests the null hypothesis of Normality against the SGED (alternative hypothesis). * indicates statistically significant at 10%. Nothing stated means that it is statistically insignificant.

1.7 Summary and Conclusions

This chapter focuses on the development of a probabilistic framework based on the skewed normal (SN) and the skewed generalized error (SGED) distributions to model the managerial biases of overconfidence and unrealistic optimism, their counterparts of underconfidence and pessimism, as well as the interrelationship between overconfidence and anchoring and adjustment heuristic.

The probabilistic framework is used to analyse each psychological bias: overconfidence, underconfidence, optimism and pessimism. The probabilistic framework also compares the differences and similarities of these biases and examine their impact on the expected value and risks (overall risk, downside risk, value–at–risk and expected shortfall) of economic variables, e.g., the investment's return, the future cash-flows, and others.

Psychological theory claims that an overconfident manager overestimates the probability of favourable events. At the same time, he/she narrow the tails of the probability distribution of an economic variable under consideration. This miscalibration leads to (a) positively skewed probability distributions for the economic variables under consideration, (b) overestimation of their expected values and (c) underestimation of their overall risk, downside risk, value-at-risk and expected shortfall.

An underconfident manager underestimates the probability of favourable events and imposes wider tails of the probability distribution of an economic variable (negatively skewed distribution).

Optimistic managers underestimate the probability of undesirable events and overestimate the probability of desirable events. Nothing is stated with regards to how optimists view the tails of the probability distribution. This miscalibration will lead to (a) a positively skewed subjective probability distribution for the variables under consideration, (b) an overestimation of their expected values and (c) an underestimation of the downside risk, value-at-risk and expected shortfall. An optimistic manager will overestimate the overall risk of the economic variable under consideration.

On the other hand, a pessimistic manager underestimates the probability of desirable events, overestimates the probability of undesirable events, underestimates their expected value, and overestimates the downside risk, value-at-risk and expected shortfall. In this case, this miscalibration leads to a negatively skewed distribution.

Analytical formulas presented in this chapter showed that anchoring and adjustment and overconfidence bias share an interconnection. In this chapter, a mathematical framework that explains the professional forecaster’s behaviour under an adaptation process has also been developed. The empirical application based on the SGED (Skewed Generalized Error Distribution) distribution. However, any other distributions, such as Skewed Generalized T, Skewed Laplace, etc., can also fit well.

Using Monte Carlo simulations, this chapter showed and proved the above conclusions for the perceptions of the overconfident, underconfident, optimistic, pessimistic, and rational managers on expected mean and the risk measures: overall risk, downside risk, value-at-risk, and the expected shortfall (ES).

Empirically, the three vintages data of BEA (first, second, and third) and the most recent revision are used, and they have shown that forecasting from the actual exhibit skewness and kurtosis characteristics. This result leads to the conclusion that the probability distribution of the underlying variable (e.g. GDP) distorts from the forecasting across time. Therefore, the finding that the vintage data has more than two moments characteristics (skewness and kurtosis) indicate an overconfident/underconfident behaviour.

This chapter has explained the behaviour of forecasters based on the sign of the estimated asymmetry parameter of the skewed distribution. When the asymmetry parameter is positive, it generates a positively skewed distribution and therefore an overconfident behaviour. A key probabilistic characteristic of a positive skewness distribution is that there are more probabilities for an event to occur right of the mode rather than left (more probability mass for good events than bad). The opposite is true when the asymmetry parameter is negative. A negative sign of the asymmetry parameter indicates an underconfident behaviour and more probabilities occur left the mode rather than right (more probability mass left the mode than right). In the empirical application on the professional forecasters, the results show that a forecaster’s probability distribution is negatively skewed (consequently underconfident); therefore, they underestimate the nominal and real GDP. Noteworthy is also the fact that the understanding of this interrelationship is important for all participants because forecasting in GDP affects a lot of people in their decisions such as bankers, investors, consumers, businesses, etc.

Nevertheless, this chapter is subjected to some limitations that can be addressed for future research. For instance, there are many other psychological biases that behavioural finance literature uses to explore various topics in the empirical studies. Currently, the first chapter of this dissertation explains the most common psychological biases of overconfidence and optimism (and their counterparts) to clarify these biases and the differences between them. However, this study provides the foundation for further investigation of the behaviour of agents using alternative biases.

Furthermore, the empirical application provides important implications for those who take into account these estimates (e.g., people, policymakers, investors, bankers, and others) in their decisions. For example, in a potential recession, businesses and consumers will be conservative in their decisions (e.g., hiring employees, getting mortgage loans, and others). In addition, other distributions such as the skewed Laplace can be used in a probabilistic framework to explore the perceptions of agents on mean and risk about a decision-making variable. Specifically, other distributions of the SGT family can accommodate more characteristics, therefore, can be used in a similar framework.

2 Exploring the Stochastic Behaviour of Bitcoin under an Asymmetric Framework

2.1 Introduction

Through the years, the development of digital currencies has been rapid and extraordinary. Bitcoin is a virtual currency introduced by Nakamoto (2008) as a peer-to-peer cash system. Up to today, there are more than 2,000 cryptocurrencies with Bitcoin being the most popular followed by Ethereum, Ripple, Tether, Bitcoin Cash, Litecoin, etc.

The popularity of Bitcoin is emphatic in figures 2.1 and 2.2. Figure 2.1 showing that the current number of Bitcoin's daily transactions is extremely high compared to eight years before. In 2018, the daily bitcoin transactions have peaked at approximately 400,000 (see figure 2.1).[15] Also, the multi-billion-dollar market capitalization of Bitcoin is what makes it the most popular cryptocurrency. The market capitalization of Bitcoin is on the top spot of the cryptocurrencies' total. [16]

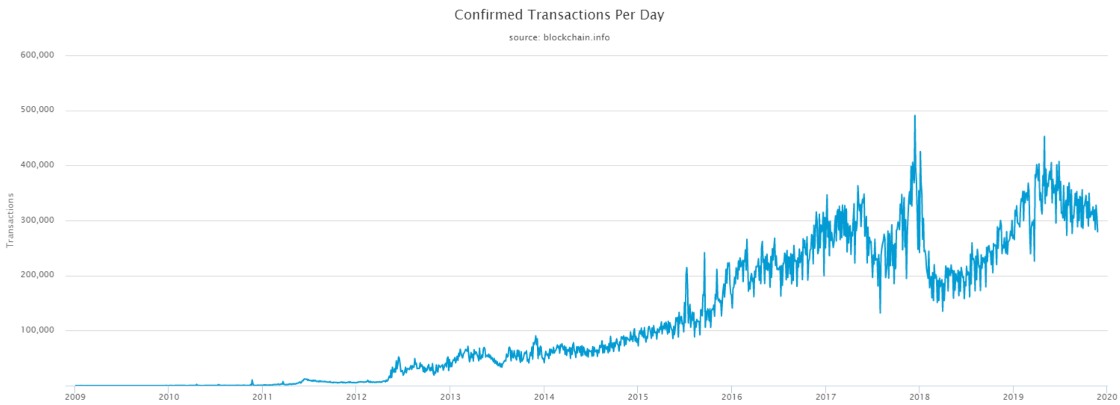


Figure 2.1. The number of daily confirmed Bitcoin transactions.[17]

A characteristic of Bitcoin is that allows online payments from one party to the other without an intermediary, e.g. financial institution. Therefore, there is no government or monetary policy. Figure 2.2 presents daily bitcoin prices and trading volume. Prices and trading volume follow the same pattern. Bitcoin prices in 2011 started at the price of 0.06 cents and reached 8,000 dollars in 2019.

---

[15] Cited in https://blockchain.info/charts/n-transactions (accessed: April 2021).
[16] See https://data.bitcoinity.org (accessed: April 2021).

[17] Source: https://blockchain.info/charts/n-transactions (accessed: April 2021).

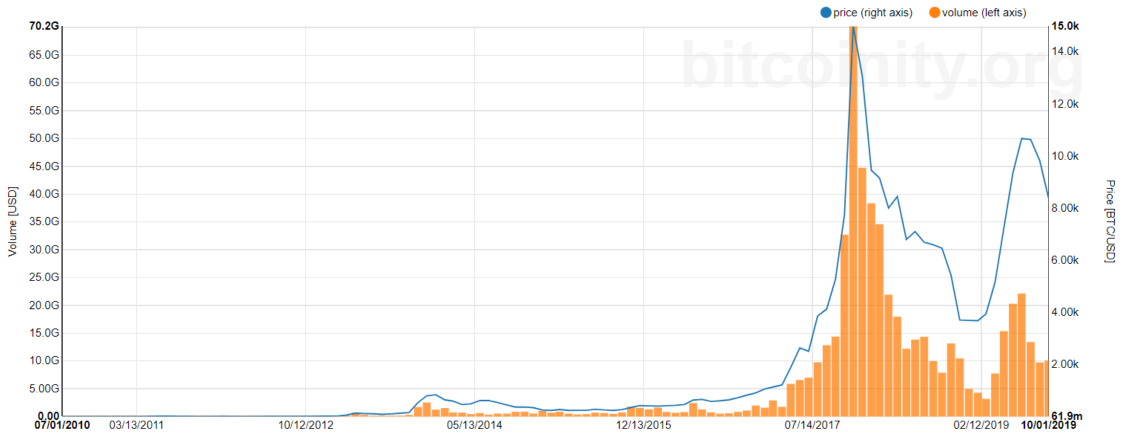


Figure 2.2. Bitcoin Prices and Trading Volume.[18]

Empirical evidence suggests that Bitcoin behaves in a very different way compared to the rest of the assets (e.g., exchange rates, bonds, equities, etc.). Hence, investors opt to use Bitcoin in their portfolios to reduce their risk. The investigation of the relation between such currencies (henceforth Bitcoin) and exchange rates is important for investments, trading, and hedging strategies. More specifically, the understanding of the co-movement between Bitcoin and exchange rates is useful for investors since it will give them a hint on how to diversify their portfolios. It is also of interest to the traders of forex markets since they investigate the behaviour of various currencies. Finally, it is of interest to the general audience since Bitcoin can be used as a money substitute.

This chapter investigates the stochastic behaviour of Bitcoin and exchange rates under a flexible framework that accounts for a time-varying skewness and kurtosis price of risk. This is the first time the literature will examine the dynamic behaviour of Bitcoin using a skewness-kurtosis price model. More specifically, the contributions of this chapter are the following: Firstly, it investigates the stochastic properties of bitcoin and exchange rates (such as first and second-moment dependencies and non-linearities). Secondly, it links the time-varying skewness-kurtosis price of risk to downside and upside volatility using the ST-GJR–GARCH model (Savva and Theodossiou, 2018) under the SGED distribution. Thirdly, it examines the spillover effects of Bitcoin and exchange rates. Fourthly, the forecasting accuracy of Bitcoin's prices is computed using the ST-GJR-GARCH-SGED model and is compared to the rest assets. Finally, by examining the

[18] Source: https://data.bitcoinity.org/ (accessed: April 2021).

behaviour of Bitcoin, this study sheds light on the trading and hedging capabilities helping investors to decide whether to incorporate it in their portfolios or not.

Empirically, skewness and kurtosis characteristics leading to the rejection of the Gaussian distribution can be found in all return series. This prevents the necessity for higher-moment probability distributions. The asymmetric and kurtosis characteristics are more profound in the case of Bitcoin. Also, higher-order dependencies are presented in the series. Empirical findings in the univariate and bivariate analysis suggest that the skewness\kurtosis price of risk has an important role in the model (especially in the case of Bitcoin). The findings suggest that there are implications for those who use Bitcoin as a financial asset. The importance of the time-varying skewness and shape parameters proved the presence of asymmetric behaviour. In the bivariate analysis (spillover effects), the common exchange rates affect the conditional mean and volatility of Bitcoin more than the reverse, Bitcoin has no effect on the conditional mean and volatility of the other exchange rates, and the shape of the Bitcoin's probability distribution is not affected when spillover effects are presented. Overall, the findings reveal a negligible relationship between Bitcoin and exchange rates.

Accordingly, section 2.2 presents a literature review about Bitcoin and its statistical behaviour compared to other assets. Section 2.3 presents the time-varying ST-GARCH-GJR-SGED model and explains in detail the conditional mean and variance equations. In addition, there is a presentation of the conditional asymmetry and shape parameters as time-varying parameters to investigate if the distribution change over time. The distribution used is the skewed generalized error distribution (SGED). Section 2.4 analyses the model estimation technique while section 2.5 analyses data using preliminary statistics and tests for non-linearities (higher-moment dependencies). Moreover, there is a presentation of the unconditional results of the return series of this distribution. Section 2.5 also presents the results of the risk and return relationship and shows the impact that the time-varying skewness and kurtosis parameters have on the returns (for a univariate case as well as on a bivariate case to examine the spillover effects from Bitcoin to exchange rates and vice versa). Summary and conclusions are presented in section 2.6.

2.2 Literature Review

The History of Bitcoin

Decentralized cryptocurrencies and specifically Bitcoin is the most popular digital currency over the more than 2,000 altcoins that exist.[19] Nakamoto (2008), introduced Bitcoin and the blockchain as a peer–to–peer cash system. If computer users can solve pre-specified mathematical problems, new bitcoins are created, and this process is called 'mining'. The technology behind Bitcoin, blockchain, public accounting ledger, and others was explained by Yermack (2015). An anonymous address is created and transfers bitcoins through a network.

In short, Bitcoin has the following particularities:

- There is no central authority for Bitcoin.
- Bitcoin's network is peer-to-peer consequently there is no central server.
- There is no central storage; the bitcoin ledger is distributed.
- Anybody can use ledger as a public store.
- Anybody can be a 'miner'.
- Creating a bitcoin address is an easy procedure. There is no need for any approval from a bank or any other financial institution.
- The transaction can be sent without any approval.

The above characteristics drive a lot of investors to increase their interest in the cryptocurrency market.

Bitcoin as a Financial Asset

As for the economic and financial effects of Bitcoin, several scholars have investigated them in various ways. Authors have pointed out that Bitcoin apart from an online payment method, it is also a financial and speculative asset (e.g., Yermack, 2015; Baur et al., 2016; Kristoufek, 2015). Dyhrberg (2016a) showed that Bitcoin is somewhere between a currency and a commodity, but it will never behave exactly as a currency.

The high returns and volatility of Bitcoin lead Briere et al. (2015) to incorporate Bitcoin into a portfolio with other financial assets. They found that the correlations between Bitcoin and the other assets are significantly low and the benefits of including Bitcoin in portfolio improve its risk-return characteristics. Therefore, the portfolios'

[19] See http://coinmarketcap.com/ (accessed: April 2021).

diversification can be achieved using Bitcoin as a suitable asset to reduce the potential risk (Bouri et al., 2017b). Dyhrberg (2016a, 2016b) using GARCH models investigated the hedging and asset capabilities of Bitcoin. The hedging capabilities of Bitcoin against stocks have been found to be important.

The increasing interest of people in cryptocurrencies drives researchers to investigate their statistical behaviour in comparison to traditional assets. Statistically, researchers showed that Bitcoin doesn't follow the normal distribution. Osterrieder and Lorenz (2017) investigated the tail risk characteristics (value–at–risk and expected shortfall) of Bitcoin and compared it to the G10 currencies. They concluded that Bitcoin's probability distribution is not normal, and it exhibits characteristics such as six to seven higher standard deviations than the other currencies and heavier tails. Osterrieder (2016), Chu et al. (2017), and Phillip et al. (2018) proved that cryptocurrencies, among others, follow a heavy-tailed distribution (such as Student t distribution, generalized hyperbolic distribution). Takaishi (2018) finds that the distribution of Bitcoin's return distribution is leptokurtic. A statistical view about cryptocurrencies was investigated by Osterrieder et al. (2017). They found that Bitcoin is the least risky cryptocurrency of the others.

Bitcoin has gained the attention of many people, leading the cryptocurrencies' values, in many cases, to reach extreme levels. This behaviour needs a probability distribution triggered by skewness and kurtosis characteristics. The family of skewed generalized t distribution (SGT) includes the skewed generalized error distribution (SGED), the Skewed t (ST), the Skewed Normal (SN), the Skewed Cauchy (SC), the Skewed Laplace (SL), etc as special cases. In this chapter of the dissertation, the focus is on the SGED; the distribution proven to fit well in financial data (Theodossiou, 2015).

Mean and Volatility Spillovers

The asymmetry in volatility and volatility spillovers is another aspect that has been examined extensively in financial markets literature (e.g. Theodossiou and Lee, 1993; Theodossiou, 1994; Yang and Doong, 2004; Savva et al., 2009; Savva and Aslanidis, 2010; among others). Nevertheless, this kind of analysis is very limited in the case of cryptocurrencies.

Barndorff–Nielsen et al. (2010) proposed an alternative measure of risk to investigate volatility. They introduced the downside risk measure which is based on the semivariance (negative variance). Baruník et al. (2015, 2016, and 2017), in order to quantify asymmetries in spillovers, they used the semi-variances to estimate the volatility spillovers due to bad and good volatility. They found, through the use of a spillover asymmetry measure, that the asymmetry in spillover is more due to bad volatility. Reboredo et al. (2016) investigated the risk spillovers in downside and upside terms using exchange rates and stock price data. They found an asymmetric risk spillover effect from exchange rates to stock prices and vice versa. The asymmetric risk spillover effect is found to be greater in the downside risk than in the upside.

However, the investigation of the asymmetry volatility spillover in Bitcoin is restricted. The role of volatility spillover on cryptocurrencies was closely investigated by Corbet et al. (2018). Corbet et al. (2018) used three cryptocurrencies as well as gold, bond, equities, and VIX and found that there is an interconnection between the cryptocurrencies; however, this is not true for the other assets. They found that the cryptocurrency market is isolated from the financial markets.

Bouri et al. (2018) also examined the asymmetry return and volatility spillover effect between Bitcoin and other assets using the VAR-GARCH–in–mean model. They found that there are spillover effects between Bitcoin and the other assets under different market conditions (bear and bull markets), Bitcoin and other assets are more closely linked to return versus volatility, and Bitcoin imparts little variability as opposed to what it receives. Kurka (2019) also examined the asymmetric transmissions between Bitcoin and traditional assets and found a negligible connectedness between them confirming the diversifier and hedge properties of Bitcoin. Symitsi and Chalvatzis (2018) found significant return spillovers from technology and energy stocks to Bitcoin, and Koutmos (2018) concluded that among 18 cryptocurrencies, Bitcoin is the key transmission contributor of return and volatility spillovers. The chapter of this dissertation attempts to investigate the behaviour of Bitcoin using a flexible time-varying skewness and kurtosis price of risk model. This model allows examining the stochastic behaviour of Bitcoin and understanding in more detail the inter-relationship between Bitcoin and other financial traditional assets.

2.3 Asymmetric Framework

GARCH-M with Dynamic Skewness and Kurtosis Parameters

A common modelling method that investigates the variation of an asset can be done based on the Autoregressive Conditional Heteroscedasticity (GARCH) models. A lot of papers examined Bitcoin using GARCH models.[20,21] This is the first time Bitcoin and the relationship between Bitcoin and other assets is examined using the ST-GJR–GARCH framework (Savva and Theodossiou, 2018) that follows the SGED distribution. This is a reasonable development since literature proved that Bitcoin does not follow the normal distribution but a heavy-tailed distribution.

In this section, there will be a presentation of the parameters of the conditional variance and conditional mean, the conditional asymmetry and shape equations of Bitcoin's returns (BTC) and exchange rates (Euro, Japanese Yen, Canadian Dollar, British Pound). All rates are expressed in US dollar log-price changes. The log-price changes have been computed using the equation

$$r_{i,t} = 100 \cdot \left(\ln X_{i,t} - \ln X_{i,t-1} \right),$$

where $X_{i,t}$ is the US dollar price of currency i at time t and i = BTC, EUR, JPY, CAN, and GBP.

Volatility clustering and asymmetric volatility characteristics have been showed to exist in financial and currency markets including bitcoin prices, e.g., Corbet et al. (2018). Volatility clustering refers to the fact that large price shocks tend to be followed by large price shocks but of either sign. Respectively, (negative) asymmetric volatility phenomenon is the tendency of the volatility to be higher when the market downs than when the market is rising. These characteristics are triggered by skewness and excess kurtosis in the distribution of return series, e.g., Theodossiou (2015).

These phenomena can be modelled using asymmetric GARCH specifications. The conditional variance equation is a function of past squared errors and past conditional variances. Furthermore, the conditional mean of an asset's returns is a function of the conditional standard deviation (GARCH-M models). The GJR-GARCH-M model with time-varying conditional asymmetry and shape parameters is employed in this chapter to investigate the stochastic properties of the Bitcoin in relation to those of the exchange rates: Euro, Japanese yen, Canadian dollar, and British pound.

[20] See for example Katsiampa, (2017), Cermak, (2017), Bouri et al. (2017a), and Guesmi et al. (2019).

[21] The examination of volatility spillovers has been also done using GARCH models. e.g., Theodossiou and Lee (1993) and Yang and Doong (2004).

2.3.1 Distribution of Returns and Moments (Mean and Variance) - SGED

The pdf of BTC and exchange rates is modelled using the Generalized Error Distribution (GED – non centered) computed by

$$f\left(r_{i,t}\middle|I_{t-1}\right)=\frac{k_{i,t}^{-\frac{1}{k_{i,t}}+1}}{2\phi_{i,t}}\Gamma\left(\frac{1}{k_{i,t}}\right)^{-1}\exp\left(-\frac{1}{k_{i,t}}\frac{\left|r_{i,t}-m_{i,t}\right|^{k}}{\left(1+sgn(r_{i,t}-m_{i,t})\lambda_{i,t}\right)^{k_{i,t}}\varphi_{i,t}^{\;k_{i,t}}}\right) \tag{2.1}$$

where $\lambda_{i,t}$, $k_{i,t}$ and $m_{i,t}$ are respectively time-varying asymmetry, shape and the mode parameters of the distribution of $r_{i,t}$ and $\Gamma(\cdot)$ is the gamma function. The computations of the moments (mean and variance) and further statistics are shown in chapter 1, therefore, in this chapter, the equations for the SGED are shown directly.

The mean and variance of Bitcoin and exchange rates using the SGED are respectively

$$\mu_{i,t}=m_{i,t}+M_{1,i,t}=m_{i,t}+2\lambda_{i,t}G_{1,i,t}\phi_{i,t}=m_{i,t}+\delta_{i,t}\sigma_{i,t} \tag{2.2}$$

$$\sigma_{i,t}^{\;2}\equiv\operatorname{var}\left(r_{i,t}\right)=M_{2,i,t}-M_{1,i,t}^{2}=\left(\left(3\lambda_{i,t}^{\;2}+1\right)G_{2,i,t}-4\lambda_{i,t}^{\;2}G_{1,i,t}^{2}\right)\phi_{i,t}^{\;2}=\theta^{-2}\varphi^{2} \tag{2.3}$$

where for s = 1, $M_{1,i,t}=2\lambda_{i,t}G_{1,i,t}\varphi_{i,t}$ and for s = 2, $M_{2,i,t}=\left(3\lambda_{i,t}^{2}+1\right)G_{2,i,t}\varphi_{i,t}^{2}$,[22]

$$\theta\equiv1\Big/\sqrt{\left(3\lambda_{i,t}^{2}+1\right)G_{2}-4\lambda_{i,t}^{2}G_{1,i,t}^{2}} \tag{2.4}$$

$$G_{i,1,t}=k_{i,t}^{\frac{1}{k_{i,t}}}\Gamma\left(\frac{2}{k_{i,t}}\right)\Gamma\left(\frac{1}{k_{i,t}}\right)^{-1} \tag{2.5}$$

and

$$G_{i,2,t}=k_{i,t}^{\frac{2}{k_{i,t}}}\Gamma\left(\frac{3}{k_{i,t}}\right)\Gamma\left(\frac{1}{k_{i,t}}\right)^{-1}. \tag{2.6}$$

The skewness price of risk is computed by

$$\delta_{i,t}=\frac{\mu_{i,t}-m_{i,t}}{\sigma_{i,t}}=2\lambda_{i,t}G_{i,1}\theta_{i,t}. \tag{2.7}$$

where all notations are explained above.

[22] Note that $-\left(1-\lambda\right)^{2}+\left(1+\lambda\right)^{2}=4\lambda$ and $\left(1-\lambda\right)^{3}+\left(1+\lambda\right)^{3}=2\left(3\lambda^{2}+1\right)$.

The returns for Bitcoin and exchange rates are modelled using the centered SGED (Skewed Generalized Error Distribution). That is,

$$f_r\left(r_{i,t}\left|I_{t-1}\right.\right)=\frac{1}{2\theta_{i,t}\sigma_{i,t}}k_{i,t}^{1-\frac{1}{k_{i,t}}}\Gamma\left(\frac{1}{k_{i,t}}\right)^{-1}$$

$$\exp\left(-\frac{1}{k_{i,t}}\left|\frac{r_{i,t}-\mu_{i,t}+\delta_{i,t}\cdot\sigma_{i,t}}{\left(1+sgn(r_{i,t}-\mu_{i,t}+\delta_{i,t}\cdot\sigma_{i,t})\lambda_{i,t}\right)\theta_{i,t}\cdot\sigma_{i,t}}\right|^{k_{i,t}}\right), \tag{2.8}$$

where $\mu_{i,t}$, $\sigma_{i,t}$, $\delta_{i,t}$, $\theta_{i,t}$, $\lambda_{i,t}$, $k_{i,t}$ and $\mu_{i,t}$ are as defined previously. The SGED has been used in the literature to model the time-series behaviour of returns of currencies, stock indices, freight rates, and others. The Skewed Generalized Error Distribution of Theodossiou (2015) yields to the skewed Laplace for $k_{i,t}$ = 1, to the Laplace for $k_{i,t}$ = 1 and $\lambda_{i,t}$ = 0, to the well-known Skewed Normal distribution commonly used in many articles (e.g., Feunou et al., 2012) for $k_{i,t}$ = 2, to the Normal distribution for $k_{i,t}$ = 2 and $\lambda_{i,t}$ = 0, to the uniform distribution for $k_{i,t} = \infty$ and others.

2.3.2 Conditional Variance, Mode, and Mean in GJR-GARCH-M

The conditional variance and conditional mean of the returns of BTC, EUR, JPY, CAN and GBP are modelled using the Glosten et al. (1993) GARCH-M model. Using the GJR GARCH-M, a conditional time-series asymmetry and conditional shape parameters are also included in the model. This framework follows that of Savva and Theodossiou (2018). For the rest of this chapter, this model will be mentioned to as the ST-GJR GARCH-M model.

Using the GJR GARCH-M model, the two volatility phenomena (volatility clustering and asymmetric volatility) for each of the five currencies of returns are modelled through the following conditional variance equation

$$\sigma_{i,t}^2=\operatorname{var}\left(r_{i,t}\left|I_{t-1}\right.\right)=v_i+\left(a_{N,i}N_{i,t-1}+a_i\right)\varepsilon_{i,t-1}^2+\beta_i\sigma_{i,t-1}^2, \tag{2.9}$$

where

$$\varepsilon_{i,t}=r_{i,t}-\mu_{i,t}$$

is the error term excess from its conditional mean and i = BTC, EUR, JPY, CAN, and GBP. The indicator variable $N_{i,t}$ takes the value of one for negative error values of $\varepsilon_{i,t}$, and zero otherwise. That is,

$$N_{i,t} = 1 \text{ for } \varepsilon_{i,t} < 0 \text{ and } N_{i,t} = 0 \text{ for } \varepsilon_{i,t} \geq 0$$

Note that the conditional variance of returns at time t is based on the information set at time $t-1$ (I_{t-1}). The coefficients $a_{N,i}$, a_i and β_i are indicative of asymmetric volatility, volatility clustering and persistence. The coefficient $a_{N,i}$ measures the impact of past shocks (negative) on volatility at time t (current). The coefficient β_i measures the persistence of volatility in the market i.

The conditional mode and mean of returns for each currency are linear functions of past returns as well as their conditional standard deviations. That is,

$$m_{i,t} = mode\left(r_{i,t} \middle| I_{t-1}\right) = m_{0,i} + b_i\, r_{i,t-1} + c_i \sigma_{i,t} \tag{2.10}$$

and

$$\mu_{i,t} = E\left(r_{i,t} \middle| I_{t-1}\right) = m_{i,t} + \delta_{i,t}\sigma_{i,t}$$

$$= m_{0,i} + b_i\, r_{i,t-1} + \left(c_i + \delta_{i,t}\right)\sigma_{i,t}, \tag{2.11}$$

where $m_{0,i}$ is a regression intercept and b_i is an autoregressive coefficient. As will be discussed in the next equations, the conditional mode plays a key role in the definition of upside and downside markets for currencies. For each currency, the time-varying sum $\xi_{i,t} = c_i + \delta_{i,t}$, is the total price of risk and measures the impact of risk on mean returns. The coefficients c_i and $\delta_{i,t}$ are the pure and the skewness price of risk (equation 2.7), respectively. This decomposition introduced on the paper of Theodossiou and Savva (2016).

2.3.3 Conditional Asymmetry Parameter

The conditional asymmetry parameter of the distribution of $r_{i,t}$ is computed as

$$\lambda_{i,t} = asym\left(r_{i,t} \middle| I_{t-1}\right) = 1 - \frac{2}{1+e^{h_{i,t}}}, \tag{2.12}$$

where

$$h_{i,t} = \gamma_{0,i} + \gamma_{N,i}u^{-}_{i,t-1} + \gamma_{P,i}u^{+}_{i,t-1} + \gamma_{h,i}h_{i,t-1} \tag{2.13}$$

$u^{-}_{i,t-1} = \left|u_{i,t-1}\right|$ for negative shocks (zero otherwise),

$u^{+}_{i,t-1} = u_{i,t-1}$ for positive shocks (zero otherwise)

and

$$u_{i,t-1} \equiv \frac{r_{i,t-1} - m_{i,t-1}}{\sigma_{i,t-1}} \tag{2.14}$$

where u_{t-1} are the standardized returns excess the conditional mode and $u_{i,t-1} = -\left|u^-_{i,t-1}\right| + u^+_{i,t-1}$.

The absolute values of the negative shocks $u_{i,t}$'s (downside shocks) are used to investigate the downside markets. The positive values (upside shocks) are used to investigate the upside markets. A negative intercept $\gamma_{0,i}$ leads to a negative impact on the asymmetry parameter $\lambda_{i,t}$. This also leads to a negative impact on the conditional price of risk $\delta_{i,t}$. The downside coefficient $\gamma_{N,i}$ measures the impact of negative shocks on the asymmetry index $h_{i,t}$. The asymmetry parameter $\lambda_{i,t}$ is a function of the asymmetry index, therefore, the downside coefficient also measures the impact of negative shocks on this parameter. A positive downside coefficient means that past downside shocks will have a positive impact on these two parameters (asymmetry index and asymmetry parameter). The opposite occurs in the case of a negative downside coefficient. On the other hand, the upside coefficient $\gamma_{P,i}$ measures the impact of past positive shocks on the asymmetry index and consequently on the asymmetry parameter. A positive upside coefficient shows that past upside shocks have a positive impact on these parameters (asymmetry index and asymmetry parameter). The opposite also occurs in the case of a negative upside coefficient. At the same time, the coefficient $\gamma_{h,i}$ measures the persistence of past shocks (downside and upside) on the asymmetry parameters.

2.3.4 Conditional Shape Parameter

The dynamic behaviour of the shape parameter of the distribution of returns is examined using an equation that allows for a time-varying tail parameter.[23] That is,

$$k_{i,t} = k_U - \frac{\left(k_U - k_L\right)}{1 + e^{g_{i,t}}}, \tag{2.15}$$

where

$$g_{i,t} = d_{0,i} + d_{N,i} u^-_{t-1} + d_{P,i} u^+_{t-1} + d_{h,i} \cdot g_{i,t-1}$$

u^-_t and u^+_t are as defined in sub-section 2.3.3. Equation (2.15) depends on the k_L and k_U parameters. These parameters are the minimum and maximum bounds of the time varying shape parameter $k_{i,t}$. In the cases of the Laplace and the Normal distributions, these parameters are set to k_L = 1 and k_U = 2, respectively. In special cases, k_L can be lower than

[23] See also Mazur and Pipien (2018).

one, e.g. the distribution of Bitcoin returns. Importantly, zero values for $d_{N,i}$ and $d_{P,i}$ indicate that the shape parameter $k_{i,t}$ is not time-varying. The downside ($d_{N,i}$) and upside ($d_{P,i}$) parameters control the shape of the distribution left and right the mode $m_{i,t}$.

2.3.5 Downside and Upside Probabilities

The probabilities for downside and upside markets can be computed using respectively the following equations

$$P\left(r_{i,t} \leq m_{i,t}\right) = \int_{-\infty}^{m_{i,t}} f_r\left(r_{i,t}\right) dr_{i,t} = \left(1-\lambda_{i,t}\right)/2 = \frac{1}{1+e^{h_{i,t}}} \tag{2.16}$$

and

$$P\left(r_{i,t} > m_{i,t}\right) = \left(1+\lambda_{i,t}\right)/2 = \frac{1}{1+e^{-h_{i,t}}} \tag{2.17}$$

; see also Savva and Theodossiou, (2018). Equations (2.16) and (2.17) are functions of the asymmetry parameter $\lambda_{i,t}$, and the asymmetry index $h_{i,t}$. These equations can be used to calculate the probabilities of downside and upside markets (excess the mode). For instance, equation (2.16) computes the probability of the returns to be below the mode. Respectively, equation (2.17) computes the probability of the returns to be above the mode. The difference between upside and downside probability gives the asymmetry parameter. To avoid repeating further computations, see chapter 1 equations (1.1) – (1.10). Also, for the computation of the conditional mean, variance, Pearson skewness see equations (1.23)-(1.30) (chapter 1).

The previous analysis is extended on a bivariate context to investigate the mean and volatility spillovers from Bitcoin to exchange rates and vice versa.

2.3.6 Conditional Mean and Variance – Spillover Effects

The conditional mean and conditional variance of bitcoin rates and exchange rates (log-bitcoin and exchange rates changes) are modelled using the following bivariate GJR specification (Glosten et al., 1993):

$$\mu_{i,t} = E\left(r_{i,t} \middle| I_{t-1}\right) = m_0 + b_i\, r_{i,t-1} + \left(c_i + \delta_{i,t}\right)\sigma_{i,t} + \sum_{j\neq i=1}^{M} b_{i,j}\, r_{j,t-1} + \sum_{j\neq i=1}^{M} \xi_{i,j}\sigma_{j,t} \tag{2.18}$$

and

$$\sigma_{i,t}^2 = \operatorname{var}\left(r_{i,t} \middle| I_{t-1}\right) = v_i + \left(a_{N,i} N_{i,t-1} + a_i\right)\varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 + \sum_{j\neq i=1}^{M} \left(a_{i,j} N_{j,t-1} + \beta_{i,j}\right)\varepsilon_{j,t-1}^2 \tag{2.19}$$

where i, j = BTC, EUR, JPY, CAN, and GBP and $N_{i,t} = 0$ for $\varepsilon_{i,t} > 0$ and $N_{i,t} = 1$ for $\varepsilon_{i,t} < 0$ and $r_{i,t} = \mu_{i,t} + \varepsilon_{i,t}$ and $i = 1, 2,\ldots, M$. The time varying skewness price of risk $\delta_{i,t}$ is given by equation (2.7).

All coefficients are explained above with the exception of the additional parameters that investigate the spillover effects. The coefficient $\xi_{i,j}$ measures the impact of risk (standard deviation) in market j on the mean of rates in the market i. The coefficients $a_{i,j}$ and $\beta_{i,j}$ (for $i \neq j$) measure asymmetric volatility and volatility spillovers from j market into the i market.

2.3.7 Forecasting Bitcoin Prices

The expected price of currency i (time t) is [24]

$$E\left(X_{i,t}\right) = X_{i,t-1} E e^{r_{i,t}} = X_{i,t-1} \int_{-\infty}^{\infty} e^{\mu_{i,t} + \sigma_{i,t} \cdot z_{i,t}} f_z\left(z_{i,t}\right) dz_{i,t} = X_{i,t-1} e^{\mu_{i,t} + \ln E_{i,t}^{z}},$$

where

$$E_{i,t}^{z} \equiv \int_{-\infty}^{\infty} e^{\sigma_{i,t} \cdot z_{i,t}} f_z\left(z_{i,t}\right) dz_{i,t} \tag{2.20}$$

and $z_{i,t}$ is a standardized return for currency i. The probability density function for $z_{i,t}$ is computed from equation (2.8) by substituting $\mu_{i,t} = 0$ and $\sigma_{i,t} = 1$.

Prices will be martingale processes in the absence of arbitrage opportunities. The rate of return of the period is

$$\hat{r}_{i,t} = k_{f,i,t} + \rho_{i,t},$$

where $k_{f,i,t}$ is the risk-free rate (conditional) in country i and $\rho_{i,t}$ a risk premium (conditional) for currency i. In this case, the expected value discounted by this rate of return (time t) gives

$$E\left(X_{i,t}\right) e^{-\hat{r}_{i,t}} = X_{i,t-1} \cdot e^{\mu_{i,t} + \ln E_{i,t}^{z} - k_{f,i,t} - \rho_{i,t}} = X_{i,t-1}.$$

This equality leads to the conclusion that

$$\hat{r}_{i,t} = k_{f,i,t} + \rho_{i,t} = \mu_{i,t} + \ln E_{i,t}^{z} \tag{2.21}$$

and

$$\hat{X}_{i,t} = X_{i,t-1} \cdot e^{\hat{r}_{i,t}}, \tag{2.22}$$

[24] See also Theodossiou et al. (2020).

where $E^{z}_{i,t}$ is given by equation (2.20).

2.4 Model Estimation

The parameters of the conditional mean and conditional variance equations as well as the parameters of the asymmetry and shape equations are obtained using an optimization procedure (Berndt et al., 1974) to a conditional log-likelihood function. That is,

$$L\left(\boldsymbol{\theta}_i\right)=\sum_{t=1}^{T}\log f_r\left(\boldsymbol{\theta}_i\left|r_{i,t},I_{t-1}\right.\right)=\sum_{t=1}^{T}L_t\left(\boldsymbol{\theta}_i\right), \tag{2.23}$$

where $f_y(\boldsymbol{\theta}_i \mid r_{i,t}\, \mathrm{I}_{t-1})$ is the conditional likelihood function for currency i returns (equation 2.8) and $\boldsymbol{\theta}_i$ is a column that includes the maximum likelihood estimated parameters. The dynamic skewness price of risk $\delta_{i,t}$ is computed using the substitution of the maximum likelihood estimates for $k_{i,t}$ and $\lambda_{i,t}$ into equation (2.7). Robust standard errors for the maximum likelihood estimates denoted by $\tilde{\boldsymbol{\theta}}_i$ are computed by the following equation

$$\mathrm{var}\left(\tilde{\boldsymbol{\theta}}_i\right)=\left(\sum_{t=1}^{T}\frac{\partial^2 L_t\left(\tilde{\boldsymbol{\theta}}_i\right)}{\partial\boldsymbol{\theta}_i\,\partial\boldsymbol{\theta}_i'}\right)^{-1}\sum_{t=1}^{T}\frac{\partial L_t\left(\tilde{\boldsymbol{\theta}}_i\right)}{\partial\boldsymbol{\theta}_i}\frac{\partial L_{t+1}\left(\tilde{\boldsymbol{\theta}}_i\right)}{\partial\boldsymbol{\theta}_i'}\left(\sum_{t=1}^{T}\frac{\partial^2 L_t\left(\tilde{\boldsymbol{\theta}}_i\right)}{\partial\boldsymbol{\theta}_i\,\partial\boldsymbol{\theta}_i'}\right)^{-1}. \tag{2.24}$$

The parameters of the conditional variance and mean, the conditional asymmetry, and the conditional kurtosis equations are estimated (equations 2.9-2.15) by maximizing the skewed generalized error log-likelihood of the return series in each exchange rate (Euro, Japanese Yen, Canadian Dollar and British pound) and Bitcoin.

To further investigate the spillover effects, the parameters of equations (2.18) and (2.19) of bitcoin rates and exchange rates are computed using a two-stage maximum likelihood estimation technique. Firstly, the maximum likelihood method is used to compute the conditional mean and conditional variance equations of each currency without accounting for the spillover effects from other currencies (univariate analysis). Furthermore, the estimated volatility shocks ($\varepsilon^{2}_{j,t}$) and the estimated conditional risk (standard deviations) of returns ($\sigma_{j,t}$) are used in the mean and variance multivariate analysis. More specifically, in stage two, the estimated parameters in each currency are computed by using the estimated shocks and the estimated conditional risk in stage one from the other currencies in equations (2.18) and (2.19). The skewness price of risk $\bar{\delta}_i$ (average) is calculated using the equation (2.7) and the dynamic conditional asymmetry ($\lambda_{i,t}$) and shape ($k_{i,t}$) equations. The results are reliable after a few iterations.

2.5 Empirical Findings

This section presents the preliminary statistics of data, the estimated distributional and GJR-GARCH parameters under the SGED discussed in section 2.3, as well as the forecasting performance of Bitcoin's prices using the ST-GJR-GARCH-SGED model and compares it to other models' specifications.

2.5.1 Summary Statistics

Data is collected from DATASTREAM for the period July 18, 2011 to June 1, 2020 and include daily US dollar prices of Bitcoin (BTC), Euro (EUR), Japanese yen (JPY), Canadian dollar (CAN) and British pound (GBP). Figure 2.3 presents the plots of the prices of Bitcoin and the four currencies over the data period. The time-series plot shows a sharp upward price trend for the BTC. The return series of the five currencies (figure 2.4) are characterized by extreme spikes. Furthermore, the range of Bitcoin's returns compared to those of the other series is about 10 times larger, depicting the high volatility of Bitcoin.

Table 2.1 presents the summary statistics (expected value, standard deviation, skewness, kurtosis) and the Normality Bera-Jarque test for the daily returns.[25] Consistent with its steep upward trend, Bitcoin's expected value of returns is 0.345. Furthermore, the expected values of the other four currencies are negative (and lower compared to BTC). The standard deviation for Bitcoin returns is 5.473 (10 times larger than the standard deviations of returns of the other series, e.g. 0.513 in the case of EURO). The Pearson's skewness and kurtosis statistics are estimated using the equations $sk= m_3 / m_2^{3/2}$ and $ku= m_4 / m_2^2$, where m_j is the j^{th} moment around the mean. Negative skewness is found to be in BTC, and CAN return series and positive for the rest assets. Leptokurtosis is present in the return series of all five currencies. However, the kurtosis parameter for BTC is about three to four times larger than that of the normal distribution, which is three. Also, the kurtosis parameter is about two times larger than those of the other currencies. The Bera-Jarque test rejects the null hypothesis of normality (Gaussian). This indicates the presence of skewness and/or kurtosis in the data.

Therefore, the use of a higher moment distribution that accounts for skewness and kurtosis characteristics is dictated by the data. Table 2.2 presents the correlation matrix

[25] Daily log-returns are winsorized to ±5 standard deviations from the means.

for Bitcoin and exchange rates returns. The correlation between Bitcoin and the other exchange rates is extremely low. In all cases, the correlation between Bitcoin and exchange rates is positive except in the case between Bitcoin (BTC) and Japanese Yen (JPY) which is extremely negative.

Table 2.1. Summary Statistics for Bitcoin and Currency Log-Price Changes (Returns)

Statistics	BTC	EUR	JPY	CAN	GBP
Descriptive Statistics:					
Mean	0.345	-0.010	-0.014	-0.013	-0.009
Std	5.473	0.513	0.561	0.481	0.5520
SK–Skewness	-0.292	0.033	0.070	-0.022	0.040
KU–Kurtosis	9.852	5.010	6.612	5.230	5.830
Normality Test:					
Bera–Jarque	8,957*	2,309*	4,022*	2,515*	3,126*

Notes. All prices rates are expressed in USD. Daily returns are continuously compounded. The data covers the period from July 18, 2011 to June 1, 2020 (2,207 observations). Returns are winsorized to ±5 standard deviations from their means. The skewness and kurtosis statistics are computed by $SK = m_3 / m_2^{3/2}$ and $KU = m_4 / m_2^2$, where m_j is the j^{th} moment around the mean. The Bera–Jarque test statistics for normality are calculated using the equation $BJ = (T/24)(4SK^2 + KU^2)$. * denotes statistical significance at 1%.

Table 2.2. Correlation Matrix for Bitcoin and Exchange Rates Log-Price Changes (Returns)

	BTC	**EUR**	**JPY**	**CAN**	**GBP**
BTC	1.0000				
EUR	0.0180	1.0000			
JPY	-0.0007	0.3477	1.0000		
CAN	0.0371	0.4380	0.1230	1.0000	
GBP	0.0278	0.5416	0.1654	0.4511	1.0000

Notes. The table presents the correlation between Bitcoin (BTC) and the four exchange rates log returns: Euro (EUR), Japanese Yen (JPY), Canadian Dollar (CAN) and British Pound (GBP).

Figure 2.3. US Dollar Currency Prices – Daily Frequency

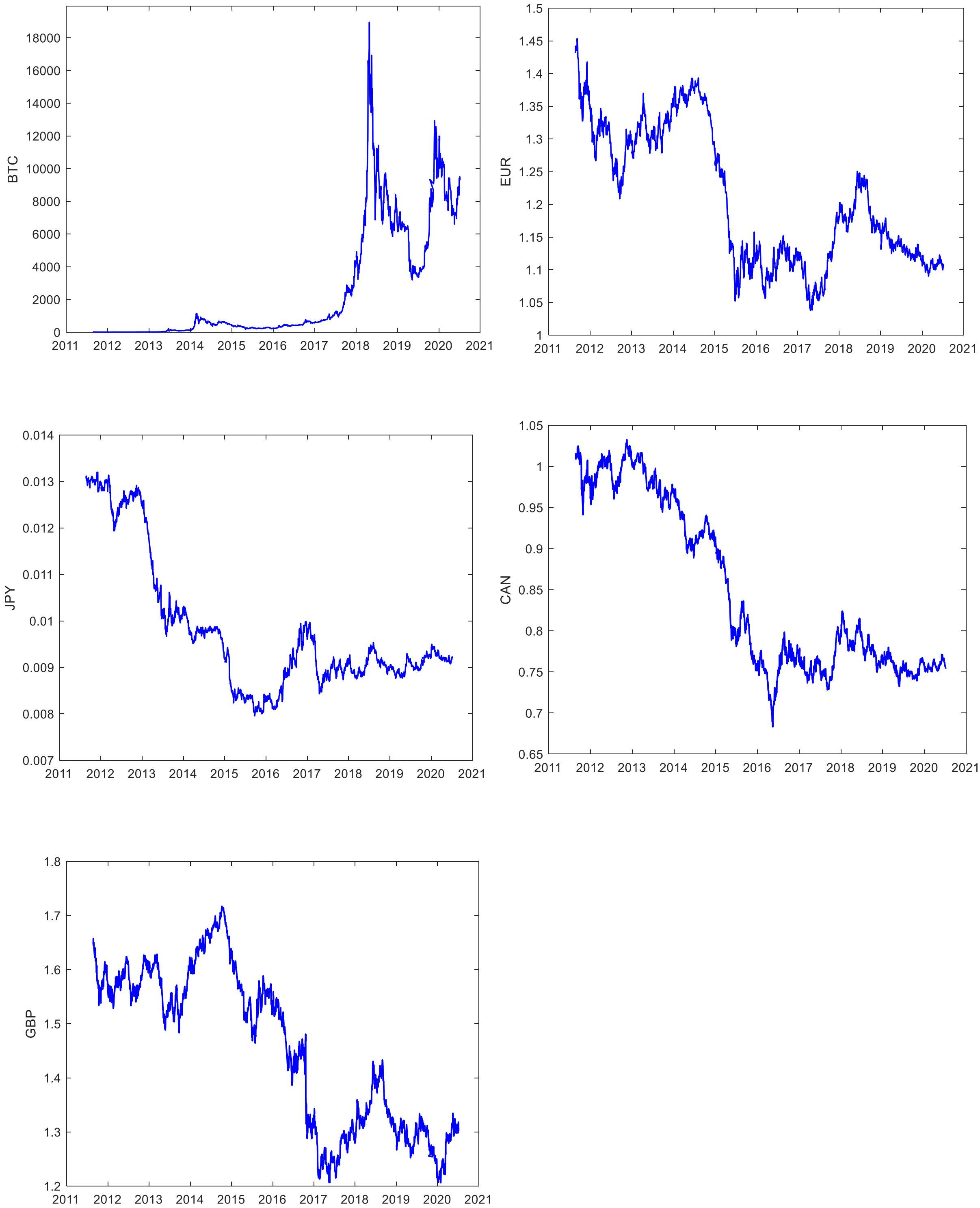
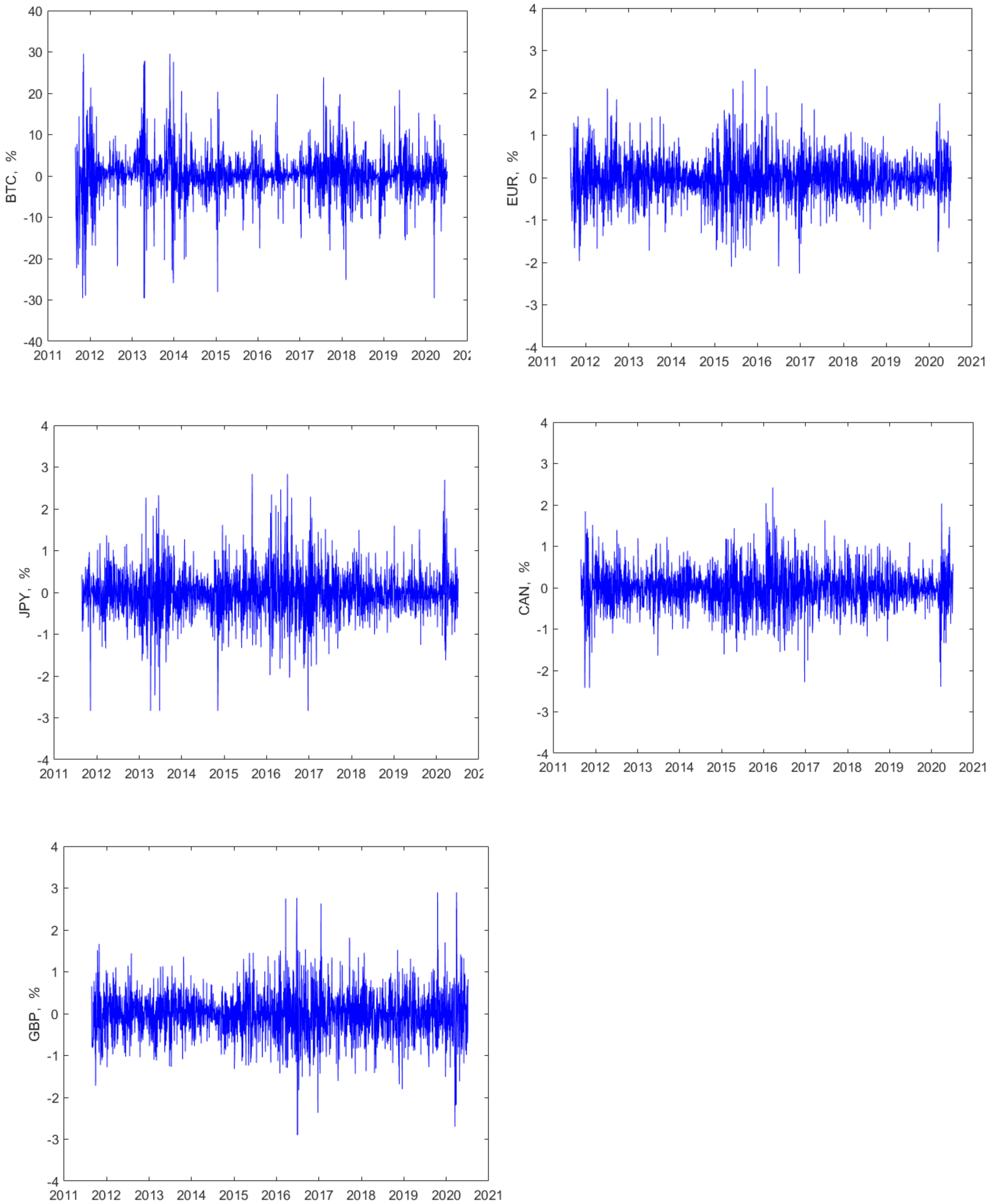


Figure 2.4. Daily Returns (Log-Price Changes)



2.5.2 Unconditional Distribution of Returns

Table 2.3 presents the estimated parameters of the unconditional Skewed Generalized Error Distribution (SGED). The resulting SGED distributions of each series and the normal distribution (dotted curve) are presented in figure 2.5.

The estimated parameters for the first two moments (expected values and standard deviations) of all the series are very similar to those of Table 2.1. The asymmetry parameter is positive for BTC, EURO, JPY and negative for CAN and GBP. The estimated shape parameter of the Bitcoin is lower than one (excess kurtosis). This is not true for the other assets. All the rest assets present kurtosis larger than one. This is a key characteristic of Bitcoin's returns, a highly peaked probability distribution. That is, Pearson's kurtosis for the BTC is 12.38. This means that the kurtosis of Bitcoin is about 2.5 to 3 times larger than that of the other currencies and four times larger than the kurtosis of the normal distribution ($KU = 3$). For example, the EURO's kurtosis is 4.81 which is three times lower compared to that of BTC.

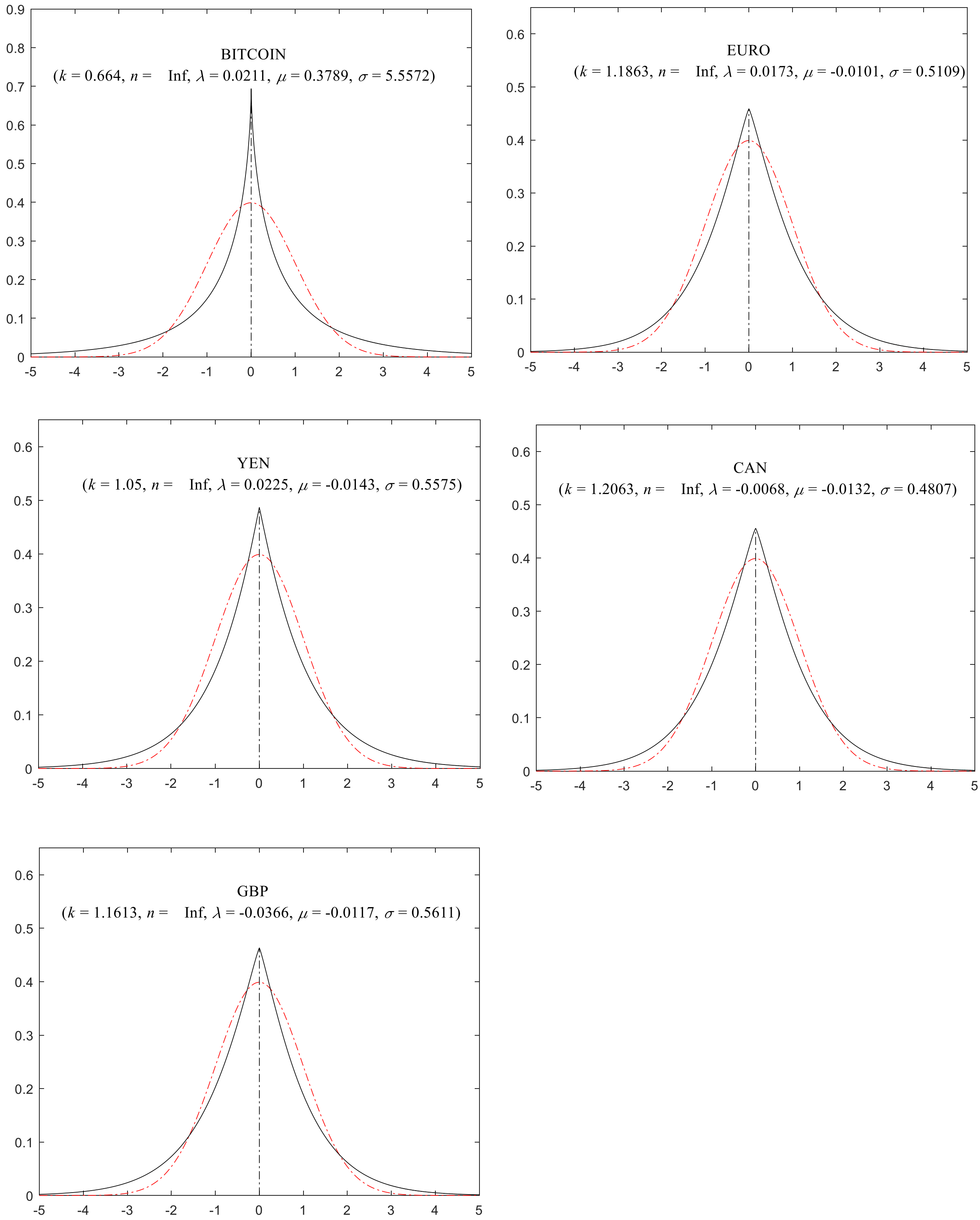
These findings lead to the following conclusions. Firstly, Bitcoin's mean and volatility is quite larger in relation to the other assets. Secondly, it exhibits a completely different probabilistic behaviour due to the high uncertainty presented in the data. This is a contradictory finding compared to the other assets. Therefore, these findings lead to a further investigation of the properties of Bitcoin's series in relation to the other assets using a model that accounts for skewness and kurtosis characteristics.

Table 2.3. Estimated Parameters of the Unconditional SGED Distribution of Returns

Parameters	BTC	EUR	JPY	CAN	GBP
μ	0.3789	-0.0101	-0.0143	-0.0132	-0.0117
	(0.1727)**	(0.0145)	(0.0157)	(0.0137)	(0.016)
σ	5.5572	0.5109	0.5575	0.4807	0.5611
	(0.1586)**	(0.0102)**	(0.0113)**	(0.0091)**	(0.0104)**
k	0.664	1.1863	1.05	1.2063	1.1613
	(0.028)**	(0.0533)**	(0.0427)**	(0.0498)**	(0.0379)**
λ	0.0211	0.0173	0.0225	-0.0068	-0.0366
	(0.0875)	(0.0193)	(0.016)*	(0.0193)	(0.0178)**
SK	0.1626	0.0577	0.0889	-0.0222	–0.1255
KU	12.3817	4.8101	5.6143	4.7149	4.9428
Log–L	-6,407.50	-1,566.80	-1,707.20	-1,438.80	-1,764.60
LR	1,290.20**	180.80**	342.251**	183.638**	322.617**

Notes. The table presents the estimated distributional parameters of the unconditional distribution of Bitcoin and exchange rate returns obtained from the maximization of an unconditional log-likelihood function based on the unconditional SGED distribution (all skewed generalized error distribution parameters are modelled as fixed). The parameters μ, σ, λ and k represent the mean, standard deviation, the asymmetry, and the shape parameters of the SGED distribution. Standard errors are included in the parentheses. The *SK* and *KU* are the Pearson's skewness and the kurtosis parameters, respectively. Log-L is the maximum likelihood value and LR is the log-likelihood ratio test statistic for normality. *, and ** indicate statistically significant at 10%, and 5%.

Figure 2.5. Unconditional Distributions of Daily Returns Based on the SGED



2.5.3 Higher-Order – Dependencies

This sub-section presents further statistics to investigate whether daily log–returns exhibit higher-order moment dependencies such as, conditional heteroscedasticity, asymmetric volatility, and other dependencies; see Theodossiou (2015).[26]

Table 2.4 reports the statistics given by equations (B7) to (B12) in Appendix II and their standard errors for the returns. The findings reveal the statistical behaviour of the daily log–returns indicating the consistency of the earlier results with regards to the stochastic nature of all the series. Statistics based on equations (B13) and (B14) show volatility clustering and asymmetric volatility, respectively. These statistics are reported in Table 2.4 and the results reveal the presence of both. Asymmetric volatility is presented in the cases of BTC, JPY, CAN, and GBP. The volatility is higher in the bear than in the bull market when the statistics are negative. Conditional heteroscedasticity is presented in all series. Table 2.4 also presents more complex higher-order dependencies (non–linearities). The findings show that higher-order dependencies are presented in the case of BTC followed by JPY and GBP.

---

[26] See also mathematical proofs and test statistics in Appendix II.

Table 2.4. Higher-Order Moment Dependencies

Estimates	BTC	EUR	JPY	CAN	GBP
$z_t^2 z_{t-1}$	-0.331	0.032	0.057	-0.055	-0.185
	(0.103)***	(0.048)	(0.059)	(0.051)	(0.105)*
$z_t^2 z_{t-2}$	-0.25	0.043	-0.003	-0.029	0.007
	(0.103)**	(0.048)	(0.059)	(0.051)	(0.105)
$z_t^2 z_{t-3}$	0.17	0.066	0.099	-0.093	-0.182
	(0.103)*	(0.048)	(0.059)*	(0.051)*	(0.105)*
$z_t z_{t-1} z_{t-2}$	0.504	-0.013	0.000	0.005	0.079
	(46.979)	(46.979)	(46.979)	(46.979)	(46.979)
$z_t^3 z_{t-1}$	2.283	-0.078	-0.028	0.286	-0.271
	(0.897)**	(0.153)	(0.261)	(0.206)	(1.348)
$z_t^3 z_{t-2}$	-1.445	0.182	0.354	-0.289	-0.203
	(0.897)	(0.153)	(0.261)	(0.206)	(1.348)
$z_t^3 z_{t-3}$	0.122	-0.072	0.100	0.034	0.209
	(0.897)	(0.153)	(0.261)	(0.206)	(1.348)
$z_t^2 z_{t-1}^2 - 1$	5.265	0.444	0.671	0.418	4.055
	(0.5)***	(0.107)***	(0.161)***	(0.124)***	(0.516)***
$z_t^2 z_{t-2}^2 - 1$	5.919	0.273	0.585	0.454	0.987
	(0.5)***	(0.107)**	(0.161)***	(0.124)***	(0.516)*
$z_t^2 z_{t-3}^2 - 1$	3.719	0.484	0.459	0.310	1.213
	(0.5)***	(0.107)***	(0.161)***	(0.124)**	(0.516)**
$z_t^2 z_{t-1} z_{t-2}$	-1.249	-0.024	-0.017	0.037	-0.204
	(0.103)***	(0.048)	(0.059)	(0.051)	(0.105)*
$z_t z_{t-1} z_{t-2} z_{t-3}$	0.036	-0.030	-0.159	-0.017	0.48
	(0.021)*	(0.021)	(0.059)***	(0.021)	(0.021)***

Notes. All daily prices are expressed in USD. Returns are continuously compounded and standardized using the equation $z_{i,t} = \left(r_{i,t} - \bar{r}_i\right)/\bar{\sigma}_i$ where $\bar{r}_i$ is the sample mean and $\bar{\sigma}_i$ the standard deviation of each currency's returns. This table presents the test statistics such as asymmetric volatility, heteroscedasticity, and higher order linearities. The standard errors are included in the parentheses. *,**, and *** indicates statistical significance at the 10%, 5%, and 1%.

2.5.4 Estimation of the Relationship: Risk and Return - Univariate Estimation

The parameters of the conditional variance, conditional mean, conditional asymmetry, and conditional shape equations (equations 2.9-2.15) are estimated through the maximization of the SGED log-likelihood of the return series in each Bitcoin and exchange rate.

Conditional Variance and Mean

Table 2.5 presents the results of the estimated parameters, v_i, $\alpha_{N,i}$, a_i, and β_i for the conditional variance computed by equation (2.9) and $m_{0,i}$, b_i, $\overline{\xi}_{i,t}$, c_i, and $\overline{\delta}_{i,t}$ for the conditional mean computed by equation (2.11).

Table 2.5 Panel A presents the estimated parameters of the conditional variance equation. The parameter $\alpha_{N,i}$ is positive and significant for the cases of CAN and GBP, positive and insignificant for the case of EUR, and negative and insignificant for the case of BTC and JPY. Also, the negative asymmetric volatility is in line with the literature. For example, Baur and Dimpfl (2018) found that most cryptocurrencies present a negative asymmetric volatility parameter. The asymmetric volatility of EUR and JPY is found to be insignificant; this is due to the two-sided effect of the exchange rates (Theodossiou, 1994). The coefficient a_i is positive and significant for all series. The parameter β_i is positive and significant for all cases indicating that volatility is persistent over time.

Figure 2.6 presents the conditional time-varying volatility of the returns. The horizontal line represents the unconditional standard deviation given in Table 2.3. For the case of BTC, it ranges from 0 to 25, much higher compared to the currencies, due to the great volatility observed in the prices of Bitcoin.

Table 2.5 Panel B reports the estimated parameters of the conditional mean equation. The estimated parameters of the coefficients that measure the total price of risk (the pure price plus the skewness price of risk) given as the mean value of $\xi_{i,t} = c_i + \delta_{i,t}$ ($\overline{\xi}_{i,t}$), are insignificant for all cases except BTC (positive and significant). This suggests that there is a significant relationship between the volatility and the conditional mean returns. This is also the case for the pure price of risk c_i (positive and significant only for the case of BTC). The average estimated coefficients of the skewness price of risk, $\overline{\delta}_{i,t}$, are negative and significant for all cases except the case of EURO and JPY (positive and significant). Overall, the above findings highlight the importance of the skewness on the model to investigate the risk and return relationship, e.g., León et al. (2005), Theodossiou and Savva (2016), and Savva and Theodossiou (2018).

The estimated parameter b_i is negative and significant in all cases apart from EUR (positive and significant) suggesting that these currencies are predicted by their own lag values. Figure 2.7 illustrates the behaviour of the conditional time-varying expected value with the horizontal line denoting the unconditional mean (as estimated in Table 2.3). From the plots, it is inferred that BTC ranges between -2 to 5 which is higher compared to the rest of the assets.

Table 2.5. Estimation of the Conditional Variance and Mean

	BTC	EUR	JPY	CAN	GBP
Panel A.					
Conditional variance: $\sigma_{i,t}^2 = v_i + \left(a_{N,i} N_{i,t-1} + a_i\right)\varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2$					
v_i	1.072	0.001	0.003	0.001	0.002
	(0.214)**	(0.000)**	(0.001)**	(0.001)*	(0.001)**
α_i	0.190	0.030	0.064	0.025	0.026
	(0.034)**	(0.007)**	(0.013)**	(0.009)**	(0.011)**
$\alpha_{N,i}$	-0.026	0.009	-0.008	0.036	0.040
	(0.036)	(0.010)	(0.016)	(0.012)**	(0.014)**
β_i	0.799	0.963	0.930	0.953	0.947
	(0.022)**	(0.007)**	(0.012)**	(0.010)**	(0.010)**
Panel B.					
Conditional mean: $\mu_{i,t} = m_{0,i} + b_i\, r_{i,t-1} + \left(c_i + \delta_{i,t}\right)\sigma_{i,t}$					
$m_{0,\mathrm{i}}$	-0.035	-0.053	0.009	-0.024	-0.033
	(0.084)	(0.029)*	(0.035)	(0.036)	(0.034)
b_i	-0.097	0.024	-0.081	-0.088	-0.074
	(0.019)**	(0.012)**	(0.020)**	(0.024)**	(0.023)**
$\overline{\xi}_{i,t}$	0.080	0.093	-0.054	0.020	0.046
	(0.030)**	(0.066)	(0.076)	(0.112)	(0.066)
c_i	0.095	0.064	-0.073	0.066	0.095
	(0.030)**	(0.066)	(0.076)	(0.112)	(0.066)
$\overline{\delta}_{i,t}$	-0.015	0.029	0.019	-0.046	-0.049
	(0.002)**	(0.001)**	(0.002)**	(0.002)**	(0.002)**

Notes. The sum of the pure price of risk (c_i) and the skewness price of risk ($\delta_{i,t}$) denoted by $\xi_{i,t} = c_i + \delta_{i,t}$, measures the impact of conditional risk on mean returns. The coefficients $a_{N,i}$, a_i and β_i are indicative of asymmetric volatility, volatility clustering and persistence. *, and ** indicate significant higher-order moment dependencies at 10%, and 5%, respectively.

Conditional Asymmetry and Shape Indexes

Table 2.6 presents the results of the estimated parameters $\gamma_{0,i}$, $\gamma_{N,i}$, $\gamma_{P,i}$, and $\gamma_{h,i}$ for the asymmetry index and conditional asymmetry parameter calculated by equations (2.12-2.14), $d_{0,i}$, $d_{N,i}$, $d_{P,i}$, and $d_{h,i}$ for the shape index and conditional shape parameter calculated by equation (2.11) and $\bar{\lambda}$ and $\bar{k}$ (sample averages) with their standard errors of all financial assets. Panel A of Table 2.6 reports the maximum likelihood estimated parameters of the conditional asymmetry index. The results show that the downside asymmetry coefficient, $\gamma_{N,i}$, is negative in all cases (but statistically significant in the cases of JPY, CAN, and GBP). The upside asymmetry coefficient, $\gamma_{P,i}$, is significant only for BTC and JPY, suggesting that the past upside shocks have a positive impact on the asymmetry index. Furthermore, this coefficient is greater for all cases compared to the downside asymmetry coefficient apart from the case of EUR. The estimated parameter $\gamma_{0,i}$ is insignificant for all cases except BTC (negative and significant) while the persistence of past price shocks, $\gamma_{h,i}$, (downside and upside) is significant only for the cases of BTC, and EUR.

Panel B of Table 2.6 presents the estimated values of $d_{0,i}$, $d_{N,i}$, $d_{P,i}$, and $d_{h,i}$ of the conditional shape index $g_{i,t}$ given by equation (2.15) and their standard errors. The constant shape parameter ($d_{0,i}$) is negative and significant for BTC, positive and significant for CAN and GBP, and positive but insignificant for EURO and JPY. The parameter $d_{N,i}$ is positive and significant only for BTC and negative and significant for EURO while $d_{P,i}$ is positive and significant only for BTC indicating the importance of the downside and upside shocks decomposition in the conditional time-varying shape specification for these currencies. The parameter $d_{h,i}$ is statistically significant at 5% level for all cases except EURO and JPY.

Panel C of Table 2.6 reports the average values of the conditional asymmetry and shape parameters and their standard errors.[27] The conditional asymmetry parameter, $\bar{\lambda}$, is negative and significant for all cases apart from EURO and JPY (positive and significant) indicating a negatively (positively) skewed distribution. The conditional shape parameter, $\bar{k}$, is highly significant for all cases suggesting a leptokurtic empirical distribution (lower than one for the case of BTC). Focusing on the BTC, the shape

[27] $\lambda_{i,t}$ (asymmetry) and $k_{i,t}$ (shape) are time – varying parameters, therefore, the average values of $\bar{\lambda}$ and $\bar{k}$ are presented.

parameter suggests a highly peaked distribution in relation to the other currencies. Figures 2.8 and 2.9 present the time-varying conditional asymmetry and shape parameters, $\lambda_{i,t}$ and $k_{i,t}$ (respectively), estimated by ST-GJR-GARCH model under the SGED distribution. The horizontal line represents the unconditional distributional estimated parameters (asymmetry and shape parameters) given in Table 2.3.

Panel D of Table 2.6 presents the standardized skewness and standardized kurtosis. Standardized skewness is negative for BTC, CAN, and GBP, and positive for the rest assets. The standardized kurtosis of BTC is almost two times higher (8.344) compare to the rest of the exchange rates. In this case, the volatility shocks of the BTC have wider spread than the rest assets (figure 2.10).

Table 2.7 presents the percentiles for the conditional time-varying estimated parameters (conditional variance, conditional mean, conditional asymmetry, and shape parameters) for each return series. The conditional variance of Bitcoin ranges from 5.56 to 549.22, indicating the persistence of high volatility across time which is quite larger compared to the rest assets. This is also happening in the case of the conditional mean. The conditional means take lower values compared to that of BTC (-1.57 to 5.06 in the case of BTC). Furthermore, the conditional asymmetry parameter is ranging from -0.24 to 0.52 for BTC; -0.11 to 0.06 for EUR; -0.41 to 0.36 for JPY; -0.48 to 0.21 for CAN and -0.27 to 0.20 to GBP. The conditional shape parameter of all series varied between 0.4 (k_L) and 1.6 (k_U). These are the pre-defined minimum and maximum bounds of the conditional shape parameter. The above findings reveal the importance of the use of a time-varying asymmetric model to investigate the stochastic properties of BTC compared to the rest assets.

Table 2.6. Estimation of the Asymmetry and Shape Indexes

	BTC	EUR	JPY	CAN	GBP
Panel A.					
Asymmetry parameter: $h_{i,t} = \gamma_{0,i} + \gamma_{N,i} u^{-}_{i,t-1} + \gamma_{P,i} u^{+}_{i,t-1} + \gamma_{h,i} h_{i,t-1}$, $\lambda_{i,t} = 1 - 2/\left(1 + \exp\left(h_{i,t}\right)\right)$					
$\gamma_{0,i}$	-0.059	0.018	0.037	-0.028	-0.069
	(0.035)*	(0.018)	(0.054)	(0.102)	(0.069)
$\gamma_{N,i}$	-0.054	-0.005	-0.157	-0.194	-0.112
	(0.046)	(0.022)	(0.062)**	(0.062)**	(0.055)**
$\gamma_{P,i}$	0.189	-0.033	0.117	0.088	0.091
	(0.063)**	(0.026)	(0.051)**	(0.060)	(0.060)
$\gamma_{h,i}$	0.360	0.907	0.004	-0.208	-0.288
	(0.208)*	(0.108)**	(0.226)	(0.246)	(0.220)
Panel B.					
Shape parameter: $g_{i,t} = d_{0,i} + d_{N,i} u^{-}_{t-1} + d_{P,i} u^{+}_{t-1} + d_{h,i} \cdot g_{i,t-1}$, $k_{i,t} = k_U + \left(k_L - k_U\right)/\left(1 + \exp\left(g_{i,t}\right)\right)$					
$d_{0,i}$	-0.105	0.969	1.436	4.677	1.992
	(0.036)**	(0.621)	(0.880)	(2.349)**	(0.842)**
$d_{N,i}$	0.084	-0.623	-0.352	-0.460	-0.496
	(0.045)*	(0.306)**	(0.255)	(0.511)	(0.626)
$d_{P,i}$	0.188	0.350	-0.259	-0.315	-1.494
	(0.058)**	(0.779)	(0.270)	(0.460)	(0.525)**
$d_{h,i}$	0.971	0.419	-0.330	-0.994	0.477
	(0.011)**	(0.411)	(0.858)	(0.048)**	(0.227)**
Panel C. Sample averages					
$\bar{\lambda}_{i,t}$	-0.012	0.019	0.013	-0.031	-0.032
	(0.002)**	(0.001)**	(0.002)**	(0.002)**	(0.001)**
$\bar{k}_{i,t}$	0.876	1.368	1.253	1.454	1.470
	(0.002)**	(0.001)**	(0.002)**	(0.002)**	(0.001)**
Panel D. Other					
$L(\theta)$	-6,162.554	-1,454.166	-1,594.068	-1,319.456	-1,643.271
SK	-0.090	0.054	0.036	-0.082	-0.065
KU	8.344	4.172	4.574	3.952	4.023

Notes. This table presents the estimated asymmetry and shape parameters. The parameters $\gamma_{0,i}$, $\gamma_{N,i}$, $\gamma_{P,i}$, and $\gamma_{h,i}$ measure the impact of past negative and positive shocks on the asymmetry parameter. The minimum and maximum bounds for $k_{i,t}$ set to $k_L = 0.4$ and $k_U = 1.6$, respectively. The coefficients $d_{0,i}$, $d_{N,i}$, $d_{P,i}$, and $d_{h,i}$ measure the impact of past negative and positive shocks on the shape parameter and control the shape of the distribution. $u_{i,t-1} \equiv \left(r_{i,t-1} - m_{i,t-1}\right)/\sigma_{i,t-1}$ is the standardized excess to mode return. $L(\theta)$ is the sample log-likelihood values. *SK* and *KU* are Pearson's skewness and kurtosis, respectively. The estimated parameters are statistically insignificant unless otherwise noted. *, and ** indicate significance at 10%, and 5%, respectively.

Table 2.7. Percentiles of Conditional Estimated Parameters

	Min	**0.05**	**0.1**	**0.25**	**0.5**	**0.75**	**0.9**	**0.95**	**Max**
BTC									
$\sigma^2_{i,t}$	5.56	-0.26	7.60	6.28	17.58	32.85	67.33	497.49	549.22
$\mu_{i,t}$	-1.57	-0.26	-0.10	0.12	0.30	0.56	1.03	1.40	5.06
$\lambda_{i,t}$	-0.24	-0.09	-0.07	-0.05	-0.03	0.01	0.08	0.13	0.52
$k_{i,t}$	0.54	0.62	0.66	0.76	0.86	0.97	1.11	1.21	1.47
EURO									
$\sigma^2_{i,t}$	0.07	0.08	0.10	0.16	0.22	0.36	0.50	0.57	0.78
$\mu_{i,t}$	-0.06	-0.04	-0.04	-0.03	-0.02	-0.01	0.00	0.01	0.06
$\lambda_{i,t}$	-0.11	-0.02	-0.01	0.01	0.02	0.03	0.05	0.05	0.06
$k_{i,t}$	0.65	1.17	1.23	1.32	1.39	1.44	1.47	1.49	1.56
JPY									
$\sigma^2_{i,t}$	0.08	0.11	0.13	0.17	0.24	0.39	0.63	0.78	1.43
$\mu_{i,t}$	-0.29	-0.11	-0.08	-0.05	-0.03	0.00	0.02	0.04	0.18
$\lambda_{i,t}$	-0.41	-0.10	-0.07	-0.02	0.02	0.05	0.09	0.11	0.36
$k_{i,t}$	0.75	1.14	1.18	1.23	1.27	1.29	1.31	1.32	1.40
CAN									
$\sigma^2_{i,t}$	0.05	0.08	0.09	0.14	0.19	0.30	0.43	0.51	0.97
$\mu_{i,t}$	-0.18	-0.04	-0.03	-0.01	0.01	0.04	0.07	0.09	0.30
$\lambda_{i,t}$	-0.48	-0.17	-0.13	-0.07	-0.01	0.02	0.04	0.07	0.21
$k_{i,t}$	0.83	1.22	1.30	1.41	1.48	1.53	1.56	1.57	1.59
GBP									
$\sigma^2_{i,t}$	0.08	0.12	0.15	0.20	0.26	0.35	0.49	0.63	1.71
$\mu_{i,t}$	-0.18	-0.04	-0.03	-0.01	0.01	0.04	0.07	0.09	0.30
$\lambda_{i,t}$	-0.27	-0.12	-0.10	-0.06	-0.03	0.00	0.03	0.05	0.20
$k_{i,t}$	0.41	1.21	1.33	1.46	1.52	1.54	1.56	1.56	1.57

Notes. This table presents the time-varying behaviour of each estimated parameter. The percentiles of the conditional estimated parameters of all return series are computed using the equation (2.9) for the conditional variance ($\sigma^2_{i,t}$), equation (2.11) for the conditional mean ($\mu_{i,t}$), equation (2.12) for the conditional asymmetry parameter ($\lambda_{i,t}$) and equation (2.15) for the conditional shape parameter ($k_{i,t}$).

Figure 2.6. Conditional Standard Deviation – Daily Frequency

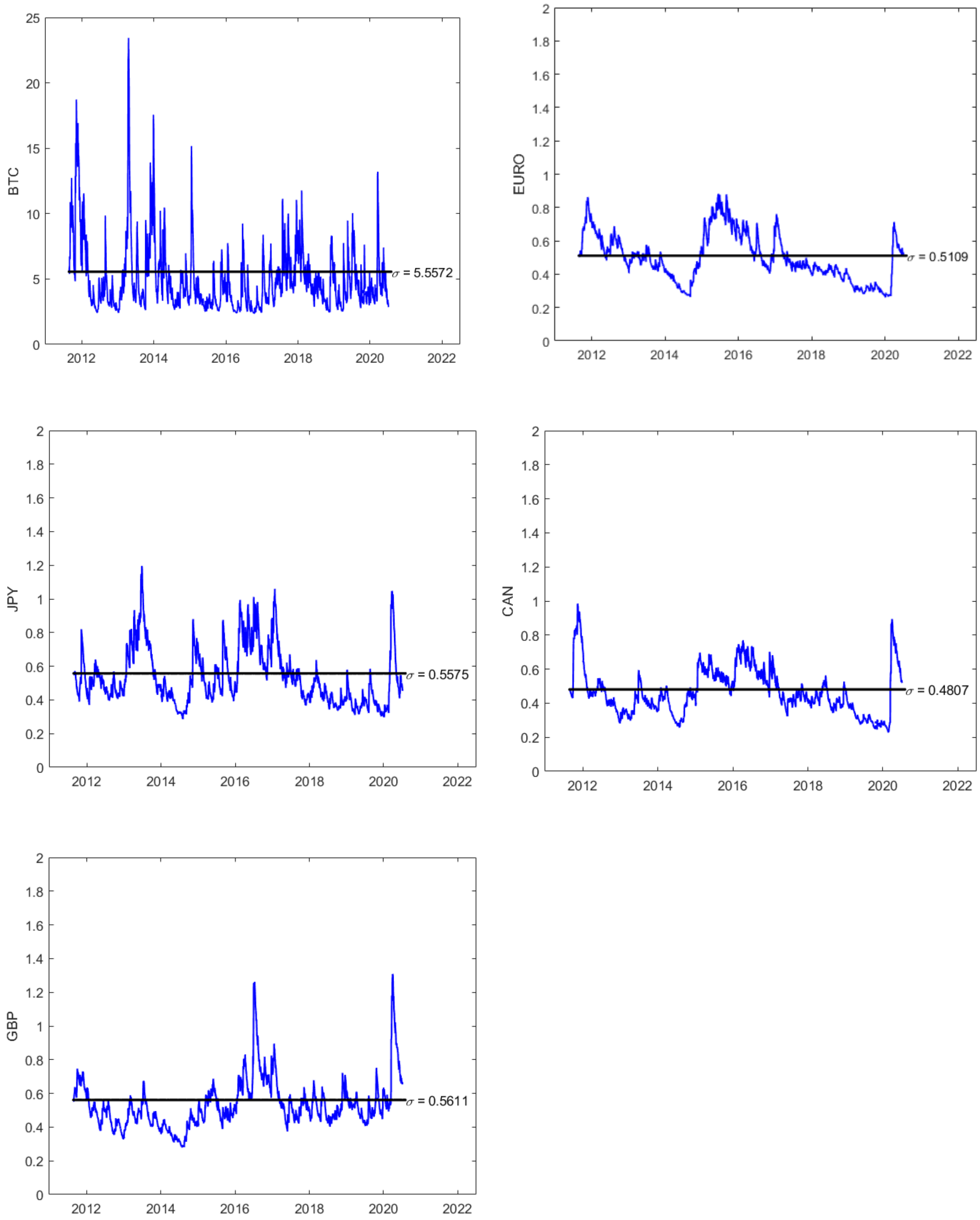


Figure 2.7. Conditional Mean – Daily Frequency

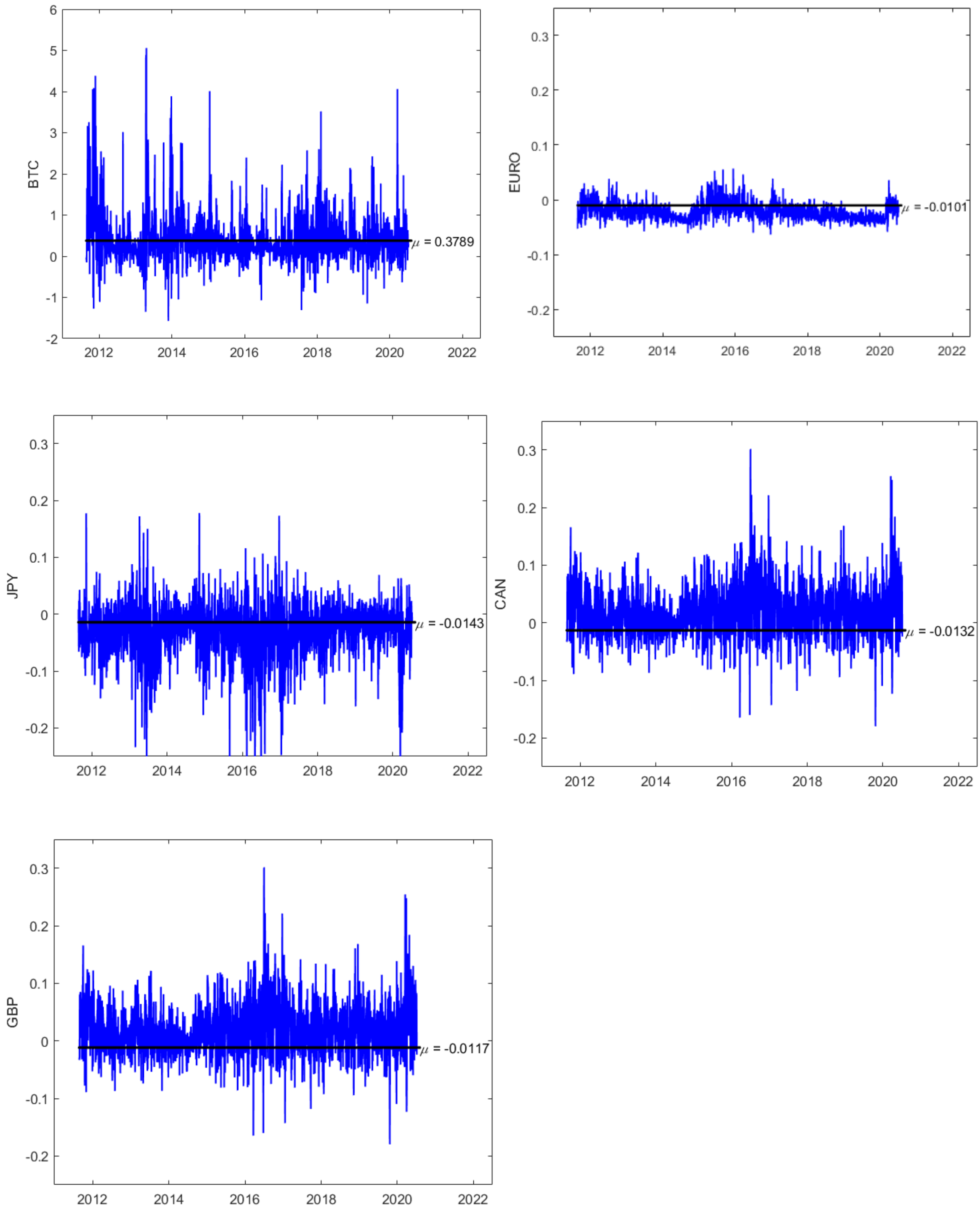


Figure 2.8. Conditional Asymmetry Parameter – Daily Frequency

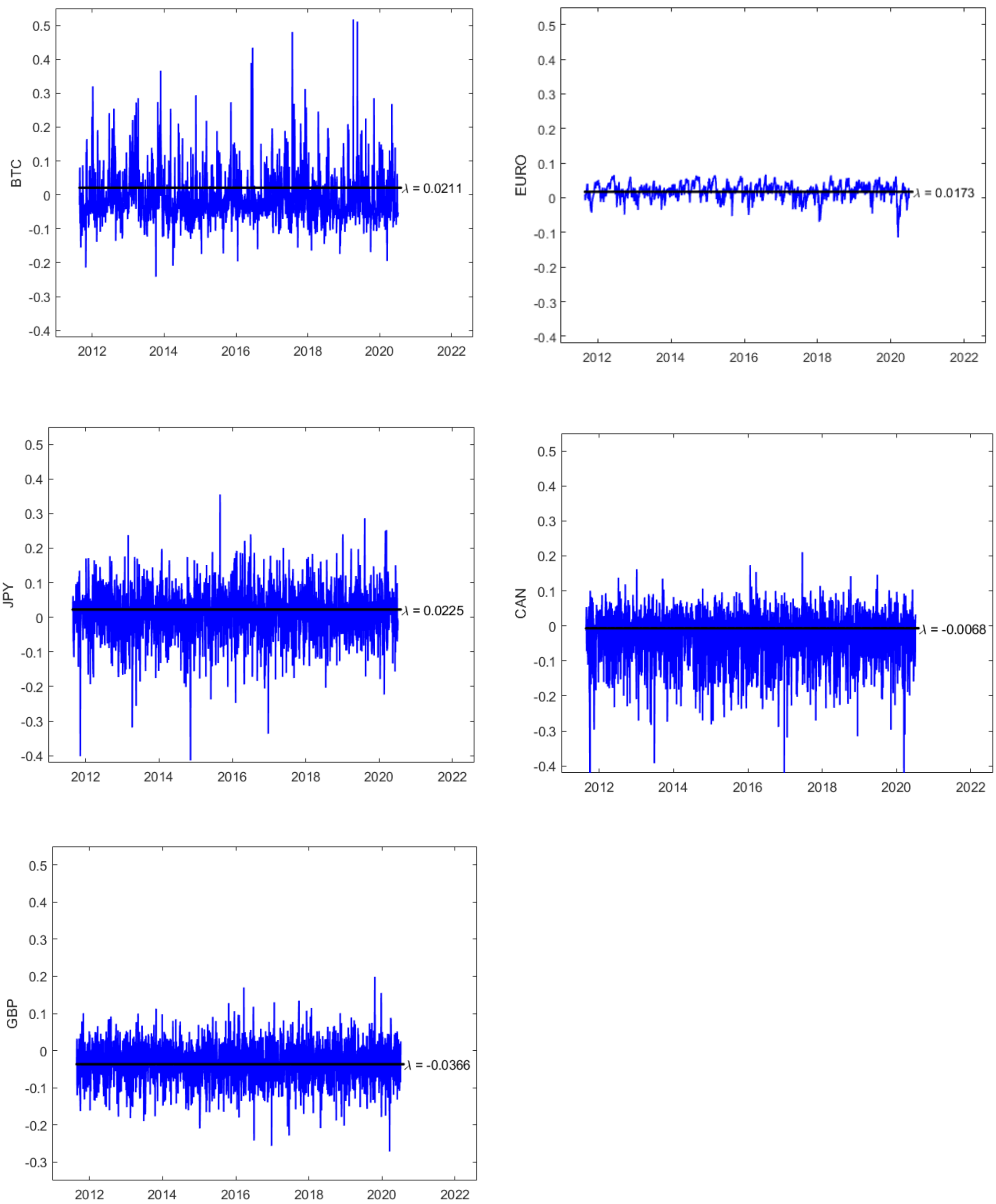


Figure 2.9. Conditional Shape Parameter – Daily Frequency

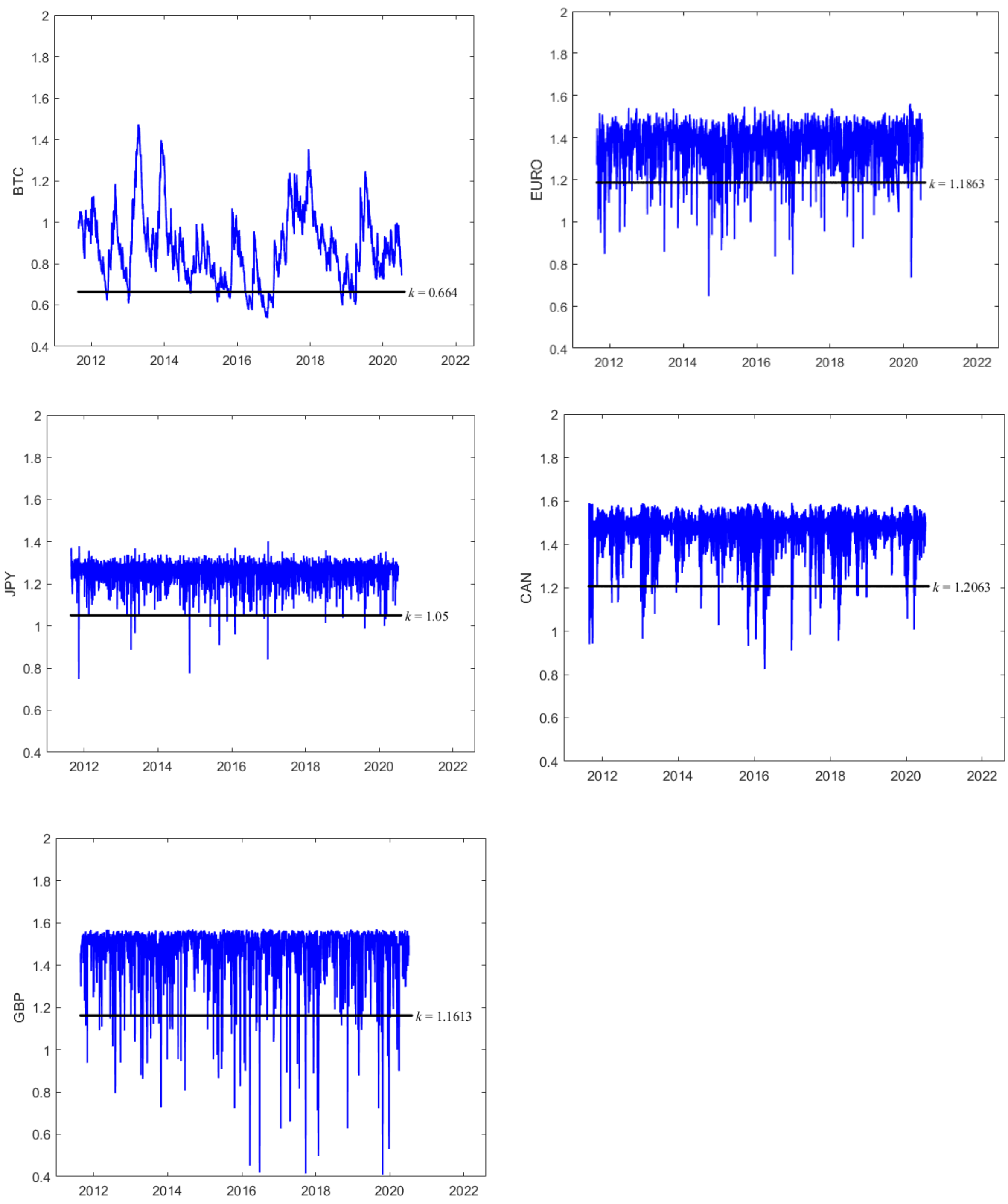


Table 2.8 presents the correlation of standardized residuals between BTC and the exchange rates. The results show that the correlation is extremely low, supporting the existing evidence (Briere et al., 2015). This also proves that the ST-GJR-GARCH-SGED model gives accurate results.

Table 2.8. Correlation Matrix of Standardized Residuals

	BTC	EUR	JPY	CAN	GBP
BTC	1				
EUR	0.0105	1			
JPY	0.0027	0.3231	1		
CAN	0.0313	0.3999	0.1080	1	
GBP	0.0081	0.5481	0.1903	0.4121	1

Notes. This table presents the correlation matrix of the standardized residuals. The standardized residuals are computed by $z_{i,t} = \left(r_{i,t} - \mu_{i,t}\right)/\sigma_{i,t} = \varepsilon_{i,t}/\sigma_{i,t}$, where $\mu_{i,t}$ and $\sigma_{i,t}$ are the expected value and standard deviation, respectively.

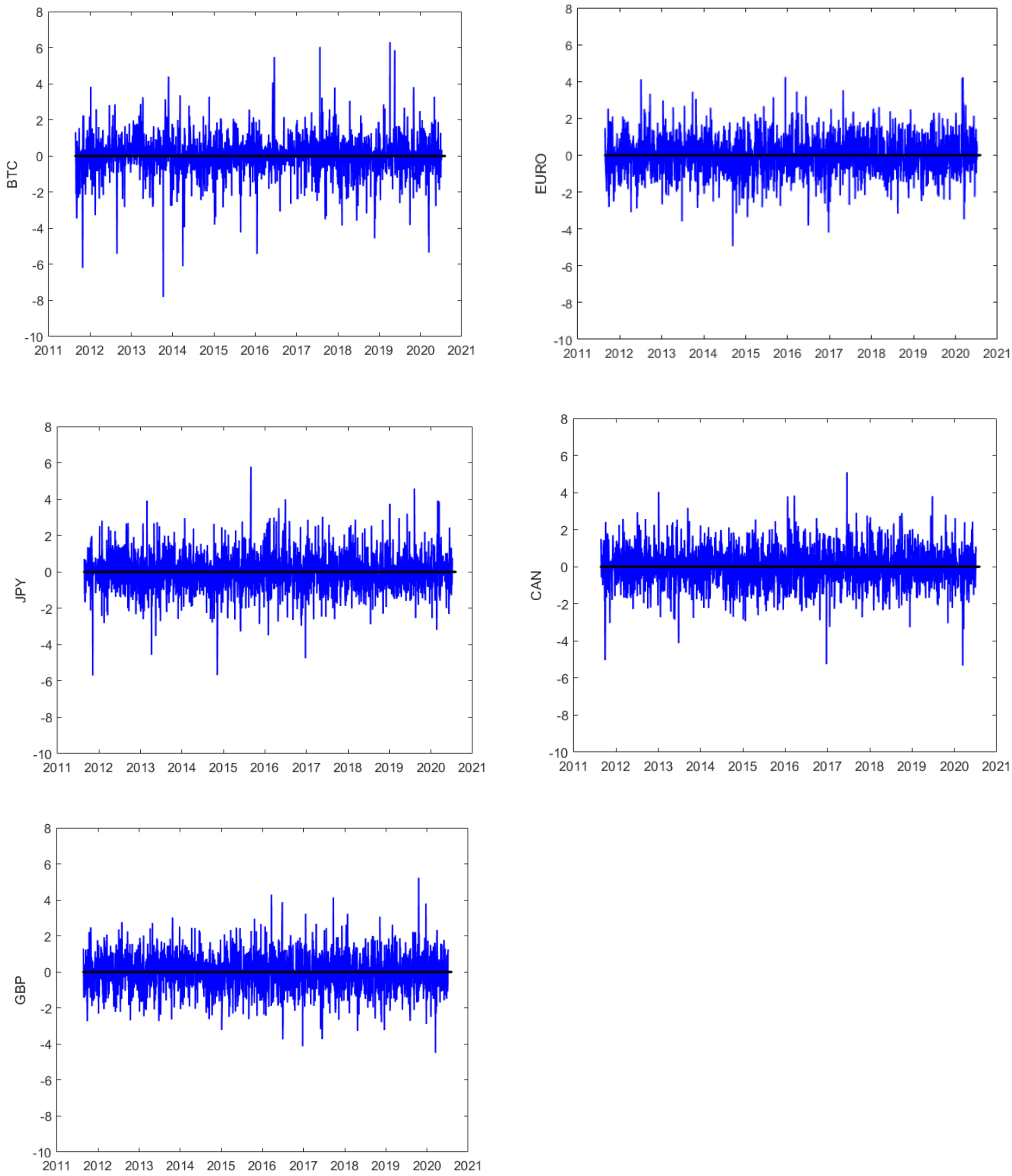
Table 2.9 reports the downside and upside mean probabilities using the equations (2.13) and (2.14). In the case of BTC, CAN, and GBP the upside probability is lower than the downside indicating that there is a higher probability of a negative shock to occur than a positive (negatively probability distribution). The difference between the upside and downside probability is the conditional asymmetry parameter. In the cases of EURO and JPY, the upside probabilities are higher than the downside probabilities, therefore, exhibiting positively probability distribution.

Table 2.9. Mean Downside and Upside Probabilities

	BTC	EUR	JPY	CAN	GBP
Downside Prob.	0.506	0.496	0.494	0.508	0.516
Upside Prob.	0.494	0.510	0.506	0.492	0.484
Diff. Prob.	-0.012	0.019	0.013	-0.016	-0.032

Notes. This table presents the downside and upside mean probabilities that are computed using the equations (2.16) and (2.17). The difference between upside and downside probability gives the asymmetry parameter (also reported in Table 2.6, Part C).

Figure 2.10. Standardized Residuals



The finding that the estimated shape parameter, k, is lower than one in the case of Bitcoin indicates more leptokurtic in Bitcoin's returns compared to the other assets (see also Takaishi, 2018). Furthermore, the results suggest that ignoring skewness and kurtosis in the estimation of the risk and return relationship, may lead to misleading findings, e.g., Savva and Theodossiou (2018).

2.5.5 Mean and Volatility Spillover – Bivariate Estimation

To investigate the mean and volatility spillovers from Bitcoin to exchange rates and vice versa the analysis was extended on a bivariate context.

Conditional Variance

Table 2.10 Panel A presents the results of the estimated parameters, v_i , α_i , $a_{N,i}$,β_i, $a_{i,j}$, and $\beta_{i,j}$ for the conditional variance calculated by equation (2.19) of BTC as the endogenous variable and other asset as exogenous. Table 2.11 reports the estimates with BTC as the exogenous variable.

As for the estimated parameters of the conditional variance presented in Panel A of Table 2.5, the estimated parameters v_i and a_i are positive and significant in all cases while the parameter $a_{N,i}$ is negative and insignificant in all cases. The parameter β_i is positive and significant for all cases indicating that volatility is persistent over time. The volatility and asymmetric volatility spillovers from exchange rate to BTC are captured by the coefficients $a_{i,j}$,and $b_{i,j}$. The estimated parameters do not reveal a statistical effect (positive or negative) indicating that BTC's volatility is not affected by the rest assets.

The reverse relationship is also investigated to show whether Bitcoin affects the behaviour of the four exchange rates (Table 2.11). The estimated parameter v_i and a_i are positive and significant in all cases. The parameter $a_{N,i}$ is negative and insignificant for EURO-BTC and JPY-BTC and positive and significant for CAN-BTC and GBP-BTC. The parameter β_i is positive and significant for all cases. The estimated parameter $a_{i,j}$ is insignificant in all cases (apart from EURO-BTC where it is positive and significant) while the parameter $\beta_{i,j}$ is negative and significant for EURO-BTC and GBP-BTC. The last two parameters are very close to zero indicating that BTC has no effect on the exchange rate's risk.

Figure 2.11 depicts the pattern of the conditional volatility (which is perceived as a proxy for risk). For the case that examined the effect of exchange rates on BTC, it ranges from 0 to 25 (blue figure). Interestingly, the conditional volatility for the case that examined the effect of BTC on exchange rates shows that conditional volatilities are quite lower indicating the weak volatility spillover effects from Bitcoin to exchange rates and vice versa (red figure).

Conditional Mean

Table 2.10 Panel B presents the results of the estimated parameters, $m_{o,i}$, b_i, $b_{i,j}$, $\xi_{i,j}$, $\overline{\xi}_{i,t}$, c_i, and $\overline{\delta}_{i,t}$ for the conditional mean calculated by equation (2.18) of BTC as the endogenous variable and other asset as exogenous. Table 2.11 Panel B reports the estimates with BTC as the exogenous variable.

Table 2.10 (Panel B) reports the estimated parameters of the conditional mean equation. The estimated parameters of the total price of risk ($\overline{\xi}_{i,t}$) computed as the average of the pure price plus skewness price of risk ($\xi_{i,t} = c_i + \delta_{i,t}$) are positive and significant for all cases. Positive and significant is also for the case of BTC in the univariate analysis. The pure price of risk c_i is positive and significant while the average estimated coefficients of the skewness price of risk, $\overline{\delta}_{i,t}$, is negative and significant in all cases.

The estimated parameter $m_{o,i}$ is negative and significant for EURO-BTC and positive and insignificant for the rest assets. The parameter b_i is negative and significant in all cases suggesting that BTC is predicted by its own lag values. The estimated parameter $b_{i,j}$ is negative and significant for BTC-JPY, negative and insignificant for BTC-GBP and positive and insignificant for the rest assets. In other words, past returns of the Japanese yen negatively affect the current returns. The estimated parameter $\xi_{i,j}$ is positive and significant for BTC-EURO.

Table 2.11 (Panel B) shows the estimated parameters of the conditional mean equation when Bitcoin is the exogenous variable. The estimated parameters of the total price of risk (pure price plus skewness price of risk) computed as the mean value of $\xi_{i,t} = c_i + \delta_{i,t}$ ($\overline{\xi}_{i,t}$), are positive and insignificant for JPY-BTC and CAN-BTC, and negative and insignificant for EURO-BTC and negative and significant for GBP-BTC. The pure price of risk c_i is negative and insignificant in the case of EURO-BTC, positive and insignificant in CAN-BTC and JPY-BTC, and negative and significant in GBP-BTC.

The average estimated coefficient of the skewness price of risk, $\bar{\delta}_{i,t}$, is positive and significant for EURO-BTC, and negative and significant for the rest assets.

The estimated parameter $m_{o,i}$ is positive and insignificant in JPY-BTC and GBP-BTC (negative and insignificant for EURO-BTC and CAN-BTC). The parameter b_i is negative and significant for JPY-BTC, negative and insignificant for CAN-BTC and GBP-BTC and positive and insignificant for EURO-BTC. The estimated parameter $b_{i,j}$ is negative and insignificant in all cases except EURO-BTC (positive and insignificant). The estimated parameter $\xi_{i,j}$ is statistically insignificant in all cases suggesting that Bitcoin's volatility is not affecting the behaviour of the rest assets.

Finally, figure 2.12 illustrates the behaviour of the conditional mean computed by equation (2.18). From the plots, it can be inferred that when the spillover effects are investigated from exchange rates to BTC, conditional means range between -3 to 6 (blue figure). However, this is not happening when the investigation of the spillover effects are from BTC to exchange rates. In this case, the values are much smaller compared to the inverse relationship (red figure).

Table 2.10. Bitcoin-Exchange Rates, Conditional Variance and Mean

	BTC-EURO	BTC-JPY	BTC-CAN	BTC-GBP
Panel A.				
Conditional variance: $\sigma^2_{i,t} = \mathrm{var}\left(r_{i,t} \mid I_{t-1}\right) = v_i + \left(a_{N,i}N_{i,t-1} + a_i\right)\varepsilon^2_{i,t-1} + \beta_i\sigma^2_{i,t-1} + \sum_{j\neq i=1}^{M}\left(a_{i,j}N_{j,t-1} + \beta_{i,j}\right)\varepsilon^2_{j,t-1}$				
v_i	0.6723	0.4101	0.2634	0.3822
	(0.2986)**	(0.1523)**	(0.1201)**	(0.1618)**
α_i	0.1852	0.1824	0.1639	0.1814
	(0.037)**	(0.0366)**	(0.023)**	(0.0385)**
$\alpha_{N,i}$	-0.0301	-0.0423	-0.0381	-0.0425
	(0.0347)	(0.037)	(0.0256)	(0.0372)
β_i	0.8288	0.8377	0.8542	0.8389
	(0.0359)**	(0.0328)**	(0.0222)**	(0.0387)**
$\alpha_{i,j}$	-0.1523	0.0408	0.0093	0.0466
	(1.3567)	(0.6837)	(0.1658)	(2.0397)
$\beta_{i,j}$	-0.2273	0.0891	-0.0175	0.0997
	(0.8795)	(0.5189)	(0.5471)	(0.9758)
Panel B.				
Conditional mean: $\mu_{i,t} = E\left(r_{i,t} \mid I_{t-1}\right) = m_0 + b_i\, r_{i,t-1} + \left(c_i + \delta_{i,t}\right)\sigma_{i,t} + \sum_{j\neq i=1}^{M} b_{i,j}\, r_{j,t-1} + \sum_{j\neq i=1}^{M}\xi_{i,j}\sigma_{j,t}$				
$m_{0,i}$	-0.3061	0.0535	0.1283	0.0527
	(0.0814)**	(0.0596)	(0.2405)	(0.1167)
b_i	-0.1195	-0.0872	-0.0641	-0.0871
	(0.0100)**	(0.0186)**	(0.0174)**	(0.0446)*
$b_{i,j}$	0.0027	-0.0713	0.0166	-0.0159
	(0.0282)	(0.0387)*	(0.0361)	(0.0657)
$\xi_{i,j}$	0.6203	0.078	0.0185	0.0473
	(0.1171)**	(0.1026)	(0.3652)	(0.1586)
$\bar{\xi}_{i,t}$	0.079	0.053	0.044	0.0577
	(0.0085)**	(0.0063)**	(0.0243)*	(0.0315)*
c_i	0.0857	0.081	0.0535	0.0776
	(0.0081)**	(0.006)**	(0.0243)**	(0.0315)**
$\bar{\delta}_{i,t}$	-0.0067	-0.028	-0.0095	-0.0199
	(0.0027)**	(0.0019)**	(0.0013)**	(0.0019)**

Notes. The sum of the pure price of risk (c_i) and the skewness price of risk ($\delta_{i,t}$) denoted by $\xi_{i,t} = c_i + \delta_{i,t}$ measures the impact of conditional risk on mean returns. The coefficients $a_{N,i}$, a_i and β_i are indicative of asymmetric volatility, volatility clustering and persistence. The coefficient $\xi_{i,j}$ measures the impact of risk in market j (exchange rates) on the mean of rates in market i (Bitcoin). The coefficients $a_{i,j}$ and $\beta_{i,j}$ (for $i \neq j$) measure asymmetric volatility and volatility spillovers from j market into the i market. * and ** statistically significant at 10%, and 5%. The standard errors are presented in parentheses.

Table 2.11. Exchange Rates-Bitcoin, Conditional Variance and Mean

	EURO-BTC	JPY-BTC	CAN-BTC	GBP-BTC
Panel A.				
Conditional variance: $\sigma_{i,t}^2 = \mathrm{var}\left(r_{i,t} \middle| I_{t-1}\right) = v_i + \left(a_{N,i}N_{i,t-1} + a_i\right)\varepsilon_{i,t-1}^2 + \beta_i\sigma_{i,t-1}^2 + \sum_{j\neq i=1}^{M}\left(a_{i,j}N_{j,t-1} + \beta_{i,j}\right)\varepsilon_{j,t-1}^2$				
v_i	0.0033	0.0069	0.0025	0.0165
	(0.0011)**	(0.0021)**	(0.001)**	(0.0045)**
α_i	0.06	0.0978	0.0362	0.0524
	(0.0127)**	(0.0167)**	(0.0118)**	(0.0159)**
$\alpha_{N,i}$	-0.0049	-0.0186	0.0363	0.0543
	(0.0147)	(0.0204)	(0.0153)**	(0.0252)**
β_i	0.9306	0.8893	0.9324	0.8663
	(0.0109)**	(0.0142)**	(0.0115)**	(0.0219)**
$\alpha_{i,j}$	0.0001	-0.0001	0.0001	0.0001
	(0.0001)*	(0.0001)	(0.0001)	(0.0001)
$\beta_{i,j}$	-0.0001	0.0000	0.0000	-0.0001
	(0.0001)*	(0.0001)	(0.0000)	(0.0001)*
Panel B.				
Conditional mean: $\mu_{i,t} = E\left(r_{i,t} \middle| I_{t-1}\right) = m_0 + b_i\, r_{i,t-1} + \left(c_i + \delta_{i,t}\right)\sigma_{i,t} + \sum_{j\neq i=1}^{M} b_{i,j}\, r_{j,t-1} + \sum_{j\neq i=1}^{M} \xi_{i,j}\sigma_{j,t}$				
$m_{0,i}$	-0.0052	0.0077	-0.0077	0.0533
	(0.0259)	(0.0255)	(0.0233)	(0.0418)
b_i	0.0182	-0.1179	-0.0089	-0.0155
	(0.0328)	(0.0329)**	(0.024)	(0.0324)
$b_{i,j}$	0.0007	-0.0025	-0.0007	-0.0007
	(0.0016)	(0.0016)	(0.0014)	(0.0017)
$\xi_{i,j}$	0.0003	-0.0061	-0.004	0.0019
	(0.0035)	(0.0041)	(0.0032)	(0.0035)
$\overline{\xi}_{i,t}$	-0.0109	0.006	0.0214	-0.1384
	(0.0468)	(0.0279)	(0.0335)	(0.0725)*
c_i	-0.0573	0.0379	0.0337	-0.1318
	(0.0468)	(0.0278)	(0.0335)	(0.0725)*
$\overline{\delta}_{i,t}$	0.0464	-0.0319	-0.0123	-0.0067
	(0.0003)**	(0.0028)**	(0.001)**	(0.0003)**

Notes. The sum of the pure price of risk (c_i) and the skewness price of risk ($\delta_{i,t}$) denoted by $\xi_{i,t} = c_i + \delta_{i,t}$ measures the impact of conditional risk on mean returns. The coefficients $a_{N,i}$, a_i and β_i are indicative of asymmetric volatility, volatility clustering and persistence. The coefficient $\xi_{i,j}$ measures the impact of risk in market j (Bitcoin) on the mean of rates in market i (exchange rates). The coefficients $a_{i,j}$ and $\beta_{i,j}$ (for $i \neq j$) measure asymmetric volatility and volatility spillovers from j market into the i market. * and ** statistically significant at 10%, and 5%. The standard errors are presented in parentheses.

Conditional Asymmetry and Shape Indexes

Panel A of Table 2.12 reports the estimated parameters of the conditional asymmetry index when the examination of the spillover effects are from exchange rates to Bitcoin. The downside asymmetry coefficient, $\gamma_{N,i}$ is negative and significant for BTC-EURO and negative and insignificant for the rest assets. The upside asymmetry coefficient, $\gamma_{P,i}$, is positive and significant for BTC-EURO and BTC-JPY suggesting that the past upside shocks have a positive impact on the asymmetry index. The persistence of past upside and downside price shocks, $\gamma_{h,i}$, is positive and insignificant in all cases except BTC-CAN (positive and significant) while the estimated parameter $\gamma_{0,i}$ is negative and insignificant in all cases.

Panel B of Table 2.12 reports the estimated values of $d_{0,i}$, $d_{N,i}$, $d_{P,i}$, and $d_{h,i}$ of the shape index $g_{i,t}$ given by equation (2.15) with their standard errors. The constant parameter, $d_{0,i}$ is negative and significant in all cases except BTC-CAN (negative and insignificant). The parameters $d_{N,i}$ and $d_{P,i}$ are positive in all cases. The parameter $d_{h,i}$ is statistically insignificant in all cases.

Panel C of Table 2.12 presents the sample averages of the conditional asymmetry and shape parameters with their standard errors.[28] The conditional sample asymmetry parameter, $\bar{\lambda}_{i,t}$, is negative and significant in all cases indicating a negatively skewed distribution. The conditional sample shape parameter, $\bar{k}_{i,t}$, is smaller than one in all cases and highly significant suggesting a leptokurtic empirical distribution. In the univariate analysis, this parameter is smaller than one only in the case of BTC.

Panel A of Table 2.13 reports the estimated parameters of the conditional asymmetry index when the examination of the spillover effects are from BTC to exchange rates. The downside asymmetry coefficient, $\gamma_{N,i}$ is negative and insignificant in EURO-BTC, negative and significant for JPY-BTC and CAN-BTC and positive and insignificant for GBP-BTC. The upside asymmetry coefficient, $\gamma_{P,i}$, is positive and significant for JPY-BTC (positive and insignificant for GBP-BTC) and negative and insignificant for the rest assets. The persistence of past upside and downside price shocks, $\gamma_{h,i}$ and the estimated parameter $\gamma_{0,i}$ are insignificant in all cases.

---

[28] Asymmetry and shape parameters are time – varying, therefore, the average values of $\bar{\lambda}$ and $\bar{k}$ are presented.

Panel B of Table 2.13 reports the estimated values of $d_{0,i}$, $d_{N,i}$, $d_{P,i}$, and $d_{h,i}$ of the shape index g_t given by equation (2.15) with their standard errors. The constant parameter, $d_{0,i}$ is significant in all cases except in the case of EURO-BTC which is negative and insignificant. The parameters $d_{N,i}$ and $d_{P,i}$ are positive and insignificant in all cases except in the case of JPY-BTC (positive and statistically significant). The parameter $d_{h,i}$ is statistically significant at 5% level in all cases.

Panel C of Table 2.13 presents the sample averages of the conditional asymmetry and shape parameters with their standard errors. The conditional sample asymmetry parameter, $\bar{\lambda}_{i,t}$, is positive and significant for EURO-BTC and negative and significant in all other cases. The conditional sample shape parameter, $\bar{k}_{i,t}$, is highly significant and slightly higher than one for all cases suggesting a leptokurtic empirical distribution.

Finally, Panel D of Tables 2.12 and 2.13 report the standardized skewness and kurtosis. Standardized skewness is negative in all the cases (Table 2.12). Standardized kurtosis is higher than three in all cases having almost the same standardized kurtosis as in the case of the univariate analysis indicating that the behaviour of BTC follows the same pattern. Standardized skewness (Table 2.13 Panel D) is negative in all cases except EURO-BTC (positive) and standardized kurtosis ranges from 4.0311 to 4.7465.

Table 2.12. Bitcoin-Exchange Rates, Conditional Asymmetry and Shape Indexes

	BTC-EURO	BTC-JPY	BTC-CAN	BTC-GBP
Panel A.				
Asymmetry parameter: $h_{i,t}=\gamma_{0,i}+\gamma_{N,i}u^{-}_{i,t-1}+\gamma_{P,i}u^{+}_{i,t-1}+\gamma_{h,i}h_{i,t-1}$, $\lambda_{i,t}=1-2/\left(1+\exp\left(h_{i,t}\right)\right)$				
$\gamma_{0,i}$	-0.062	-0.0648	-0.0113	-0.0557
	(0.0391)	(0.0475)	(0.0204)	(0.0627)
$\gamma_{N,i}$	-0.0899	-0.0362	-0.0334	-0.0352
	(0.0478)*	(0.038)	(0.0272)	(0.0395)
$\gamma_{P,i}$	0.2386	0.1525	0.0547	0.1444
	(0.0513)**	(0.077)**	(0.0367)	(0.1007)
$\gamma_{h,i}$	0.0828	0.4146	0.7364	0.4308
	(0.2112)	(0.3664)	(0.2264)**	(0.5188)
Panel B.				
Shape parameter: $g_{i,t}=d_{0,i}+d_{N,i}u^{-}_{t-1}+d_{P,i}u^{+}_{t-1}+d_{h,i}\cdot g_{i,t-1}$, $k_{i,t}=k_U+\left(k_L-k_U\right)/\left(1+\exp\left(g_{i,t}\right)\right)$				
$d_{0,i}$	-0.9787	-0.7918	-0.1841	-0.7307
	(0.2039)**	(0.2169)**	(0.206)	(0.3599)**
$d_{N,i}$	0.4768	0.2295	0.0054	0.2177
	(0.2092)**	(0.1732)	(0.0757)	(0.1947)
$d_{P,i}$	0.9766	1.8513	0.0284	1.4838
	(0.3317)**	(2.1698)	(0.1242)	(3.0457)
$d_{h,i}$	0.0465	-0.0393	0.5421	0.0052
	(0.1981)	(0.0613)	(0.556)	(0.3255)
Panel C. Sample averages				
$\bar{\lambda}_{i,t}$	-0.0067	-0.0224	-0.007	-0.0163
	(0.0019)**	(0.0013)**	(0.0009)**	(0.0013)**
$\bar{k}_{i,t}$	0.8533	0.9478	0.889	0.9407
	(0.0019)**	(0.0013)**	(0.0009)**	(0.0013)**
Panel D. Other				
$L(\theta)$	-6,180.24	-6,191.09	-6,203.75	-6,19183
SK	-0.0896	-0.1604	-0.0372	-0.1261
KU	8.2914	7.2359	7.199	7.1549

Notes. The parameters $\gamma_{0,i}$, $\gamma_{N,i}$, $\gamma_{P,i}$, and $\gamma_{h,i}$ measure the impact of past negative and positive shocks on the asymmetry parameter. The minimum and maximum bounds for $k_{i,t}$ set to k_L = 0.4 and k_U = 1.6, respectively. The coefficients $d_{0,i}$, $d_{N,i}$, $d_{P,i}$, and $d_{h,i}$ measure the impact of past negative and positive shocks on the shape parameter and control the shape of the distribution. $u_{i,t-1}\equiv\left(r_{i,t-1}-m_{i,t-1}\right)/\sigma_{i,t-1}$ is the standardized excess to mode return. $L(\theta)$ is the sample log-likelihood values. SK and KU are the Pearson's skewness and the kurtosis, respectively. The estimated parameters are statistically insignificant unless otherwise noted. * and ** statistically significant at 10%, and 5%. The standard errors are presented in parentheses.

Table 2.13. Exchange Rates-Bitcoin, Conditional Asymmetry and Shape Indexes

	EURO-BTC	JPY-BTC	CAN-BTC	GBP-BTC
Panel A.				
Asymmetry parameter: $h_{i,t} = \gamma_{0,i} + \gamma_{N,i} u_{i,t-1}^{-} + \gamma_{P,i} u_{i,t-1}^{+} + \gamma_{h,i} h_{i,t-1}$, $\lambda_{i,t} = 1 - 2 \big/ \left(1 + \exp\left(h_{i,t}\right)\right)$				
$\gamma_{0,i}$	0.0484	-0.0239	0.0431	-0.0184
	(0.0922)	(0.0532)	(0.039)	(0.0452)
$\gamma_{N,i}$	-0.0137	-0.1987	-0.0993	0.0115
	(0.0600)	(0.0672)**	(0.054)*	(0.0509)
$\gamma_{P,i}$	-0.0299	0.1549	-0.0395	0.0258
	(0.0607)	(0.056)**	(0.0546)	(0.0605)
$\gamma_{h,i}$	0.4876	0.0203	0.4087	0.5124
	(1.1904)	(0.1859)	(0.4126)	(1.4082)
Panel B.				
Shape parameter: $g_{i,t} = d_{0,i} + d_{N,i} u_{t-1}^{-} + d_{P,i} u_{t-1}^{+} + d_{h,i} \cdot g_{i,t-1}$, $k_{i,t} = k_U + \left(k_L - k_U\right) \big/ \left(1 + \exp\left(g_{i,t}\right)\right)$				
$d_{0,i}$	-0.1127	-0.1896	-0.1682	-0.1602
	(0.0763)	(0.088)**	(0.0867)*	(0.0943)*
$d_{N,i}$	0.1647	0.2216	0.3096	0.2674
	(0.161)	(0.1502)	(0.2136)	(0.1812)
$d_{P,i}$	0.3018	0.5389	0.3138	0.3706
	(0.2113)	(0.2361)**	(0.2168)	(0.2784)
$d_{h,i}$	0.9618	0.9176	0.9718	0.9682
	(0.0275)**	(0.0404)**	(0.0148)**	(0.0212)**
Panel C. Sample averages				
$\bar{\lambda}_{i,t}$	0.0308	-0.0221	-0.0078	-0.0044
	(0.0002)**	(0.0019)**	(0.0006)**	(0.0002)**
$\bar{k}_{i,t}$	1.3868	1.2631	1.4395	1.458
	(0.0002)**	(0.0019)**	(0.0006)**	(0.0002)**
Panel D. Other				
$L(\theta)$	-1,478.51	-1,619.13	-1,338.3	-1,666.58
SK	0.0871	-0.0747	-0.0132	-0.0132
KU	4.2071	4.7465	4.3579	4.0311

Notes. The parameters $\gamma_{0,i}$, $\gamma_{N,i}$, $\gamma_{P,i}$, and $\gamma_{h,i}$ measure the impact of past negative and positive shocks on the asymmetry parameter. The minimum and maximum bounds for $k_{i,t}$ are set to $k_L = 0.4$ and $k_U = 1.6$, respectively. The coefficients $d_{0,i}$, $d_{N,i}$, $d_{P,i}$, and $d_{h,i}$ measure the impact of past negative and positive shocks on the shape parameter and control the shape of the distribution. $u_{i,t-1} \equiv \left(r_{i,t-1} - m_{i,t-1}\right) \big/ \sigma_{i,t-1}$ is the standardized excess to mode return. $L(\theta)$ is the sample log-likelihood values. *SK* and *KU* are Pearson's skewness and kurtosis. The estimated parameters are statistically insignificant unless otherwise noted. *, and ** statistically significant at 10%% and 5%, respectively. The standard errors are presented in parentheses.

Figure 2.11. Conditional Standard Deviation of Daily Returns Over Time (Bivariate Analysis)

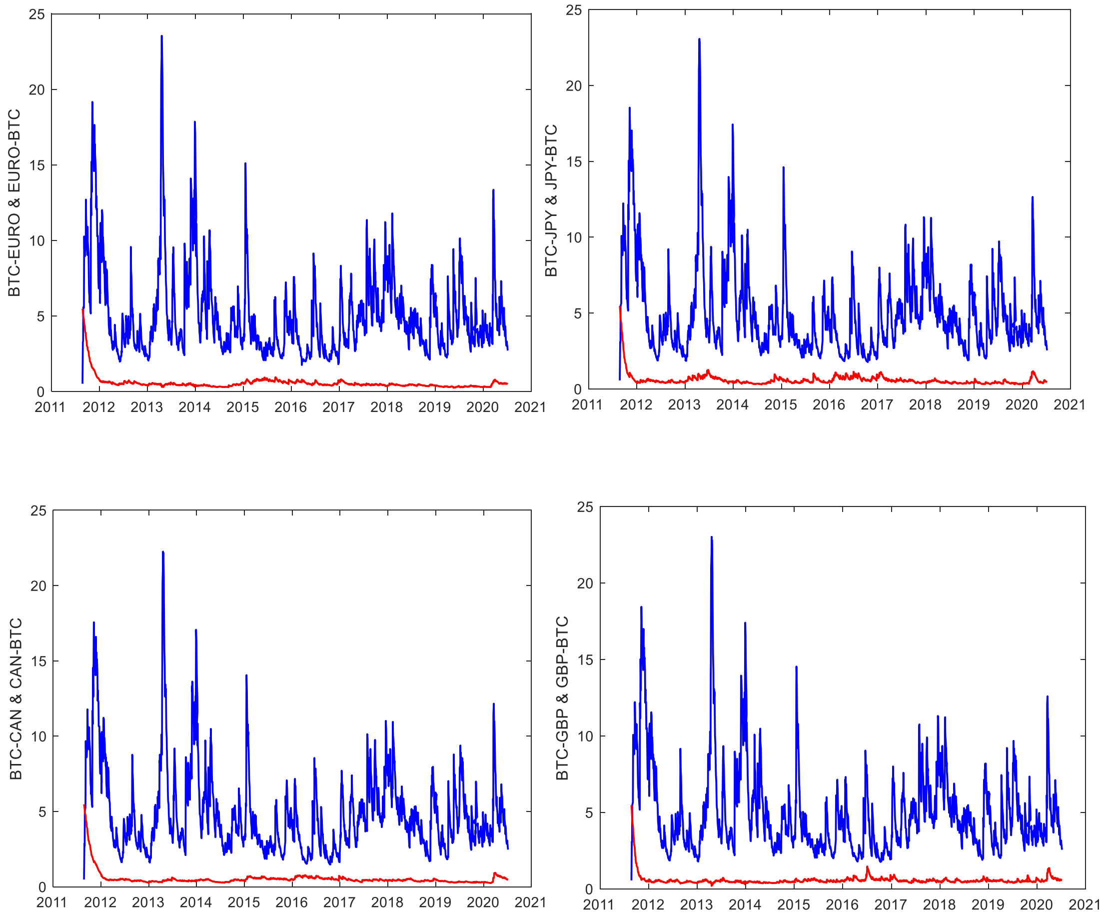
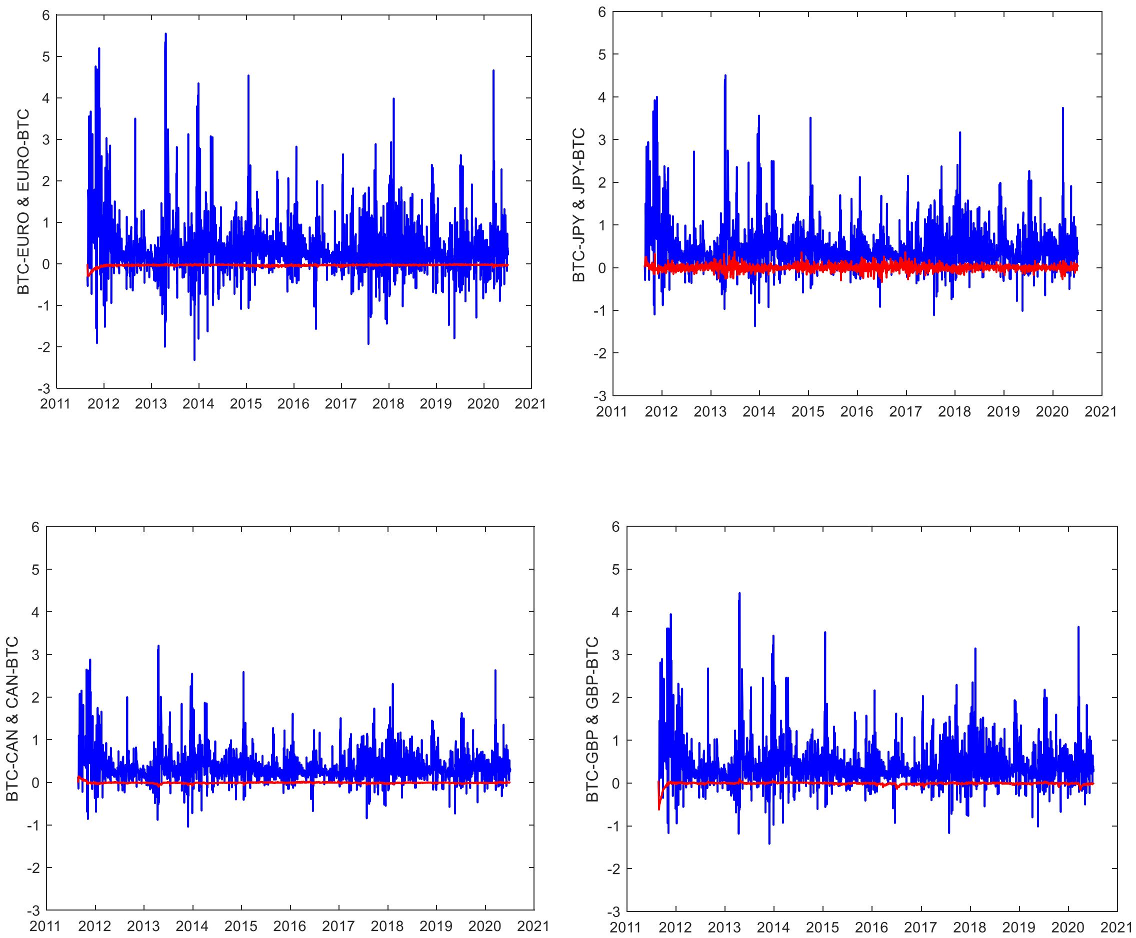


Figure 2.12. Conditional Mean of Daily Returns Over Time (Bivariate Analysis)



2.5.6 Forecasting Bitcoin's Prices

To further explore the forecasting ability of the model and compare it with other existing GARCH models, the forecasting prices of BTC are computed using the Skewed Normal, Skewed Laplace, Laplace, and the Normal probability distributions in the GARCH and GJR GARCH specifications. Table 2.14 presents the setting parameters for each GARCH specification under these different probability distributions. For example, in the case of the GARCH Normal model $a_{N,i} = 0$ in the conditional variance equation, $\delta_{i,t} = 0$ in the conditional mean equation, $\lambda_{i,t} = 0$ in the conditional asymmetry parameter, and $k_{i,t} = 2$ in the conditional shape parameter. For the rest of the GARCH specifications see Table 2.14.

Table 2.14. GARCH specifications under Different Probability Distributions.

GARCH Models	Conditional Variance	Conditional Asymmetry Parameter	Conditional Shape Parameter
GARCH Normal	$a_{N,i} = 0$	$\lambda_{i,t} = 0$	$k_{i,t} = 2$
GARCH Skewed Normal	$a_{N,i} = 0$		$k_{i,t} = 2$
GARCH Laplace	$a_{N,i} = 0$	$\lambda_{i,t} = 0$	$k_{i,t} = 1$
GARCH Skewed Laplace	$a_{N,i} = 0$		$k_{i,t} = 1$
GJR Normal		$\lambda_{i,t} = 0$	$k_{i,t} = 2$
GJR Skewed Normal			$k_{i,t} = 2$
GJR Laplace		$\lambda_{i,t} = 0$	$k_{i,t} = 1$
GJR Skewed Laplace			$k_{i,t} = 1$

Notes. Each GARCH specification under different probability distribution is computed using the set parameters in the conditional variance, conditional mean, conditional asymmetry, and shape parameters (distributional parameters) equations, (2.9) – (2.15). In the case of the GARCH Normal model $a_{N,i} = 0$ in the conditional variance equation, $\lambda_{i,t} = 0$ in the conditional asymmetry parameter, and $k_{i,t} = 2$ in the conditional shape parameter.

Table 2.15 presents the forecasting accuracy criteria of Bitcoin forecast prices using different GARCH specifications. The forecasting performance is performed using the root mean square error (RMSE), and the mean absolute error (MAE) measures. The findings suggest several conclusions. At the first sight, the GARCH Skewed Normal and GARCH Skewed Laplace are better models compared to the GARCH Normal and GARCH Laplace. This is also happening in the case of GJR-GARCH models (GJR Skewed Normal and GJR Skewed Laplace perform better compared to GJR Normal and GJR Laplace). Adding to this, the findings show that ST-GJR-GARCH under the Skewed

Generalized Error Distribution (SGED) performs better than the rest models highlighting the importance of the Skewed Generalized Error distribution to model Bitcoin returns as this distribution captures well data with leptokurtic characteristics. The reason that ST-GJR-GARCH-SGED model performs better than the other models is that it captures the shape and the tails of Bitcoin's price probability distribution at the same time.

Table 2.15. Out-of-Sample Forecasting Prices of Bitcoin under Different Model Specifications

Model	RMSE	MAE
GARCH Normal	570.1347	342.6791
GARCH Skewed Normal	387.2290	327.8108
GARCH Laplace	622.6610	344.9637
GARCH Skewed Laplace	374.8543	304.0254
GJR Normal	570.1826	342.6687
GJR Skewed Normal	387.3371	329.0468
GJR Laplace	622.7194	344.9788
GJR Skewed Laplace	376.4510	304.6615
ST-GARCH-M-GJR	373.1703	237.6083

Notes. The forecasting ability of the ST-GARCH-M-GJR is compared to other GARCH models under the Normal, Skewed Normal, Laplace, and Skewed Laplace probability distributions. The performance accuracy of each model is computed using the RMSE (root mean square error), and the MAE (mean absolute error). The RMSE is computed using the equation $RMSE = \sqrt{\frac{\sum_{t=1}^{T}\left(x_t - \hat{x}_t\right)^2}{T}}$ where x_t is the price of Bitcoin and $\hat{x}_t$ is the predicted value of Bitcoin at t = 1,2,.. The MAE is computed using the equation $MAE = \frac{\sum_{t=1}^{T}\left|x_t - \hat{x}_t\right|}{T}$. The data for the out-of-sample forecasting period are from June 2, 2020 to June 29, 2020.

2.6 Summary and Conclusions

Bitcoin is the most popular cryptocurrency over the all-digital currencies that exist. Furthermore, Bitcoin, in many cases, reaches extreme values. For this reason, it exhibits a relatively higher return and risk than the other currencies. The high returns and volatility lead authors to investigate Bitcoin in different ways, e.g. portfolio analysis. The interest to examine the stochastic behaviour of Bitcoin and exchange rates derives from the extraordinary behaviour of Bitcoin. Many scholars conclude that the behaviour of Bitcoin is unique and different.

This second chapter expands on the academic literature regarding the examination and comparison of the stochastic properties of returns of Bitcoin and four major currencies (Euro, Japanese Yen, Canadian dollar, and British pound). More specifically, it investigates the presence of higher-order moment dependencies such as conditional heteroscedasticity, asymmetric volatility and non-linearities on these currencies. Furthermore, the examination extends beyond the first two moments of a probability distribution (conditional mean and variance). To fulfil these tasks, the chapter of this dissertation used the conditional mean and conditional variance equations of the GJR-GARCH-M model to the conditional skewness and conditional kurtosis equations of Savva and Theodossiou (2018) into a dynamic framework named ST-GJR-GARCH-M. The Skewed Generalized Error Distribution (SGED) is used to incorporate the time-varying skewness and kurtosis equations in the model. This distribution proven to fit well in financial data (Theodossiou, 2015). The parameters of this framework are obtained using a maximum likelihood technique under the SGED (skewed generalized error distribution).

The framework of Savva and Theodossiou (2018) is further extended to investigate the spillover effects with the presence of time-varying asymmetry and shape parameters. Using the ST-GJR–GARCH–SGED model in a bivariate context the mean and volatility spillovers in downside and upside terms are investigated.

Analytic mathematical formulas have been tested showing the presence of higher-order dependencies including volatility clustering, asymmetric volatility, and non–linear dependencies. This result arises from the presence of an asymmetry parameter and kurtosis in all log-returns indicating the necessity of using distributions triggered by higher-order moments. Because of these dependencies, the empirical distributions of

Bitcoin’s returns exhibit skewness and extreme leptokurtosis. The shape parameter associated with leptokurtosis is found significant in all return series. Their mean values deviate between 0.876 (Bitcoin) and 1.470 (British pound). These estimates depict extreme leptokurtosis, especially in the case of Bitcoin. The latter result partially explains the extraordinary volatility of Bitcoin that leads to a higher peaked probability distribution in relation to the rest assets. Skewness is found to have a negative impact on Bitcoin, Canadian dollar and British pound and a positive impact on the two currencies examined. The findings shed light on the risk and return relationship of Bitcoin and currencies.

There is no doubt that the mean and volatility spillovers between bitcoin and exchange rates and vice versa are important. The understanding of this relationship is important for all participants. The empirical findings highlight that the exchange rates affect Bitcoin’s conditional mean and volatility more than the reverse, Bitcoin does not affect the conditional mean and volatility of the other exchange rates, Bitcoin’s behaviour is extremely leptokurtic when compared to the other assets even if spillover effects are presented, and it is a useful asset to diversify portfolio’s risk since it behaves in a very different way compared to the other assets. Overall, the findings reveal a weak inter-relationship between Bitcoin and exchange rates confirming the extremely different behaviour of Bitcoin.

The forecasting ability of the ST-GJR-GARCH-SGED model is compared to other existing GARCH models. The forecasting prices of BTC are computed using the Skewed Normal, Skewed Laplace, Laplace, and the Normal probability distributions in the GARCH and GJR GARCH specifications. The findings show that the ST-GJR-GARCH-SGED model outperforms the GARCH and GJR models indicating the importance of the model to capture the asymmetry and shape characteristics (skewness and kurtosis) of Bitcoin’s future prices.

The extremely different time-series behaviour of Bitcoin can be used in hedging and risk management by speculators and portfolio managers. Furthermore, the estimated equations for computing the conditional mean, variance, asymmetry, and kurtosis parameters provide a way to forecast future Bitcoin prices.

Bitcoin is a recent topic in the academic literature, therefore, there are several issues for future research. For instance, the empirical findings are based on the historical data of Bitcoin. Bitcoin is a new area of interest and the availability of data is limited. Therefore, in the future, the behaviour of Bitcoin may be revisited using a longer data period. A longer sampling period would help to have a clearer view of the behaviour of Bitcoin as well as the interconnection between Bitcoin and other financial traditional assets. It will be interesting to further understand the persistence of Bitcoin's returns; if the behaviour of Bitcoin is still extremely different compared to the other assets.

In addition, this chapter focuses to investigate the behaviour of Bitcoin using daily data. A possible expansion of this work will be to examine the behaviour of Bitcoin and compare it to other common assets using other frequency data (e.g., weekly, monthly). This may enhance our knowledge and understand better the diversification capabilities of Bitcoin as investors use Bitcoin in their investment strategies. Furthermore, this chapter used the GJR-GARCH-M model using time-varying skewness and kurtosis parameters under the Skewed Generalized Error Distribution (SGED). From the methodological side of view, another possible future research would be to investigate Bitcoin and other assets using alternative models that take into account time-varying skewness and kurtosis characteristics. Bitcoin exhibits extreme leptokurtosis and these models are appropriate to investigate the behaviour of such assets.

3 The Measurement of Stock Price Crashes Using a Robust Resistant Outlier Technique

3.1 Introduction

A crash can be defined as an unusually sharp drop in the firm's stock price caused by unexpected bad news. Stock price crash risk can also fall under the definition of a negative skewness of the return's distribution (Chen et al., 2001; Kim et al., 2014). This negative firm-specific shock is an extreme outlier in the distribution of returns. Jin and Myers (2006) stated that "A crash is defined as a remote outlier in a firm's residual return". Several researchers examined the firm's stock price crash risk exposure in various financial issues such as the behavioural characteristics of CEOs/CFOs (e.g., age, overconfident), corporate tax avoidance, and others (see for example Kim and Zhang, 2014, 2016; An et al., 2015; Andreou et al., 2016; Kim et al. 2016; Kim et al. 2014; Chang et al., 2017; and Andreou et al., 2021, etc).

The intuition behind the crash risk theory is that managers have incentives to withhold bad news or delay the announcement of good news. These incentives may be, among others, about compensation contracts, career prospects, improving the value of stocks, hiding fraud, maintaining their reputation, etc (see for more details Kothari, 2009). What is more, stock price crashes are more profound when agency risk among firms is high (Callen and Fang, 2015b). Therefore, the tendency of managers to conceal bad news and boost good news reinforces future crash' risk. When managers stop hiding bad news (perhaps due to limited incentives), a huge amount of negative information will be made public to the market leading to a stock price crash.

Researchers used the expanded market model for each firm and year to estimate the return's residuals (Dimson, 1979). The idea behind this model is to screen out the market crashes and only firm-specific events are considered. In this chapter, the attention is set on the binary crash risk measure that literature used to define a stock price crash. The binary measure is an indicator that takes the value of one when at least one firm-specific weekly return falls 3.09/3.20 standard deviation below the mean firm-specific weekly returns, and zero otherwise.

The literature on outliers is enormous and concerns a lot of statisticians and data analysts. A lot of authors stated definitions about outliers (e.g., Barnett, 1978; Hawkins, 1980; Barnett and Lewis, 1994). From the perspective of statisticians/econometricians, noted that a single observation is enough to dramatically change the estimated coefficients. Outliers drive OLS estimators to be biased and inconsistent, see for example Martin and Simin (2003) and Genton and Ronchetti (2008). A large body of literature used robust statistics to avoid the disadvantages of the least square estimator (OLS) technique (Huber, 1964; Huber, 1981; Hampel et al., 1986, Butler et al., 1990; McDonald et al., 2009). A wrong (biased) estimation will lead to erroneous results (Genton and Ronchetti, 2008; Martin and Simin, 2003).

The estimation of coefficients on event studies was investigated by Sorokina et al. (2013). They used OLS and robust methods to investigate the effects of coefficients. The finding was that robust methods give more accurate results when there are outliers and leverage effects on the datasets. Theodossiou and Theodossiou (2014, 2019) distinguish OLS and Huber's estimators using a model that consists of regular and outlier components. Using analytical equations, they showed that the presence of outliers in the series, drive the estimated parameters as well as the standard deviation to be contaminated and propose an outlier resistant method that corrects these issues. The chapter of this dissertation focuses on the contamination issue of uncertainty which is a statistical measure used to construct the binary crash risk measure.

The motivation behind this chapter is multi-fold. Firstly, using an analytical statistical crash risk framework, the contaminate issue on uncertainty due to outliers that presented in the return series will be shown. Notably, this contamination issue causes the binary crash risk measure to be mis-specified. Secondly, following Theodossiou and Theodossiou (2019), a robust framework is further developed to measure the percentage of crashes using a robust outlier resistant method. This is feasible due to the decomposition of returns on regular and outlier components. This methodology corrects the contamination issue and leads to more accurate findings. Furthermore, the findings are compared using the standard literature that used the logarithmic transformation of the residual returns (henceforth logarithmic transformed measure), as well as the residual returns as derived from the Dimson (1979) model (henceforth un-transformed measure). The robust methodology will illustrate that the construction of the binary measure using the untransformed robust residual returns and the corrected standard deviation, provide accurate findings. Finally, this methodology contributes in the existing literature by developing a statistical crash risk framework since a statistical theory in the crash risk literature does not exist.

Monte – Carlo simulations show that the ordinary least square methodology (OLS) detects a lower percentage of crashes relative to the robust methodology. Also, the OLS un-transformed measure detects a lower percentage of crashes relative to the OLS log transformed measure.

The empirical findings share the same conclusions as in the simulation findings. The percentage of crashes (log transformed measure) using the standard methodology (OLS) is lower in comparison to the robust methodology. More specifically, the percentage of crashes using the standard methodology is 18.21% or 30,226 firm-year observations (log-transformed measure) and 13.61% or 22,596 firm-year observations (un-transformed measure). On the other hand, the percentage of crashes using the robust methodology is 20.04% or 33,266 firm-year observations.

This chapter also presents an analysis of the commonality and differences on the percentage of crashes between the robust and ordinary least square measures. The comparison is between the robust measure and the un-transformed OLS residual returns as well as the robust measure and the log-transformed measure. The findings using the robust measure and the un-transformed measure show that 22,332 firm-year observations are common in both methodologies, 264 firm-year observations are classified as crashes in OLS and not in the robust methodology while 10,934 firm-year observations are categorized as crashes in the robust methodology and not in the OLS measure. Nevertheless, the common firm-year observations between the robust measure and the log-transformed measure are 27,121 firm-year observations while 6,145 firm-year observations are classified as crashes in the robust measure and non-crash in the logarithmic OLS measure.

The common firm-year observations between the un-transformed OLS residual returns and the logarithmic OLS residual returns are 22,596 firm-year observations. This means that all the crashes that are included using the logarithmic transformation of returns are also included in the un-transformed while 0% of crashes is detected using the un-transformed residual returns and not the logarithmic transformation of them. The number of firm-year observations that are classified as crashes using the logarithmic transformation of the residual returns and not on the un-transformed measure are 7,630.

The above findings suggest that in order to avoid the misspecification of crashes, it is better to use a robust technique that corrects the inflation on the variance driven by outliers. Further analysis of the percentage of crashes by industry using the Fama and French industry classifications is presented below as well as an analysis of the weekly specific return of firms that detected to crash under the robust technique and non-crash in ordinary least square method verifying the concerns of this chapter.

The chapter of this dissertation will be following this structure. Section 3.2 presents the literature review and section 3.3 presents the outlier crash risk framework. A robust technique is developed to define crashes showing the effect of outliers on variance as well as the estimation method. In section 3.4 there follows a presentation of the Monte-Carlo simulations showing the effects of outliers in the 52-weekly returns on the binary crash risk measures, and section 3.5 presents the empirical findings explaining the differences between standard (OLS) and robust methodologies. Summary and conclusions are presented in section 3.6.

3.2 Literature Review

Stock Price Crash Risk

The bad news hoarding stock price crash risk theory have been investigated by several empirical researchers. They examined the firm-specific stock price crashes in terms of accrual manipulation, corporate tax avoidance, religious beliefs, and the behavioural characteristics of CEOs/CFOs (e.g., age, overconfident). Chen et al. (2001) is the first study that investigated the forecasting of crashes and other determinants such as trading volume, conditional skewness, and past returns.

Jin and Myers (2006) analysed the theoretical perspective of stock price crash risk and hoarding bad news. They found that managers distort the company’s image and when they do not have any more incentives to withhold the bad news allow the information to become available to the market. This behaviour leads to a stock price crash. Hutton et al. (2009) examined the inter-relationship between stock price crash risk and accrual manipulation. Using the opacity measure as an earnings management variable, they found that opaque firms are more likely to suffer a stock price crash.

Kim et al. (2011a) and Kim et al. (2011b) examined the relationship between stock price incentives and tax avoidance and a firm's crash risk, respectively. Kim et al. (2011a) showed that the different incentives of CEOs/CFOs and firm-specific stock price crashes are correlated. Using a large dataset of U.S. companies, they found that the options portfolio value of CFOs is significantly positive with the stock price crashes. Kim et al. (2011b) exposed that corporate tax avoidance and firm-specific stock price crashes are strongly related. The explanation is that specific tools and justifications are used to accumulate bad news and mask the company's reputation for a longer period. When the firm has no other reason to withhold the bad news, the information becomes available to the market leading to a sharp fall in the firm's stock price.

What is more, the behaviour of investors has been investigated by An and Zhang (2013) and Callen and Fang (2013). An and Zhang (2013) revealed that stock price synchronicity and crashes are negatively correlated with the institutional investors (perhaps due to long-term investments that they are trying to monitor). The relationship is also negative in the case of transient investors. Callen and Fang (2013) further investigated institutional investors and stock price crashes supporting the negative relationship between the monitoring theory of institutional investors and stock price crash risk.

Kim et al. (2016) found that overconfident CEOs and stock price crashes are positively correlated. This relationship is stronger when the opinions between CEOs and investors differ more. Andreou et al. (2017) concluded that stock price crashes are more profound when CEOs are younger while Andreou et al. (2016) investigated the stock price crashes in the spirit of corporate governance (accounting opacity, managerial incentives, and others). Other papers investigated crash risk in corporate social responsibility (Kim et al., 2014), political directors (Lee and Wang, 2017), religious beliefs (Callen and Fang, 2015a), short interest (Callen and Fang, 2015b), and so on.

The empirical investigation of crash risk is based on various measures (see Habib et al.,2018 for the details). Some of these measures are the binary crash risk measures (Hutton et al., 2009), negative skewness (Chen et al., 2001), down-to-up volatility (Andreou et al. 2016), and other. The empirical analysis is a two-stage procedure. In the first stage, they used the expanded market model for each firm and year to estimate the return's residuals (Dimson, 1979). In the second stage, regression models were run using standard control variables (such as past returns, stock volatility, negative skewness, kurtosis, logarithmic of market value, detrended share turnover) and their explanatory variable to identify the determinants of stock crashes. In this chapter, the attention is set on the binary crash risk measure. The binary measure is an indicator that takes the value of one when at least one firm-specific weekly return falls 3.09/3.20 standard deviation below the mean firm-specific weekly returns, and zero otherwise.

Outliers and Robust Regression

Outlier return is an unexpected sharp decrease/increase in the series caused by a firm-specific event, such as a merger or an acquisition. These occur randomly on different magnitudes through the companies' fiscal year. In many cases, managers are masking events to satisfy their own interests (Hutton et al., 2009). When such events are no longer hidden, the information is then available in the market, leading to a stock price crash. In statistical terms, this leads to an outlier return in the 52-weekly return series.

Outliers is a widespread issue in the academic literature, especially, in statisticians and data analysts. This issue concerns a lot of researchers from a theoretical and empirical perspective. Several definitions of outliers have been noted across the years. For example, Hawkins (1980) states that an outlier is "as an observation that deviates so much from other observations as to arouse suspicion that it was generated by a different mechanism." Barnett and Lewis (1994) defines that is "an outlying observation, or outlier, is one that appears to deviate markedly from other members of the sample in which it occurs".

In real accounting and financial datasets, there are a lot of outliers. The standard ordinary least square technique (OLS) underperforms in the presence of outliers. Outliers drive OLS estimators to be biased and inconsistent. This is the reason there is a large body of literature about robust statistics (Huber, 1964; Huber, 1981; Huber, 1973; Hampel et al., 1986, Butler et al.,1990). Most papers apply robust estimation techniques to avoid the disadvantages of OLS when outliers are presented in the series.

Martin and Simin (2003) and Genton and Ronchetti (2008) illustrated that the estimated beta using an outlier-resistant technique is a better predictor of the risk-return than the ordinary least square (OLS) beta estimator. Martin and Simin (2003) explained this finding using two statistical properties. Firstly, when there are no outliers in the return-series, the outlier resistant method performs as well as the ordinary least square

(OLS). Secondly, when there are outliers in the series, the outlier resistant method minimizes the outlier beta while the OLS technique causes a large beta bias.

Sorokina et al. (2013) highlighted the importance of handling outliers appropriately since they found that a robust outlier methodology in event studies provides different results from the OLS methodology. They found that most of the outliers are within the event windows leading to the inaccurate fitting of regression. Therefore, the treatment of these outlies is important. For example, these outliers provide valuable information and their exclusion will yield to different estimators and findings. Thusly, they concluded that robust estimation techniques provide more accurate findings in the case of event studies.

Theodossiou and Theodossiou (2014) examined the presence of return outliers using the Huber's Robust M (HRM) estimation technique. They tested the impact of outliers on the OLS beta estimates compared to a robust technique. They concluded that the estimators using the OLS and HRM are similar when there are no outliers, therefore, the returns follow a normal distribution. However, when there are outliers in the return series, the estimated parameters change dramatically.

Theodossiou and Theodossiou (2019) further investigated the impact of outliers on event studies using a framework that decomposes the stock returns in regular and outlier components. Analytical equations showed the impact of outliers on the standard deviation, as well as the on the CAR statistics. Using Monte-Carlo simulations they indicated that the outlier resistant method outperforms the OLS. In this chapter, the Theodossiou and Theodossiou's (2019) model is used to investigate the measurement of stock price crash risk due to the standard deviation that is used to construct the binary crash risk measure. A robust crash risk measure will be developed based on the robust residual returns as well as on the corrected standard deviation.

3.3 Outlier Crash Risk Framework

The below framework is based on Theodossiou and Theodossiou's (2019) model to construct a measure that defines the stock price crashes using an outlier-resistant method. Following their model, the decomposition of returns in regular and outliers' components is also useful to show the effects on the residual returns, therefore, to define crashes.

3.3.1 Firm-Specific Crash Events and Outliers

A stock price crash is an unusual sharp decrease of the stock's price caused by unexpected bad news. Let assume that there is a firm-specific outlier event within a year that cause an important crash in the return series. Based on this behaviour, the company's return can be represented using a mixed return process that decomposes returns on a regular and an outlier component (Roll, 1988). The return of stock *i* in period *t* represented by

$$r_{i,t} = k_{i,t} + d_{i,t} h_{i,t} \tag{3.1}$$

for $i = 1,2, \ldots, N$ and $t = 1,2, \ldots, T$.

where $k_{i,t}$ and $h_{i,t}$ are the regular and outlier components of returns, respectively, $d_{i,t}$ is a Bernoulli and takes the value of one when outlier returns are presented and the value of zero, otherwise. Also, N represents the number of stocks and T is the sample size.

Equation (3.1) can be rewritten as

$$r_{i,t} = \mu_{k,i,t} + e_{i,t} + d_{i,t}\left(\mu_{h,i,t} + v_{i,t}\right) \tag{3.2}$$

where $\mu_{k,i,t}$ and $\mu_{h,i,t}$ are the conditional means of $k_{i,t}$ and $h_{i,t}$ and $e_{i,t}$ and $v_{i,t}$ are white noise errors with standard deviations $\sigma_{e,i}$ and $\sigma_{v,i}$, respectively.

Therefore, the conditional distribution of the return's, $r_{i,t}$, is

$$\mu_{i,t} = E\left(r_{i,t} \middle| I_t\right) = E\left(k_{i,t}\right) + E\left(d_{i,t} h_{i,t}\right) = \mu_{k,i,t} + q_i \mu_{h,i,t} \tag{3.3}$$

where $q_i = \text{prob}(d_{i,t} = 1)$.

Equation (3.3) can be rewritten as

$$r_{i,t} = \mu_{i,t} + \varepsilon_{i,t} = \mu_{k,i,t} + q_i \mu_{h,i,t} + \varepsilon_{i,t} \tag{3.4}$$

where the error term is

$$\varepsilon_{i,t} = e_{i,t} + \left(d_{i,t} - q_i\right)\mu_{h,i,t} + d_{i,t} v_{i,t} \text{ [29]} \tag{3.5}$$

The conditional mean can be decomposed in the regular conditional mean ($\mu_{i,k}$) and the outlier conditional mean ($\mu_{i,h}$). Equation (3.5) shows that the error term is a function

[29] $\varepsilon_{i,t} = r_{i,t} - \mu_{i,t} = \mu_{k,i,t} + e_{i,t} + d_{i,t}\left(\mu_{h,i,t} + v_{i,t}\right) - \mu_{k,i,t} - q_i \mu_{h,i,t} = e_{i,t} + \left(d_{i,t} - q_i\right)\mu_{h,i,t} + d_{i,t} v_{i,t}$

of the conditional mean of the outlier component returns $\mu_{h,i,t,}$ and the regular and outlier errors $e_{i,t,}$ and $v_{i,t}$.

The variance of $\varepsilon_{i,t}$ is

$$\operatorname{var}\left(\varepsilon_{i,t}\right)=\sigma_{e_i}^2+q_i\sigma_{v_i}^2+q_i\left(1-q_i\right)\mu_{h,i}^2 \text{ [30]} \tag{3.6}$$

Equation (3.6) shows that the variance of the error term is a function of the mean of the outlier component returns $\mu_{h,i,t,}$ and the variances of the regular and outlier errors $e_{i,t,}$ and $v_{i,t}$.

The mean of the regular and outlier returns at time t are specified respectively as

$$\mu_{k,i,t}=E\left(k_{i,t}\middle|I_t\right)=x_t\beta_{k,i} \tag{3.7}$$

$$\mu_{h,i,t}=E\left(h_{i,t}\middle|I_t\right)=x_t\beta_{h,i} \tag{3.8}$$

where x_t is a vector of the independent variables of the model, and $\beta_{k,i}$ and $\beta_{h,i}$ are the estimated coefficients of the regular and outlier components. The fact that there are outliers in event studies (e.g. a crash event) is captured by these equations.

The substitution of equations (3.7) and (3.8) into (3.4) yield

$$r_{i,t}=x_t\left(\beta_{k,i}+q_i\beta_{h,i}\right)+e_{i,t}+\left(d_{i,t}-q_i\right)x_t\beta_{h,i}+d_{i,t}v_{i,t}=x_t\beta_{L,i}+\varepsilon_{i,t} \tag{3.9}$$

where $\beta_{L,i}=\beta_{k,i}+q_i\,\beta_{h,i}$ it was estimated using OLS and it includes the impact of outliers, therefore, it is contaminated. To avoid this issue, a robust technique will be used.

Therefore, the OLS residual is

$$\varepsilon_{i,t}=r_{i,t}-x_t\beta_{L,i}\,.$$

This residual return is used in the crash risk literature to compute the binary crash risk measure. The estimated parameters as well as the standard deviation computed by equation (3.6) are inflated by the mean and variance of the outlier return component.

---

[30] $\operatorname{var}\left(\varepsilon_{i,t}\right)=E\left(\varepsilon_{i,t}^2\middle|d_{i,t}\right)-E\left(\varepsilon_{i,t}\middle|d_{i,t}\right)^2=\sigma_{e_i}^2+E\left(d_{i,t}\right)\sigma_{v_i}^2+E\left(d_{i,t}-q_{i,t}\right)^2\mu_{h,i}^2$
$=\sigma_{e_i}^2+q_i\sigma_{v_i}^2+\left[\left(1-q_i\right)\left(-q_i\right)^2+q_i\left(1-q_i\right)^2\right]\mu_{h,i}^2=\sigma_{e_i}^2+q_i\sigma_{v_i}^2+q_i\left(1-q_i\right)\mu_{h,i}^2$

3.3.2 Outlier Resistant Model Estimation

The following robust estimation method was introduced by Huber (1981) and extended by Theodossiou and Theodossiou (2019). Huber (1964, 1973, and 1981) proposed the minimization of the maximum likelihood given by

$$\min_{\beta_{k,i}} Q_i = \sum_{t=1}^{T} p\left(r_{i,t} - x_t \beta_{k,i} \middle| I_t\right) \tag{3.10}$$

where

$$p\left(r_{i,t} - x_t \beta_{k,i} \middle| I_t\right) = \begin{cases} 0.5\left(r_{i,t} - x_t \beta_{k,i}\right)^2 & \text{for} \left|r_{i,t} - x_t \beta_{k,i}\right| \le c\sigma_{e,i} \\ \left|r_{i,t} - x_t \beta_{k,i}\right| c\sigma_{e,i} - 0.5c^2\sigma_{e,i}^2 & \text{for} \left|r_{i,t} - x_t \beta_{k,i}\right| > c\sigma_{e,i} \end{cases}, \tag{3.11}$$

c is a positive constant[31] and T is the size of the sample. All the other coefficients are defined above. The likelihood is a mixed process of probability distribution (Normal and Laplace). The first derivative of Q_i with respect to $\boldsymbol{\beta}_{k,i}$ is

$$\frac{\partial Q_i}{\partial \beta_{k,i}} = \sum_{t=1}^{T} \frac{\partial p\left(r_{i,t} - x_t \hat{\beta}_{k,i} \middle| I_t\right)}{\partial \beta_{k,i}} = \underset{\sim}{0}. \tag{3.12}$$

where

$$\hat{e}_{i,t} = \begin{cases} \left(r_{i,t} - x_t \hat{\beta}_{k,i}\right) & \text{for} \left|r_{i,t} - x_t \hat{\beta}_{k,i}\right| \le c\hat{\sigma}_{e,i} \\ sgn\left(r_{i,t} - x_t \hat{\beta}_{k,i}\right) c\hat{\sigma}_{e,i} & \text{for} \left|r_{i,t} - x_t \hat{\beta}_{k,i}\right| > c\hat{\sigma}_{e,i} \end{cases}, \tag{3.13}$$

$\hat{e}_{i,t}$ are winsorized residual returns estimators of the regular errors $e_{i,t}$, *sgn(.)* is a sign function and $\underset{\sim}{0}$ is a zero column vector. Equation (3.13) 'corrects' the contaminate issue of the standard methodology.

Using the following recursive equation, the estimated coefficients of $\boldsymbol{\beta}_{k,i}$ are computed by

$$\hat{\beta}_{k,i}^{s+1} = \hat{\beta}_{k,i}^{s} + \sum_{t=1}^{T} x_t' \hat{e}_{i,t} \Bigg/ \left(\sum_{t=1}^{T} x_t' x_t\right), \tag{3.14}$$

[31] For the purposes of the analysis $c = 1.72$.

where s is the number of iterations. Note that the OLS estimates are used as starting values.

The estimated variance used in each next iteration to recompute the new values of $\boldsymbol{\beta}_{k,i}$. That is,

$$\hat{\sigma}^2_{e,i} = \frac{\hat{g}_i^2}{T-p-1}\sum_{t=1}^{T}\hat{e}^2_{i,t}, \tag{3.15}$$

where $\hat{g}_i = \left[1+\frac{(p+1)\hat{q}_i}{T(1-\hat{q}_i)}\right]\frac{1}{(1-\hat{q}_i)}$, $\hat{q}_i$ is the winsorized returns (proportion) and p the number of betas that are estimated in the return models. [32]

Regular and Outlier Components

Returns for regular periods can be computed by

$$\hat{\mu}_{k,i,t} = x_t\hat{\beta}_{k,i} \tag{3.16}$$

where $\widehat{\beta}_{k,i}$ are the alpha and beta outlier resistant estimators. The difference between returns and the expected returns is

$$\hat{u}_{i,t} = r_{i,t} - \hat{\mu}_{k.i.t} \tag{3.17}$$

From equation (3.13) follows that

$$\hat{u}_{i,t} = \hat{e}_{i,t} + d_{i,t}\hat{h}_{i,t} \tag{3.18}$$

where $\hat{e}_{t,i}$ represents the winsorized return residuals that belongs in the interval $\left[-c\sigma_{e,i}, c\sigma_{e,i}\right]$ and $d_{i,t}h_{i,t}$ is the outlier residual return estimator. Note that $d_{i,t}$ =1 if the residuals are trimmed, and zero, otherwise.[33] The substitution of equation (3.18) into equation (3.17) gives

$$r_{i,t} = \hat{\mu}_{k,i,t} + \hat{u}_{i,t} = \hat{\mu}_{k,i,t} + \hat{e}_{i,t} + d_{i,t}\hat{h}_{i,t} \tag{3.19}$$

---

32 The recursive estimation ends when $\max\left|\hat{\beta}^{z+1}_{k,i} - \hat{\beta}^{z}_{k,i}\right| < h$ where h = 0.0001.

33 The outlier's probability computed by $\hat{q}_{i,t} = \frac{1}{T}\sum_{t=1}^{T}\hat{d}_{i,t}$.

where $\hat{\mu}_{k,i,t} = x_t \hat{\beta}_{k,i}$. The above equation provides the decomposition of returns on two components: the regular returns and the outlier returns. Therefore, the robust residual is $\hat{u}_{i,t} = r_{t,i} - \hat{\mu}_{k,i,t}$.

OLS Estimators and Variance

The OLS estimated parameters are computed using the following equation

$$\hat{\beta}_i = \sum_{t=1}^{T} x_t' r_{i,t} \Bigg/ \left(\sum_{t=1}^{T} x_t' x_t \right). \tag{3.20}$$

Substituting the OLS equation (3.20) into (3.19) yield

$$\hat{\beta}_i = \hat{\beta}_{k,i} + \left(\sum_{t=1}^{T} x_t' e_{i,t} \Bigg/ \sum_{t=1}^{T} x_t' x_t \right) + \left(\sum_{t=1}^{T} x_t' \hat{d}_{i,t} \hat{h}_{i,t} \Bigg/ \sum_{t=1}^{T} x_t' x_t \right) = \hat{\beta}_{k,i} + \hat{q}_i \hat{\beta}_{h,i}\,^{34}. \tag{3.21}$$

The OLS variance is

$$\hat{\sigma}^2_{\varepsilon_i} = \frac{1}{T-p-1} \sum_{t=1}^{T} \hat{\varepsilon}^2_{i,t} \tag{3.22}$$

where $\hat{\varepsilon}_{i,t} = r_{i,t} - x_t \hat{\beta}_i = r_{i,t} - x_t \hat{\beta}_{k,i} - \hat{q}_i x_t \hat{\beta}_{h,i} = \hat{e}_{i,t} + \hat{d}_{i,t} \hat{h}_{i,t} - \hat{q}_{i,t} \hat{\mu}_{h,i,t}$

$= \hat{e}_{i,t} + \hat{d}_{i,t} \left(\hat{h}_{i,t} - \hat{\mu}_{h,i,t} \right) + \left(\hat{d}_{i,t} - \hat{q}_{i,t} \right) \hat{\mu}_{h,i,t}$ and $\hat{v}_{i,t} = \left(\hat{h}_{i,t} - \hat{\mu}_{h,i,t} \right)$; see also Theodossiou and Theodossiou (2019).

The substitution of $\hat{\varepsilon}_{i,t}$ into (3.22) yield

$$\hat{\sigma}^2_{\varepsilon_i} = \frac{1}{T-p-1} \sum_{t=1}^{T} \hat{\varepsilon}^2_{i,t} = \frac{1}{T-p-1} \left(\hat{e}_{i,t} + \hat{d}_{i,t} \left(\hat{h}_{i,t} - \hat{\mu}_{h,i,t} \right) + \left(\hat{d}_{i,t} - \hat{q}_{i,t} \right) \hat{\mu}_{h,i,t} \right)^2$$

$$\hat{\sigma}^2_{\varepsilon_i} = \frac{1}{\hat{g}_i^2} \hat{\sigma}^2_{e_i} + \hat{q}_i \hat{\sigma}^2_{v_i} + \left(\frac{T}{T-p-1} \right) \hat{q}_i \left(1 - \hat{q}_i \right) \bar{\hat{\mu}}^2_{h,i}\,^{35} \tag{3.23}$$

---

[34] $\sum_{t=1}^{T} x_t' \hat{e}_{i,t} = \underset{\sim}{0}$.

[35] $\hat{\sigma}^2_{\varepsilon_i} = a \left(\sum_{t=1}^{T} \hat{e}^2_{i,t} + \sum_{t=1}^{T} \hat{d}_{i,t} \hat{v}^2_{i,t} + \sum_{t=1}^{T} \left(\hat{d}_{i,t} - \hat{q}_i \right)^2 \hat{\mu}^2_{h,i,t} + 2 \sum_{t=1}^{T} \hat{e}_{i,t} \hat{d}_{i,t} \hat{v}_{i,t} + 2 \sum_{t=1}^{T} \hat{e}_{i,t} \left(\hat{d}_{i,t} - \hat{q}_i \right) \hat{\mu}_{h,i,t} + 2 \sum_{i=1}^{T} \hat{d}_{i,t} \hat{v}_{i,t} \left(\hat{d}_{i,t} - \hat{q}_i \right) \hat{\mu}_{h,i,t} \right)$

where $a = \left(T - p - 1 \right)^{-1}$. Therefore, $\hat{\sigma}^2_{\varepsilon_i} = \frac{\hat{g}_i^2}{T-p-1} \sum_{t=1}^{T} \hat{e}^2_{i,t} + \hat{q}_i \frac{1}{T-p-1} \sum_{t=1}^{T} \hat{v}^2_{i,t} + \left(\frac{1}{T-p-1} \right) \hat{q}_i \left(1 - \hat{q}_i \right) \sum_{t=1}^{T} \hat{\mu}^2_{h,i,t}$.

where all the parameters are explained previously. Equation (3.23) showed that the regression variance of the ordinary least square technique (OLS) is contaminated by the variance of the regular and outlier components and the mean of outlier returns.

3.3.3 The Estimation of the Residual Returns in the Literature

Following Hutton et al. (2009) and Kim et al. (2011a), the market index model for each firm and year was used to estimate the residuals (Dimson, 1979). The idea behind this model is to screen out the market crashes and only firm-specific events are considered. The market models computed by using the robust and OLS methods, respectively

$$\mu_{k,i,t} = a_i + b_{k,1,i} r_{m,t-2} + b_{k,2,i} r_{m,t-1} + b_{k,3,i} r_{m,t} + b_{k,4,i} r_{m,t+1} + b_{k,5,i} r_{m,t+2} \Rightarrow r_{i,t} = \mu_{k,i,t} + u_{i,t} \quad (3.24)$$

$$\mu_{L,i,t} = a_i + b_{L,1,i} r_{m,t-2} + b_{L,2,i} r_{m,t-1} + b_{L,3,i} r_{m,t} + b_{L,4,i} r_{m,t+1} + b_{L,5,i} r_{m,t+2} \Rightarrow r_{i,t} = \mu_{L,i,t} + \varepsilon_{i,t} \quad (3.25)$$

where $r_{i,t}$ is the return of stock *i* (week *t*), $r_{m,t}$ is the return on the CRSP value-weighted market index (week *t*), $u_{i,t}$ and $\varepsilon_{i,t}$ are the error terms derived from the robust and ordinary least squares estimates. Academic literature used a logarithmic transformation of the residual returns to define crashes. The transformation of the residual returns generates the question of whether this is the proper way to define crashes.

Binary Measures of Crashes

The residuals are found to be highly skewed due to crashes that are included in the series. The error term, as derived from equations (3.24) and (3.25) can be used to define crashes ($u_{i,t,}$and $\varepsilon_{i,t}$, respectively).

Researchers transformed the residual returns derived from equation (3.25) to the logarithmic of one plus the residual return. Mathematically,

$$w_{i,t} = log(1 + \varepsilon_{i,t}) \quad (3.26)$$

where $\varepsilon_{i,t}$ is explained above.

Below the different measures of crashes are presented, using the residual returns and the logarithmic transformation of the residual returns as derived from the expanded return model (under the OLS and the outlier robust regressions).

The general definition of the binary measure is the following: crash is an indicator that takes the value of one when at least one firm-specific weekly return falls 3.09/3.20

standard deviation below the mean firm-specific weekly returns and the value of zero, otherwise. A crash event under robust and OLS measures takes the value of one ($crash_{y_{i,t}} = 1$) when the following condition is met

$$y_{i,t} < \mu_y - k * \sigma_y \text{ for } t = 1,2,\ldots n$$

$y_{i,t}$ is defined in three different ways, u (the robust residual returns), ε (the OLS residual returns), and w (the logarithmic OLS residual returns). Furthermore, k = 3.09/3.20. When the inequality is not satisfied, the above condition takes the value of zero, $crash_{y_{i,t}} = 0$. More specifically, these measures are the following.

<u>1st Measure: Robust Residual Returns – ROB</u>

The first measure to define crashes is using the robust residual returns and the corrected standard deviation. Mathematically,

$$ROB_{i,t} = 1 \text{ for } u_{i,t} < \mu_u - k * \sigma_e \tag{3.27}$$

$$ROB_{i,t} = 0 \text{, otherwise}$$

where k = 3.09/3.20, u is the robust residual returns, μ_u and σ_e are the mean and standard deviation computed by equation (3.15).

<u>2nd Measure: OLS Residual Returns – OLS</u>

The second measure to define crashes is using the OLS residual returns. Mathematically,

$$OLS_{i,t} = 1 \quad \text{for } \varepsilon_{i,t} < \mu_\varepsilon - k * \sigma_\varepsilon \tag{3.28}$$

$$OLS_{i,t} = 0 \text{, otherwise}$$

where k = 3.09/3.20, ε is the OLS residual returns, μ_ε and σ_ε, are the mean and standard deviation of the residual returns ($\varepsilon_{i,t}$).

<u>3rd Measure: The Logarithmic Transform of the OLS Residual Returns – WOLS</u>

The third measure to define crashes is using the logarithmic transformation of the OLS residual returns. Mathematically,

$$WOLS_{i,t} = 1 \quad \text{for} \quad w_{i,t} < \mu_w - k * \sigma_w \tag{3.29}$$

$$WOLS_{i,t} = 0 \text{, otherwise}$$

where k = 3.09/3.20, $w_{i,t}$ is the logarithmic transformation of the OLS residual returns, μ_w and σ_w are the mean and standard deviation of $w_{i,t}$.

3.4 Monte - Carlo Simulations

This section investigates the percentage of crashes in the standard and robust approaches using Monte Carlo simulations of the log-transformation of the residual return and the un-transform residual returns under the robust and OLS measures.

Monte – Carlo simulations are derived using a single-factor model with crashes

$$r_{i,t} = a_k + \beta_k r_{m,t} + e_{i,t} + d_{i,t}\left(a_h + \beta_h r_{m,t} + v_{i,t}\right) \text{ for } t = 1, 2,\dots \text{ and } i = 1,2,\dots,T. \quad (3.30)$$

where a_k and β_k are the estimated parameters of the regular component (alpha and betas, respectively), a_h and β_h are the parameters for the outlier component, $e_{i,t}$ and $v_{i,t}$ are the errors, $d_{i,t}$ is an indicator which takes the value of one in the case of crash event (outlier) and zero otherwise, and $r_{m,t}$ is the excess market return.

The percentage of crashes is computed using the standard and robust methodologies. Monte–Carlo simulations of T = 100,000 samples of 52 weekly randomly excess market returns investigate the three different binary crash risk measures. Data is downloaded from French's website.[36] The sample period is from 10/01/1992 to 26/06/2020. The errors of the regular and outlier components are computed using the standard deviation of each random sample of 52 excess market returns multiplied by random normal numbers, $e_{i,t} = \sigma_{e,i}\, z_{e,i,t}$ and $v_{i,t} = \sigma_{v,i}\, z_{v,i,t}$ where $z_{e,i,t}$ and $z_{v,i,t}$ are random normal numbers, $\sigma_{e,i} = 0.8\text{x } \sigma_{m,i}$ and $\sigma_{v,i} = 1.6\text{x } \sigma_{m,i}$ are the standard deviations of the regular and outlier errors, respectively. The returns, $r_{i,t}$, are generated using the equation (3.30).

To generate the samples, the parameter alpha of the regular component is set to a_k = 0 and the betas of the regular and outlier component returns are set to $\beta_k = 1$ and $\beta_h = 0$, respectively. The outlier intercept is set in the following way. The intercept, a_h, is set to take values in the interval (-0.20,-0.06). This means that a_h = -20% is an extreme negative outlier return in the 52-weekly return series.

[36] https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html (accessed: April 2021).

The robust residuals and ordinary least square (OLS) are drawn from each simulation to compute the binary crash risk measures, ROB_i for the robust technique, and OLS_i for the ordinary least square technique where i = 1, 2. The transformation of the ordinary least square residual returns is also examined, namely, $WOLS_i$. These measures take the value of one when at least one firm-specific weekly returns falling 3.09 (i = 1) or 3.20 (i = 2) standard deviation below the mean firm-specific weekly returns.

Percentage of Crashes

Table 3.1 presents the percentage of crashes of the robust technique (ROB_1 and ROB_2), and ordinary least square technique. For the OLS technique, the measures are derived using the un-transform residual returns (OLS_1 and OLS_2), and the transformation of the residual returns ($WOLS_1$ and $WOLS_2$).

All results are similar in the aspect that OLS and WOLS measures detect lower percentages of crashes than the ROB methodology. This is attributed to the fact that the presence of outliers in the 52-weekly return series miscomputed the standard deviation of the series. Table 3.1 shows that when a negative outlier equals -10% is presented in the series, the percentage of crashes using 3.09 (3.20) in the OLS is 77.71% (OLS_1) and 75.85% (OLS_2), respectively. In this case, the ROB findings are 81.92% (ROB_1) and 80.82% (ROB_2). Robust methodology detects 81.92% – 77.71% = 4.20% and 80.82% – 75.85%= 4.97% higher percentage of crashes.

Table 3.1 also shows the percentage of crashes using the WOLS measure. The WOLS methodology detects a lower percentage of crashes than the robust methodology. Interestingly, this methodology detects a higher percentage of crashes than OLS. For example, in the case that a -10% negative outlier exists in the 52-weekly return series, $WOLS_1$ = 79.18% and $WOLS_2$ = 77.35%. This means that 79.18% - 77.71% = 1.47% and 77.35% - 75.77% = 1.50% higher percentage of crashes is detected using the log-transformation measure (WOLS) in comparison to the un-transform (OLS).

Comparing the ROB findings with the WOLS approach, this means that 81.92% – 79.18% = 2.74% (ROB_1-$WOLS_1$) and 80.82%-77.35% = 3.47% (ROB_2-$WOLS_2$) higher crashes are detected using the outlier resistant method (ROB) compared to the OLS logarithmic transformation technique (WOLS).

The findings using Monte-Carlo simulations lead to two main conclusions. The first one is that the OLS technique detects a lower percentage of crashes compared to the outlier robust technique. The second conclusion is related to the un-transform measure. Using the residual returns as derived from the market model, the percentage of crashes are lower when compared to the log transformation of them.

Table 3.1. A Negative Outlier Return – $ROB_{1,2}$, $OLS_{1,2}$ and $WOLS_{1,2}$

α_h	ROB_1	ROB_2	OLS_1	OLS_2	$WOLS_1$	$WOLS_2$
-0.2	97.78	97.58	96.56	96.12	97.05	96.64
-0.19	97.29	97.05	95.91	95.38	96.47	96.02
-0.18	96.67	96.40	95.13	94.53	95.72	95.20
-0.17	95.89	95.59	94.18	93.48	94.83	94.24
-0.16	94.98	94.57	92.94	92.07	93.72	92.99
-0.15	93.71	93.23	91.40	90.48	92.28	91.41
-0.14	92.43	91.86	89.79	88.71	90.76	89.79
-0.13	90.44	89.83	87.59	86.37	88.70	87.55
-0.12	88.10	87.28	84.78	83.33	85.95	84.62
-0.11	85.51	84.57	81.68	80.06	83.09	81.45
-0.10	81.92	80.82	77.71	75.85	79.18	77.35
-0.09	77.44	76.11	72.77	70.55	74.36	72.26
-0.08	72.19	70.61	66.90	64.50	68.61	66.26
-0.07	65.55	63.75	59.81	57.04	61.59	58.92
-0.06	57.23	55.22	51.06	48.14	52.97	49.97

Notes. Monte–Carlo simulations of T = 100,000 samples of 52 weekly randomly excess market returns investigate the three different binary crash risk measures. The data period is from 10/01/1992 to 26/06/2020. The parameter a_h is setting to take values in the interval (-0.20,-0.06). This means that a_h = -20% is a large negative outlier return in the return series. The robust residuals and ordinary least square (OLS) are drawn from each simulation to compute the binary crash risk measures, namely, ROB_1, ROB_2, OLS_1, OLS_2, $WOLS_1$, and $WOLS_2$. The binary measure is an indicator that takes the value of one when at least one firm-specific weekly returns falling 3.09/3.20 standard deviation below the mean firm-specific weekly returns and the value of zero, otherwise. ROB_1 (3.09 standard deviation from the mean), ROB_2 (3.20 standard deviation from the mean), OLS_1 (un-transform residual returns, 3.09 standard deviation from the mean), OLS_2 (un-transform residual returns, 3.20 standard deviation from the mean), $WOLS_1$ (transformation of residual OLS returns, 3.09 standard deviation from the mean), and $WOLS_2$ (transformation of residual OLS returns, 3.20 standard deviation from the mean).

For the case of ROB measures, the standard deviation of equation (3.15) is taken to avoid the inflation of the outlier returns.

3.5 Empirical Findings

3.5.1 Data and Sample

The sample is constructed in the following way. The crash risk measures are computed using firms in the Center for Research in Security Prices (CRSP) for the period 1992-2018. Following the existing literature (e.g., Kim et al., 2011a) from the sample are excluded those firms with a share price lower than $2.5 and those with less than twenty-six weeks of return data (Hutton et al, 2009). After the data filtering, the final sample includes 165,985 firm-year observations. This corresponds to 20,390 firms from different industries.

Empirical Findings

3.5.2 Sample's Distribution and Statistics of Crashes

Tables 3.2 and 3.4 show the sample's distribution and statistics of the $ROB_{1,}$ OLS_1 and $ROB_{2,}$ OLS_2 measures using 3.09 and 3.20-times standard deviation from the mean. Tables 3.3 and 3.5 show the sample's distribution and statistics of the $ROB_{1,}$ $WOLS_1$ and $ROB_{2,}$ $WOLS_2$ measures using 3.09 and 3.20-times standard deviation from the mean.

Table 3.2 shows that in ROB_1, 33,266 firm-year observations, or 20.04% are categorized as crashes. The OLS_1 concludes that 22,596 firm-year observations or 13.61% are classified as crashes while $WOLS_1$ finds that 30,226 firm-year observations or 18.21% are categorized as crashes (Table 3.3). Figures 3.1 and 3.3 show the percentage of crashes under the ROB_1 and OLS_1, and ROB_1 and $WOLS_1$ (respectively) by year while figures 3.2 and 3.4 present the number of firms with stock price crashes. The figures show that robust methodology follows the same pattern as OLS and WOLS but with a higher percentage of crashes and number of firms with stock price crashes.

Table 3.4 shows that in ROB_2, 29,595 firm-year observations, or 17.83% are categorized as crashes. The OLS_2 concludes that 18,903 firm-year observations or 11.39% are classified as crashes while $WOLS_2$ finds that 25,544 firm-year observations or 15.39% are categorized as crashes (Table 3.5). Figures 3.5 and 3.7 show the percentage of crashes under the ROB_2 and OLS_2, and ROB_2 and $WOLS_2$ (respectively) by year while figures 3.6 and 3.8 present the number of firms with stock price crashes. The findings show that the robust methodology detects higher percentage of crashes compared to the OLS technique.

Table 3.6 shows the differences between ROB_1 and OLS_1, and ROB_2 and OLS_2 in more detail. The findings show that 10,670 firm-year observations or 6.43% higher percentage of crashes are observed under the ROB_1 in comparison to the OLS_1. The highest difference between the percentage of crashes is observed in 2008 (12.63%). On the other hand, the lowest difference between the percentage of crashes is observed in the years 1993 and 1996 (4.21%).

The above findings show that the percentage of crashes using the robust measure (ROB_1 and ROB_2) is higher in relation to the OLS_1, OLS_2, $WOLS_1$, and $WOLS_2$.

Table 3.2. Sample's Distribution and Statistics of Crashes (ROB_1 and OLS_1)

	ROB_1			OLS_1	
Year	**Number of Observations**	**Number of Robust Crashes**	**Percentage of Robust Crashes**	**Number of Crashes**	**Percentage of Crashes**
1992	5,336	843	15.8	569	10.66
1993	6,083	859	14.12	603	9.91
1994	6,575	991	15.07	685	10.42
1995	6,801	932	13.7	637	9.37
1996	7,283	966	13.26	659	9.05
1997	7,342	1,072	14.6	716	9.75
1998	7,006	1,261	18.00	783	11.18
1999	6,914	1046	15.13	662	9.57
2000	6,182	982	15.88	648	10.48
2001	5,909	1,229	20.80	722	12.22
2002	5,438	1,307	24.03	837	15.39
2003	5,908	1,183	20.02	806	13.64
2004	5,920	1,431	24.17	999	16.88
2005	5,873	1,303	22.19	928	15.8
2006	5,978	1,298	21.71	958	16.03
2007	5,985	1,384	23.12	888	14.84
2008	5,320	1,455	27.35	783	14.72
2009	5,526	1,112	20.12	621	11.24
2010	5,738	1,070	18.65	749	13.05
2011	5,738	1,272	22.17	888	15.48
2012	5,794	1,413	24.39	1,085	18.73
2013	5,986	1,280	21.38	932	15.57
2014	6,153	1,469	23.87	1,055	17.15
2015	6,223	1,363	21.90	939	15.09
2016	6,286	1,754	27.90	1,244	19.79
2017	6,342	1,582	24.94	1,191	18.78
2018	6,346	1,409	22.20	1,009	15.9
Total	165,985	33,266	20.04	22,596	13.61

Notes. This table presents the number of stock price crashes, the percentage of crashes (yearly) and the final sample statistics of the stock price crashes of the measures ROB_1 and OLS_1. The sample covers the period from 1992 to 2018. A stock price crash is an indicator that takes the value of one when at least one firm-specific weekly returns falls 3.09 standard deviation below the mean firm-specific weekly returns, and zero otherwise.

Table 3.3. Sample's Distribution and Statistics of Crashes (ROB_1 and $WOLS_1$)

	ROB_1			$WOLS_1$	
Year	**Number of Observations**	**Number of Robust Crashes**	**Percentage of Robust Crashes**	**Number of Crashes**	**Percentage of Crashes**
1992	5,336	843	15.80	801	15.01
1993	6,083	859	14.12	846	13.91
1994	6,575	991	15.07	971	14.77
1995	6,801	932	13.70	884	13.00
1996	7,283	966	13.26	976	13.40
1997	7,342	1,072	14.60	1,039	14.15
1998	7,006	1,261	18.00	1,177	16.80
1999	6,914	1046	15.13	999	14.45
2000	6,182	982	15.88	1,046	16.92
2001	5,909	1,229	20.80	1,108	18.75
2002	5,438	1,307	24.03	1,165	21.42
2003	5,908	1,183	20.02	1,077	18.23
2004	5,920	1,431	24.17	1,273	21.50
2005	5,873	1,303	22.19	1,187	20.21
2006	5,978	1,298	21.71	1,207	20.19
2007	5,985	1,384	23.12	1,160	19.38
2008	5,320	1,455	27.35	1,168	21.95
2009	5,526	1,112	20.12	987	17.86
2010	5,738	1,070	18.65	966	16.84
2011	5,738	1,272	22.17	1,119	19.50
2012	5,794	1,413	24.39	1,281	22.11
2013	5,986	1,280	21.38	1,138	19.01
2014	6,153	1,469	23.87	1,291	20.98
2015	6,223	1,363	21.90	1,195	19.20
2016	6,286	1,754	27.90	1,517	24.13
2017	6,342	1,582	24.94	1,418	22.36
2018	6,346	1,409	22.20	1,230	19.38
Total	165,985	33,266	20.04	30,226	18.21

Notes. This table presents the number of stock price crashes, the percentage of crashes (yearly) and the final sample statistics of the stock price crashes of the measures ROB_1 and $WOLS_1$. The sample covers the period from 1992 to 2018. A stock price crash is an indicator that takes the value of one when at least one firm-specific weekly returns falls 3.09 standard deviation below the mean firm-specific weekly returns, and zero otherwise.

Table 3.4. Sample's Distribution and Statistics of Crashes (ROB_2 and OLS_2)

	ROB_2			OLS_2	
Year	**Number of Observations**	**Number of Robust Crashes**	**Percentage of Robust Crashes**	**Number of Crashes**	**Percentage of Crashes**
1992	5,336	735	13.77	463	8.68
1993	6,083	767	12.61	485	7.97
1994	6,575	857	13.03	573	8.71
1995	6,801	822	12.09	552	8.12
1996	7,283	857	11.77	539	7.40
1997	7,342	933	12.71	585	7.97
1998	7,006	1,123	16.03	650	9.28
1999	6,914	917	13.26	554	8.01
2000	6,182	872	14.11	531	8.59
2001	5,909	1,076	18.21	583	9.87
2002	5,438	1,188	21.85	697	12.82
2003	5,908	1,042	17.64	656	11.10
2004	5,920	1,273	21.50	861	14.54
2005	5,873	1,164	19.82	794	13.52
2006	5,978	1,173	19.62	825	13.8
2007	5,985	1,224	20.45	738	12.33
2008	5,320	1,310	24.62	642	12.07
2009	5,526	984	17.81	514	9.30
2010	5,738	940	16.38	634	11.05
2011	5,738	1,121	19.54	733	12.77
2012	5,794	1,265	21.83	935	16.14
2013	5,986	1,143	19.09	768	12.83
2014	6,153	1,313	21.34	876	14.24
2015	6,223	1,220	19.60	795	12.78
2016	6,286	1,579	25.12	1,044	16.61
2017	6,342	1,419	22.37	1,013	15.97
2018	6,346	1,278	20.14	863	13.60
Total	165,985	29,595	17.83	18,903	11.39

Notes. This table presents the number of stock price crashes, the percentage of crashes (yearly) and the final sample statistics of the stock price crashes of the measures ROB_2 and OLS_2. The sample covers the period from 1992 to 2018. A stock price crash is an indicator that takes the value of one when at least one firm-specific weekly returns falls 3.20 standard deviation below the mean firm-specific weekly returns and the value of zero, otherwise.

Table 3.5. Sample's Distribution and Statistics of Crashes (ROB_2 and $WOLS_2$)

	ROB_2			$WOLS_2$	
Year	**Number of Observations**	**Number of Robust Crashes**	**Percentage of Robust Crashes**	**Number of Crashes**	**Percentage of Crashes**
1992	5,336	735	13.77	672	12.59
1993	6,083	767	12.61	707	11.62
1994	6,575	857	13.03	788	11.98
1995	6,801	822	12.09	750	11.03
1996	7,283	857	11.77	801	11.00
1997	7,342	933	12.71	851	11.59
1998	7,006	1,123	16.03	1,005	14.34
1999	6,914	917	13.26	847	12.25
2000	6,182	872	14.11	893	14.45
2001	5,909	1,076	18.21	927	15.69
2002	5,438	1,188	21.85	1,009	18.55
2003	5,908	1,042	17.64	907	15.35
2004	5,920	1,273	21.50	1,080	18.24
2005	5,873	1,164	19.82	1,020	17.37
2006	5,978	1,173	19.62	1,038	17.36
2007	5,985	1,224	20.45	961	16.06
2008	5,320	1,310	24.62	999	18.78
2009	5,526	984	17.81	807	14.60
2010	5,738	940	16.38	819	14.27
2011	5,738	1,121	19.54	922	16.07
2012	5,794	1,265	21.83	1,127	19.45
2013	5,986	1,143	19.09	960	16.04
2014	6,153	1,313	21.34	1,088	17.68
2015	6,223	1,220	19.60	1,013	16.28
2016	6,286	1,579	25.12	1,293	20.57
2017	6,342	1,419	22.37	1,213	19.13
2018	6,346	1,278	20.14	1,047	16.50
Total	165,985	29,595	17.83	25,544	15.39

Notes. This table presents the number of stock price crashes, the percentage of crashes (yearly) and the final sample statistics of the stock price crashes of the measures ROB_2 and $WOLS_2$. The sample covers the period from 1992 to 2018. A stock price crash is an indicator that takes the value of one when at least one firm-specific weekly returns falls 3.20 standard deviation below the mean firm-specific weekly returns and the value of zero, otherwise.

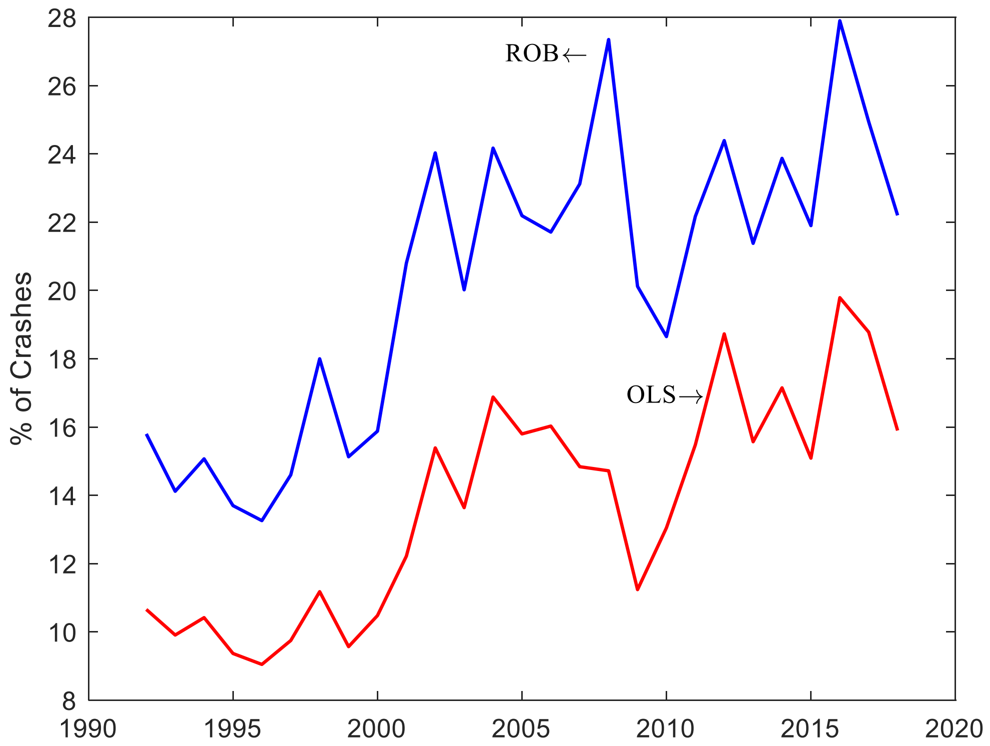


Figure 3.1. Percentage of Crashes under ROB_1 and OLS_1

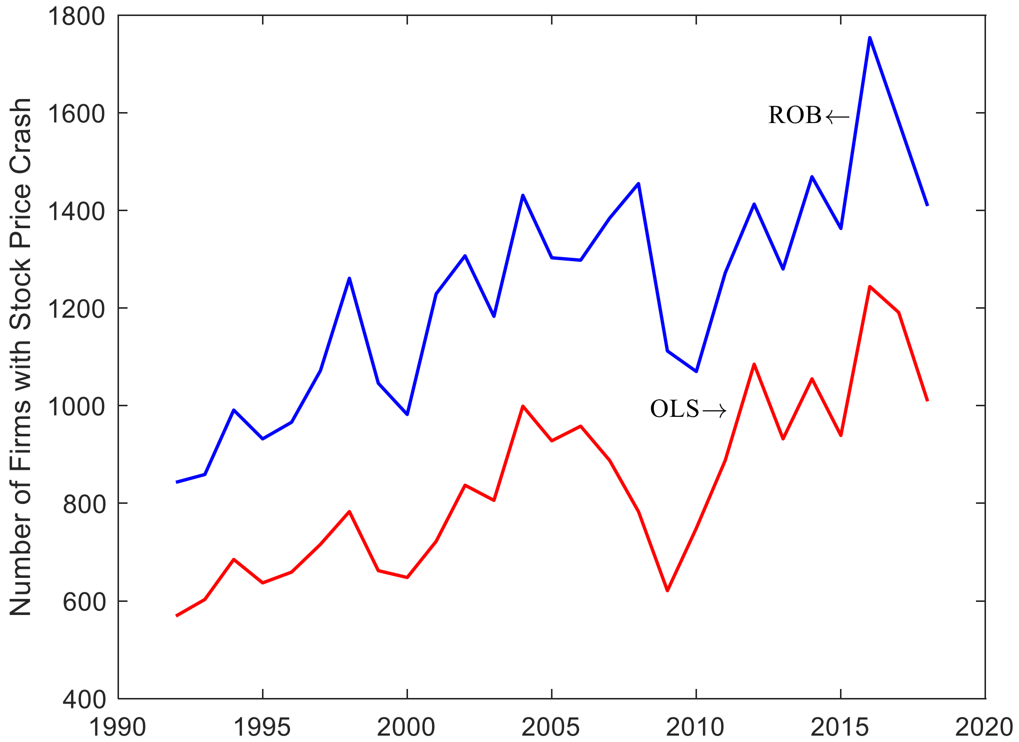


Figure 3.2. Number of Firms with Stock Price Crash under ROB_1 and OLS_1

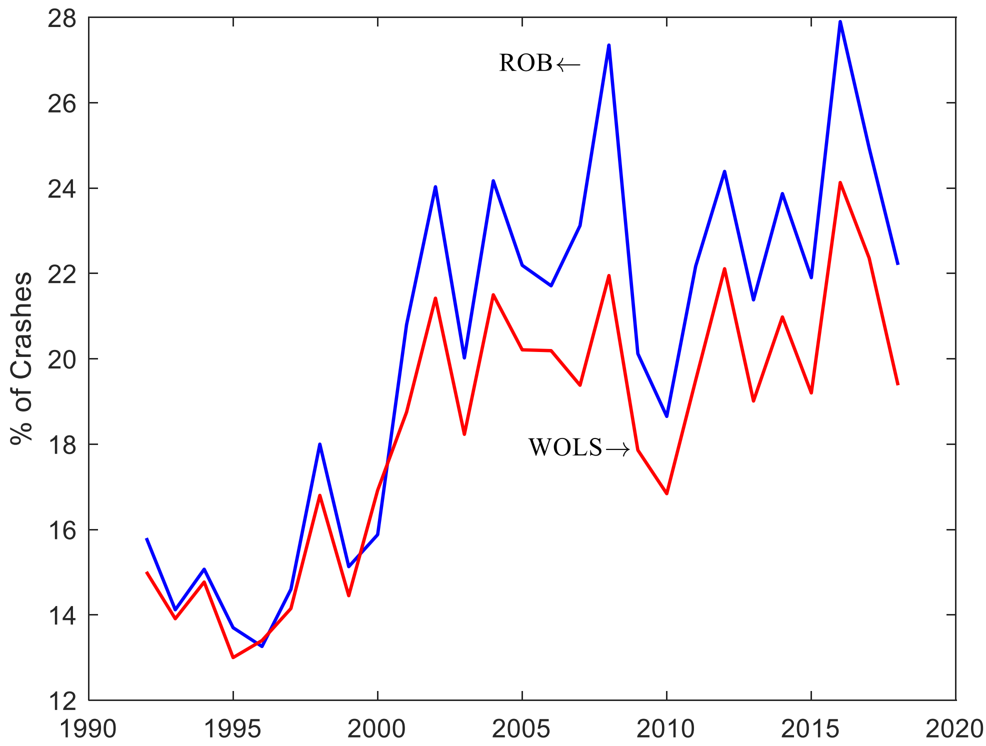


Figure 3.3. Percentage of Crashes under ROB_1 and $WOLS_1$

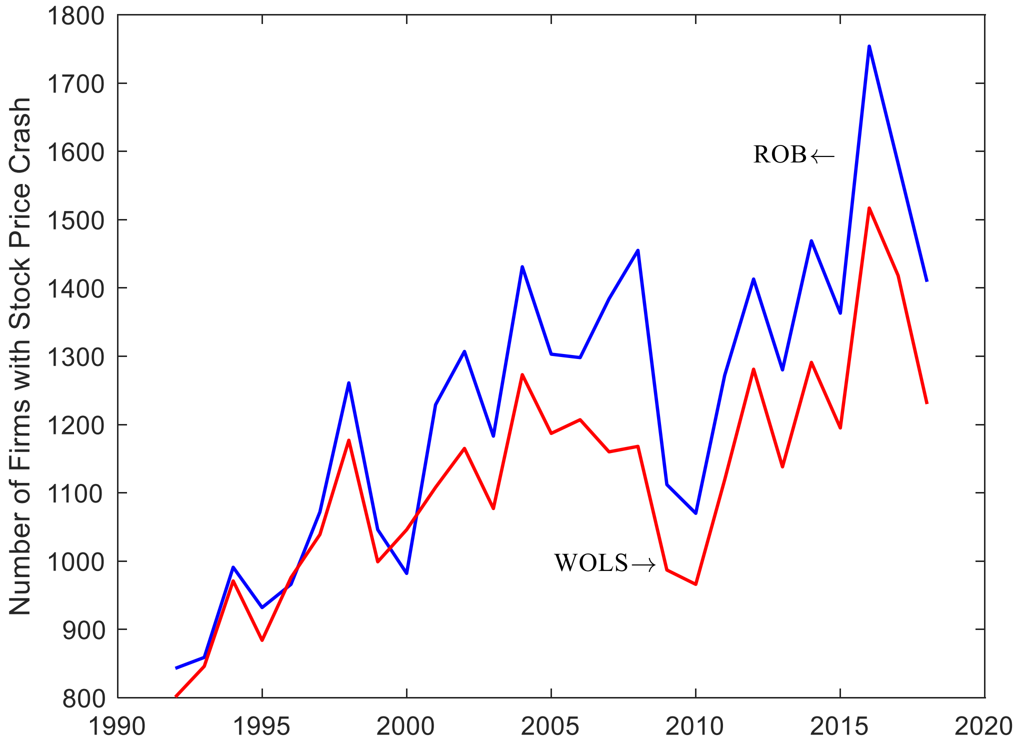


Figure 3.4. Number of firms with Stock Price Crash under ROB_1 and $WOLS_1$

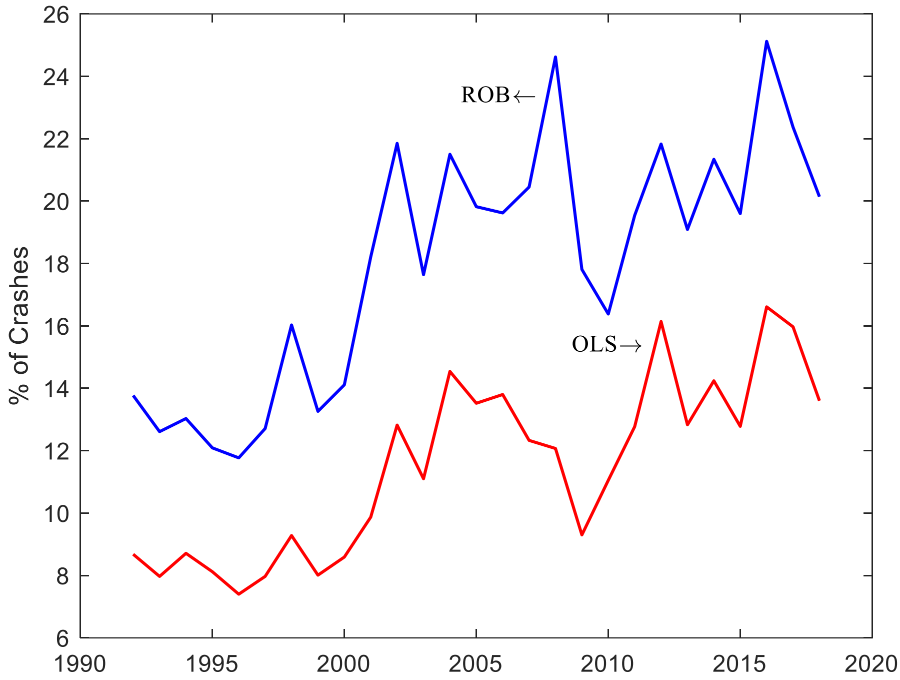


Figure 3.5. Percentage of Crashes under ROB_2 and OLS_2

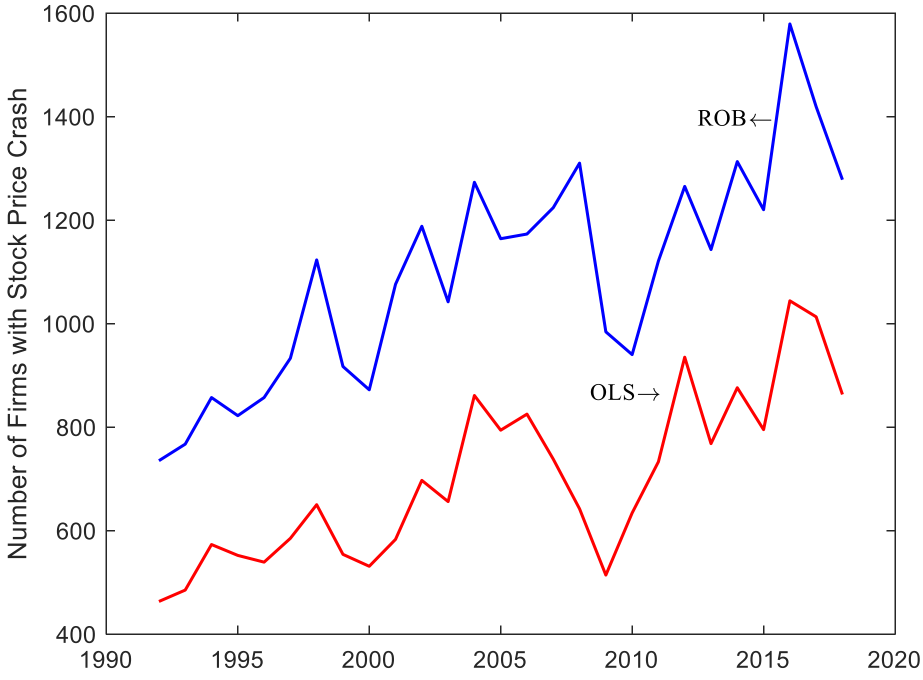


Figure 3.6. Number of Firms with Stock Price Crash under ROB_2 and OLS_2

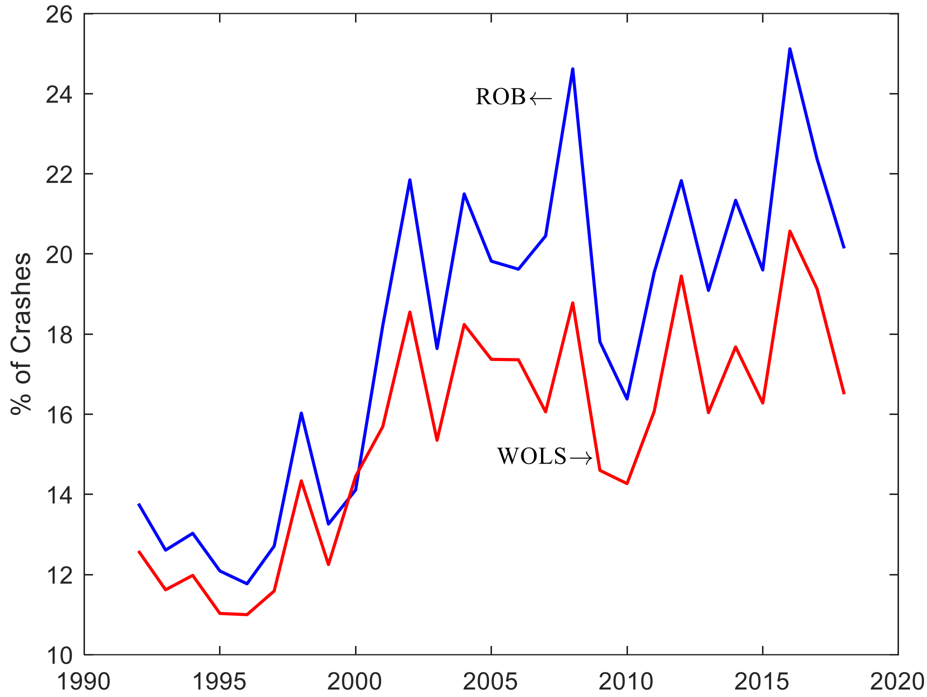


Figure 3.7. Percentage of Crashes under ROB_2 and $WOLS_2$

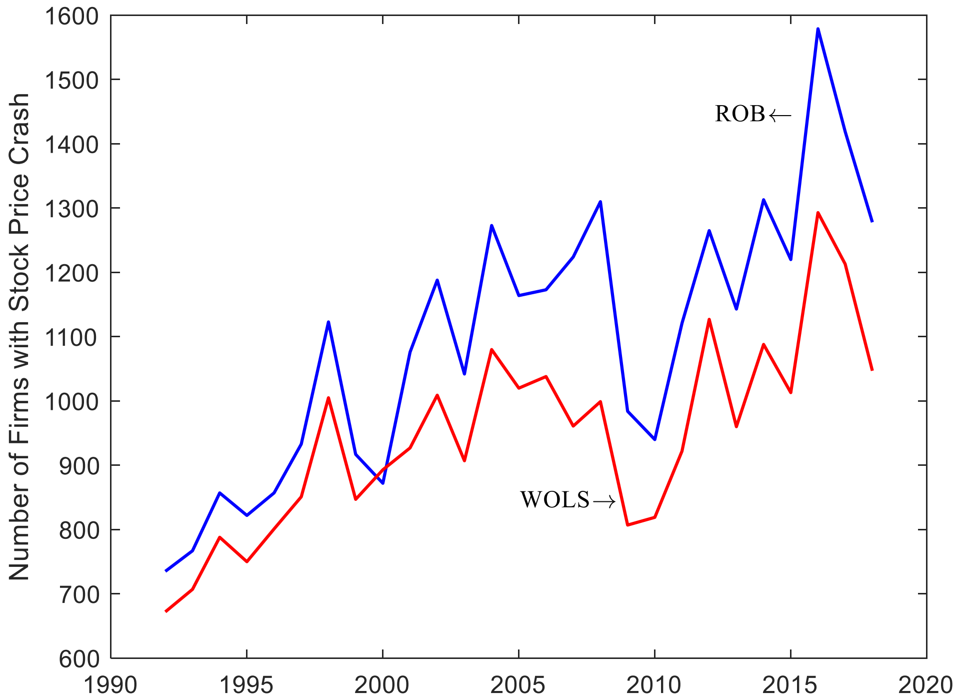


Figure 3.8. Number of Firms with Stock Price Crash under ROB_2 and $WOLS_2$

Table 3.6. Differences between $ROB_{1,2}$ and $OLS_{1,2}$

Year	ROB_1-OLS_1	ROB_1-OLS_1 (%)	ROB_2-OLS_2	ROB_2-OLS_2 (%)
1992	274	5.14	272	5.09
1993	256	4.21	282	4.64
1994	306	4.65	284	4.32
1995	295	4.33	270	3.97
1996	307	4.21	318	4.37
1997	356	4.85	348	4.74
1998	478	6.82	473	6.75
1999	384	5.56	363	5.25
2000	334	5.4	341	5.52
2001	507	8.58	493	8.34
2002	470	8.64	491	9.03
2003	377	6.38	386	6.54
2004	432	7.29	412	6.96
2005	375	6.39	370	6.30
2006	340	5.68	348	5.82
2007	496	8.28	486	8.12
2008	672	12.63	668	12.55
2009	491	8.88	470	8.51
2010	321	5.60	306	5.33
2011	384	6.69	388	6.77
2012	328	5.66	330	5.69
2013	348	5.81	375	6.26
2014	414	6.72	437	7.10
2015	424	6.81	425	6.82
2016	510	8.11	535	8.51
2017	391	6.16	406	6.40
2018	400	6.30	415	6.54
Total	10,670	6.43	10,692	6.44

Notes. This table presents the differences between the $ROB_{1,2}$-$OLS_{1,2}$ measures. More specifically, the table presents the differences regarding the number of crashes and the percentage of crashes under the robust technique compared to the standard OLS method. The table shows the number of stock price crashes, the percentage of crashes (yearly) and final sample statistics of the stock price crashes of the un-transform residual returns as derived from the Dimson (1979) model using the OLS and the outlier-resistant methodologies. The sample covers the period from 1992 to 2018. A stock price crash is an indicator that takes the value of one when at least one firm-specific weekly returns falls 3.09/3.20 standard deviation below the mean firm-specific weekly returns and the value of zero, otherwise.

3.5.3 Commonalities and Differences Between Crashes under the Measures

Table 3.7 summarizes the percentages of crashes and the number of observations of the three measures $ROB_{1,2}$, $OLS_{1,2}$, and $WOLS_{1,2}$ for 3.09- and 3.20-times standard deviation from the mean. These findings lead to a further investigation of the commonalities and differences between the measures.

Table 3.7. Percentage of Crashes

Panel A. 3.09 std from the mean			
Measures	ROB_1	OLS_1	$WOLS_1$
% of Crashes	20.04%	13.61%	18.21%
Observations	33,266	22,596	30,226
Panel B. 3.20 std from the mean			
Measures	ROB_2	OLS_2	$WOLS_2$
% of Crashes	17.83%	11.39%	15.39%
Observations	29,595	18,903	25,544

Notes. This table presents the percentage of crashes using the measures namely $ROB_{1,2}$, $OLS_{1,2}$, and $WOLS_{1,2}$. Part A (Part B) presents the percentage of crashes and number of observations using 3.09 (3.20) times standard deviation from the mean.

A further investigation of the commonalities and differences between the $ROB_{1,2}$ and, $OLS_{1,2}$ is presented in Table 3.8 Panel A. Panel A of Table 3.8 (column two) presents the analysis of the percentage of crashes between ROB_1 and OLS_1 while column three the analysis between ROB_2 and OLS_2. The common percentage of crashes between the two measures ($ROB_{1,2} \cap OLS_{1,2}$) is 13.45% (11.32%) or 22,332 (18,784) firm-year observations. The percentage of crashes that belongs on $OLS_{1,2}$ and not on $ROB_{1,2}$ ($OLS_{1,2} \notin ROB_{1,2}$) is 0.16% (0.07%) or 264 (119) firm-year observations while 6.59% (6.51%) or 10,934 (10,811) firm-year observations are classified as crashes in $ROB_{1,2}$ and not in $OLS_{1,2}$ ($ROB_{1,2} \notin OLS_{1,2}$). These findings lead to the conclusion that $OLS_{1,2}$ measure is almost a subset of the $ROB_{1,2}$ measure.

Table 3.8. Percentage of Common Crashes between ROB and OLS

Panel A. ROB and OLS		
% of Crashes and Firm-year observations	3.09 std from the mean	3.20 std from the mean
ROB∩OLS	13.45%	11.32%
Observations	22,332	18,784
OLS∉ROB	0.16%	0.07%
Observations	264	119
ROB ∉OLS	6.59%	6.51%
Observations	10,934	10,811
Panel B. ROB and WOLS		
% of Crashes and Firm-year observations	3.09 std from the mean	3.20 std from the mean
ROB∩WOLS	16.34%	14.04%
Observations	27,121	23,302
WOLS∉ROB	1.87%	1.35%
Observations	3,105	2,242
ROB ∉WOLS	3.70%	3.79%
Observations	6,145	6,293

Notes. This table presents the commonality or difference between the $ROB_{1,2}$ and $OLS_{1,2}$ and $ROB_{1,2}$ and $WOLS_{1,2}$. OLS represents the binary measure using the un-transform OLS residual returns while ROB using the un-transform robust residual returns under the corrected standard deviation derived from equation (3.15). $WOLS_{1,2}$ is the binary measure using the logarithmic transformation of the OLS residual returns.

Panel B of Table 3.8 (column two) presents the analysis of the percentage of crashes between ROB_1 and $WOLS_1$ while column three the analysis between ROB_2 and $WOLS_2$. The common percentage of crashes between the two measures ($ROB_{1,2}$∩$WOLS_{1,2}$) is 16.34% (14.04%) or 27,121 (23,302) firm-year observations. The percentage of crashes that belongs to $WOLS_{1,2}$ and not to $ROB_{1,2}$ ($WOLS_{1,2} \notin ROB_{1,2}$) is 1.87% (1.35%) or 3,105 (2,242) firm-year observations while 3.70% (3.79%) or 6,145 (6,293) firm-year observations are classified as crashes in $ROB_{1,2}$ and not in $WOLS_{1,2}$ ($ROB_{1,2} \notin WOLS_{1,2}$).

3.5.4 Preliminary Statistics of the Measures

Table 3.9 presents the expected value, standard deviation, and the quantiles (minimum, 25%, median, 75%, and a maximum) of the binary crash measures. Panel A of Table 3.9 shows the descriptive statistics of the measures using 3.09 standard deviation from the mean, namely, ROB_1 , OLS_1 ,and $WOLS_1$. Panel B of Table 3.9 presents the preliminary statistics of the measures using 3.20 standard deviation from the mean.

The expected value and standard deviation of crashes in ROB_1 (ROB_2) are 0.20 (0.178) and 0.40 (0.383), respectively. Conversely, the expected value and standard deviation of crashes in OLS_1 (OLS_2) are 0.136 (0.114) and 0.343 (0.318), respectively. Finally, the expected value and standard deviation in $WOLS_1$ ($WOLS_2$) are 0.182 (0.154) and 0.386 (0.361), respectively.

The higher expected value and uncertainty of $ROB_{1,2}$ compared to the $OLS_{1,2}$ and $WOLS_{1,2}$ is due to the higher percentage of crashes that detected in $ROB_{1,2}$ compared to the other two measures.

Table 3.9. Descriptive Statistics

Variable	N	Mean	Std	Min	Q1 (25%)	Median (50%)	Q3 (75%)	Max
Panel A. 3.09 std from the mean								
ROB_1	165,985	0.200	0.400	0	0	0	0	1
OLS_1	165,985	0.136	0.343	0	0	0	0	1
$WOLS_1$	165,985	0.182	0.386	0	0	0	0	1
Panel B. 3.20 std from the mean								
ROB_2	165,985	0.178	0.383	0	0	0	0	1
OLS_2	165,985	0.114	0.318	0	0	0	0	1
$WOLS_2$	165,985	0.154	0.361	0	0	0	0	1

Notes. This table presents the preliminary statistics for the stock price crash risk measures $ROB_{1,2}$, $OLS_{1,2}$ and $WOLS_{1,2}$ from 1992 to 2018.

3.5.5 Sample's Distribution and Statistics of Crashes by Industry

Tables 3.10 - 3.13 present the sample's distribution and statistics of crashes for each one of the Fama and French 49 industry classifications. The findings reveal that the percentage of crashes differs across industries. Figures 3.9 and 3.10 show the percentage of crashes between ROB_1 and OLS_1 and $WOLS_1$. Overall, Tables 3.10 and 3.11 show that 20.12% (or 24,718 firm-year observations) are classified as crashes using ROB_1, 13.80% (or 16,952 firm-year observations) using OLS_1, and 19.14% or 23,505 firm-year observations using $WOLS_1$. Tables 3.12 and 3.13 present the sample's distribution and statistics of crashes by industry of the measures ROB_2 and OLS_2 and $WOLS_2$. The analysis of the findings is similar; therefore, it is omitted.

The industry that exhibits the lowest percentage of crashes in comparison to the other industries is 27 (Gold, Precious Metals). On the other hand, industries that present high percentage of crashes are 33 (Personal Services), and 36 (Computer Software). Industries with more than 1,000 firm-year observations are Pharmaceutical (classification number 13), Business services (classification number 34), Computer software (classification number 36), Electronic equipment (classification number 37), Retail (classification number 43), Banking (classification number 45), Insurance (classification number 46), and Trading (classification number 48).

Overall, the findings show that 7,766 (=24,718-16,952) firm-year observations or 8.32% (=20,12% - 13.80%) higher percentage of crashes is detected as crashes in ROB_1 compared to OLS_1 while 1,213 firm-year observations (=24,718-23,505) or 0.98% (=20.12%-19.14%) higher percentage of crashes is classified as crashes in ROB_1 and not in $WOLS_1$.

To have a clearer view, a further investigation of the commonalities and differences between the $ROB_{1,2}$ and, $OLS_{1,2}$ by industry is presented in Table 3.14 Panel A. Panel A of Table 3.14 (column two) presents the analysis of the number of observations with stock price crashes between ROB_1 and OLS_1 by industry while column three the analysis between ROB_2 and OLS_2. The common number of firm-year crash observations ($ROB_{1,2} \cap OLS_{1,2}$) is 16,765 (14,233) firm-year observations. The number of crash observations that belongs to $OLS_{1,2}$ and not to $ROB_{1,2}$ ($OLS_{1,2} \notin ROB_{1,2}$) is 187 (91) firm-year observations while 7,953 (7,834) firm-year observations are classified as crashes in $ROB_{1,2}$ and not in $OLS_{1,2}$ ($ROB_{1,2} \notin OLS_{1,2}$).

Panel B of Table 3.14 (column two) presents the analysis of the number of common firm-year observations between ROB_1 and $WOLS_1$ while column three the analysis between ROB_2 and $WOLS_2$. The common number of firm-year observations between the two measures ($ROB_{1,2} \cap WOLS_{1,2}$) is 20,775 (18,040) firm-year observations. The number of firm-year observations that belongs to $WOLS_{1,2}$ and not to $ROB_{1,2}$ ($WOLS_{1,2} \notin ROB_{1,2}$) is 2,730 (2,010) firm-year observations while 3,943 (4,027) firm-year observations are classified as crashes in $ROB_{1,2}$ and not in $WOLS_{1,2}$ ($ROB_{1,2} \notin WOLS_{1,2}$).

Table 3.10. Sample's Distribution by Industry, ROB_1, OLS_1 and $WOLS_1$ (%)

Industry	Number of Firms by Industry	ROB_1 (%)	OLS_1 (%)	$WOLS_1$ (%)
1	372	19.35	10.75	15.86
2	1,899	21.43	14.85	19.69
3	298	18.79	13.42	17.79
4	449	19.15	12.92	17.82
5	184	19.02	14.67	19.02
6	802	22.57	15.09	21.32
7	1,611	16.76	11.42	16.95
8	986	20.79	14.50	18.66
9	1,702	22.86	16.27	21.62
10	1,513	23.79	16.13	22.74
11	1,978	22.19	15.42	21.74
12	3,512	22.35	16.00	21.87
13	7,042	21.00	14.73	21.26
14	2,178	20.57	14.51	19.10
15	876	24.89	15.87	20.78
16	515	20.97	13.79	19.03
17	2,087	20.41	13.85	18.69
18	1,431	18.8	13.42	18.59
19	1,663	16.78	10.64	16.54
20	366	19.4	11.20	18.03
21	3,805	16.85	10.78	15.82
22	1,617	21.4	13.85	19.11
23	1,705	18.89	12.96	18.59
24	570	18.77	14.21	18.07
25	251	21.12	14.34	19.52
26	222	16.22	9.91	13.06
27	307	9.12	7.17	11.07
28	432	15.28	10.19	13.89
29	237	13.08	7.59	15.61
30	4,243	13.03	9.00	13.46
31	3,743	18.27	12.61	15.66
32	4,201	15.90	10.78	15.62
33	1,297	24.83	17.58	23.75
34	5,757	23.22	17.11	22.48
35	2,805	22.17	16.58	23.85
36	8,597	21.94	15.49	22.57
37	6,802	20.71	13.92	21.07
38	2385	19.75	13.71	18.36
39	1,407	20.68	14.36	18.41
40	379	20.32	11.87	16.62
41	3562	19.09	12.91	18.36
42	4120	22.06	15.29	20.51
43	5909	23.08	16.36	22.56
44	2137	18.62	12.77	17.17
45	10,269	18.13	11.33	15.6
46	4,787	21.50	15.02	19.3
47	1,022	20.25	11.74	16.54
48	8,075	19.79	13.16	17.29
49	717	19.25	13.39	17.99
Total	122,824	20.12	13.80	19.14

Notes. This table presents the percentage of crashes of the measures: ROB_1, OLS_1, and $WOLS_1$ for each of the Fama and French 49 industry classifications. The number of observations is 122,824 and 43,161 missing observations (overall 165,985).

Table 3.11. Number of Firms with Crashes by Industry, ROB_1, OLS_1 and $WOLS_1$

Industry	Number of Firms by Industry	ROB_1	OLS_1	$WOLS_1$
1	372	72	40	59
2	1,899	407	282	374
3	298	56	40	53
4	449	86	58	80
5	184	35	27	35
6	802	181	121	171
7	1,611	270	184	273
8	986	205	143	184
9	1,702	389	277	368
10	1,513	360	244	344
11	1,978	439	305	430
12	3,512	785	562	768
13	7,042	1,479	1,037	1,497
14	2,178	448	316	416
15	876	218	139	182
16	515	108	71	98
17	2,087	426	289	390
18	1,431	269	192	266
19	1,663	279	177	275
20	366	71	41	66
21	3,805	641	410	602
22	1,617	346	224	309
23	1,705	322	221	317
24	570	107	81	103
25	251	53	36	49
26	222	36	22	29
27	307	28	22	34
28	432	66	44	60
29	237	31	18	37
30	4,243	553	382	571
31	3,743	684	472	586
32	4,201	668	453	656
33	1,297	322	228	308
34	5,757	1,337	985	1,294
35	2,805	622	465	669
36	8,597	1,886	1,332	1,940
37	6,802	1,409	947	1,433
38	2,385	471	327	438
39	1,407	291	202	259
40	379	77	45	63
41	3,562	680	460	654
42	4,120	909	630	845
43	5,909	1,364	967	1333
44	2,137	398	273	367
45	10,269	1,862	1,163	1,602
46	4,787	1,029	719	924
47	1,022	207	120	169
48	8,075	1,598	1,063	1,396
49	717	138	96	129
Total	122,824	24,718	16,952	23,505

Notes. This table presents the number of stock price crashes of the measures: ROB_1, OLS_1, and $WOLS_1$ for each of the Fama and French 49 industry classifications (see appendix III). The number of observations is 122,824 and 43,161 missing observations (overall 165,985).

Table 3.12. Sample's Distribution by Industry, ROB_2, OLS_2 and $WOLS_2$ (%)

Industry	Number of Firms by Industry	ROB_2 (%)	OLS_2 (%)	$WOLS_2$ (%)
1	372	17.47	9.14	14.25
2	1,899	19.22	12.48	16.96
3	298	17.45	11.41	14.77
4	449	17.37	9.13	14.70
5	184	16.30	10.87	14.13
6	802	19.08	12.84	18.83
7	1,611	15.15	9.68	14.4
8	986	18.86	12.17	16.23
9	1,702	20.98	13.87	18.57
10	1,513	21.48	13.75	18.97
11	1,978	19.97	13.4	19.01
12	3,512	20.39	13.78	19.36
13	7,042	19.14	13.19	18.47
14	2,178	18.37	11.66	15.98
15	876	22.60	14.04	18.04
16	515	18.83	12.04	16.12
17	2,087	17.92	11.40	15.91
18	1,431	16.84	11.53	16.28
19	1,663	14.43	9.08	13.65
20	366	17.21	8.47	14.48
21	3,805	14.32	9.07	13.01
22	1,617	19.29	12.06	16.57
23	1,705	16.30	10.73	15.01
24	570	16.67	11.05	15.09
25	251	18.73	12.35	17.13
26	222	15.32	8.56	10.81
27	307	8.14	5.86	9.12
28	432	12.96	7.87	10.42
29	237	10.13	6.33	10.97
30	4,243	11.38	7.09	11.08
31	3,743	15.98	10.26	12.69
32	4,201	13.83	9.00	12.90
33	1,297	22.36	15.42	21.13
34	5,757	21.43	14.99	19.91
35	2,805	19.75	14.4	20.86
36	8,597	19.76	13.21	19.65
37	6,802	18.26	11.73	18.04
38	2385	17.9	11.66	16.31
39	1,407	17.98	11.87	16.06
40	379	19.00	9.50	13.46
41	3562	17.10	10.61	15.41
42	4120	20.07	13.35	17.84
43	5909	20.78	13.49	19.60
44	2137	16.75	10.81	14.32
45	10,269	15.91	9.40	12.77
46	4,787	19.55	13.08	16.69
47	1,022	18.10	8.90	13.99
48	8,075	17.19	10.70	14.18
49	717	17.57	11.58	15.48
Total	122,824	17.97	11.66	16.32

Notes. This table presents the percentage of crashes of the measures ROB_2 , OLS_2 , and $WOLS_2$ using the robust outlier-resistant and ordinary least square methods for each of the Fama and French 49 industry classifications. The number of observations is 122,824 and 43,161 missing observations (overall 165,985).

Table 3.13. Number of Firms with Crashes by Industry, ROB_2, OLS_2 and $WOLS_2$

Industry	Number of Firms by Industry	ROB_2	OLS_2	$WOLS_2$
1	372	65	34	53
2	1,899	365	237	322
3	298	52	34	44
4	449	78	41	66
5	184	30	20	26
6	802	153	103	151
7	1,611	244	156	232
8	986	186	120	160
9	1,702	357	236	316
10	1,513	325	208	287
11	1,978	395	265	376
12	3,512	716	484	680
13	7,042	1,348	929	1,301
14	2,178	400	254	348
15	876	198	123	158
16	515	97	62	83
17	2,087	374	238	332
18	1,431	241	165	233
19	1,663	240	151	227
20	366	63	31	53
21	3,805	545	345	495
22	1,617	312	195	268
23	1,705	278	183	256
24	570	95	63	86
25	251	47	31	43
26	222	34	19	24
27	307	25	18	28
28	432	56	34	45
29	237	24	15	26
30	4,243	483	301	470
31	3,743	598	384	475
32	4,201	581	378	542
33	1,297	290	200	274
34	5,757	1,234	863	1,146
35	2,805	554	404	585
36	8,597	1,699	1,136	1,689
37	6,802	1,242	798	1,227
38	2,385	427	278	389
39	1,407	253	167	226
40	379	72	36	51
41	3,562	609	378	549
42	4,120	827	550	735
43	5,909	1,228	797	1,158
44	2,137	358	231	306
45	10,269	1,634	965	1311
46	4,787	936	626	799
47	1,022	185	91	143
48	8,075	1,388	864	1,145
49	717	126	83	111
Total	122,824	22,067	14,324	20,050

Notes. This table presents the number of firms with stock price crashes of the measures ROB_2 , OLS_2 , and $WOLS_2$ using the robust outlier resistant and OLS methods for each of the Fama and French 49 industry classifications. The number of observations is 122,824 and 43,161 missing observations (overall 165,985).

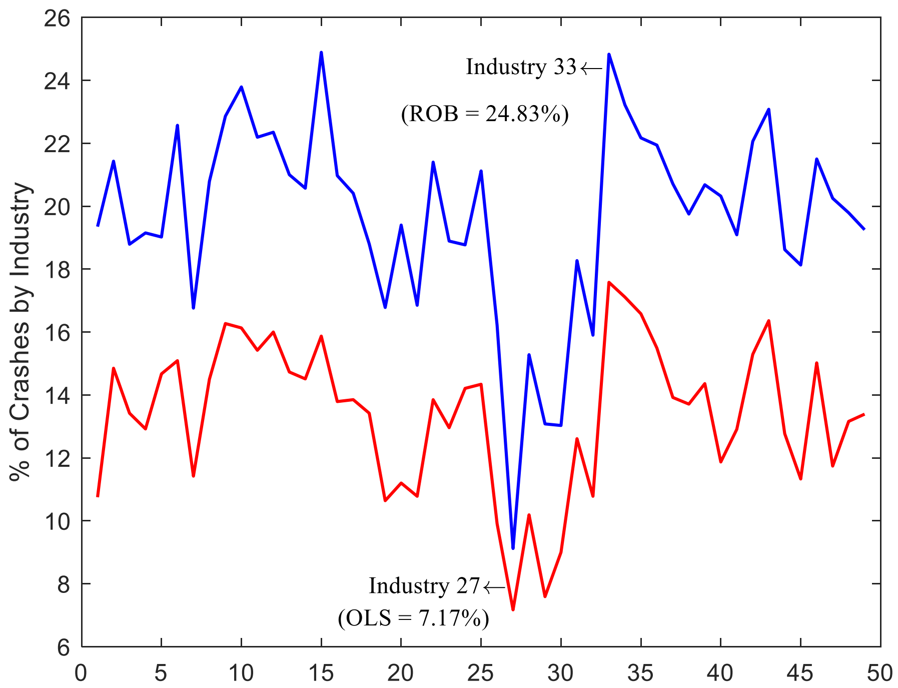


Figure 3.9. Percentage of Crashes by Industry under the ROB_1 and OLS_1

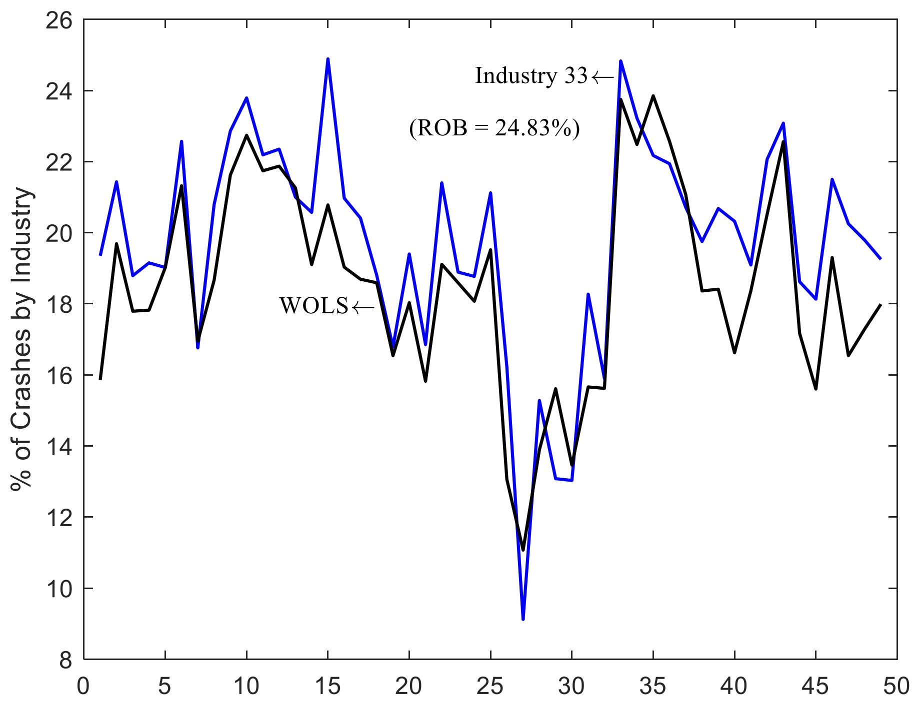


Figure 3.10. Percentage of Crashes by Industry under the ROB_1, and $WOLS_1$.

Table 3.14. Number of Common Crashes between ROB and OLS by Industry

Panel A. ROB and OLS		
Firm-year observations	3.09 std from the mean	3.20 std from the mean
ROB∩OLS	16,765	14,233
OLS∉ROB	187	91
ROB ∉OLS	7,953	7,834
Panel B. ROB and WOLS		
Firm-year observations	3.09 std from the mean	3.20 std from the mean
ROB∩WOLS	20,775	18,040
WOLS∉ROB	2,730	2,010
ROB ∉WOLS	3,943	4,027

Notes. This table presents the commonality or difference between the $ROB_{1,2}$ and $OLS_{1,2}$ and $ROB_{1,2}$ and $WOLS_{1,2}$. $OLS_{1,2}$ represents the binary measure using the un-transform OLS residual returns while ROB_1 using the un-transform residual returns under the corrected standard deviation derived from equation (3.15). $WOLS_{1,2}$ represents the binary measure using the logarithmic transformation of the OLS residual returns

3.5.6 Analysis of the OLS Percentage of Crashes

This sub-section analyses the percentage of crashes between the $OLS_{1,2}$ and $WOLS_{1,2}$ measures. Literature used the transformation of the residual returns to detect the percentage of crashes ($WOLS_{1,2}$). Table 3.15 presents the common percentage of crashes between the two measures as well as the differences between them. Table 3.15 column two presents the analysis of the percentage of crashes between OLS_1 and $WOLS_1$ while column three the analysis between OLS_2 and $WOLS_2$.

The common percentage of crashes between the two measures ($OLS_{1,2}$∩$WOLS_{1,2}$) is 13.61% (11.39%) or 22,596 (18,903) firm-year observations. The percentage of crashes that belongs on $OLS_{1,2}$ and not on $WOLS_{1,2}$ ($OLS_{1,2}$∉$WOLS_{1,2}$) is 0% (0%) or zero firm-year observations while 4.60% (4%) or 7,630 (6,641) firm-year observations are classified as crashes in $WOLS_{1,2}$ and not in $OLS_{1,2}$ ($WOLS_{1,2}$ ∉$OLS_{1,2}$). These findings lead to the conclusion that $OLS_{1,2}$ measure is a subset of the $WOLS_{1,2}$ measure.

Table 3.15. Percentage of Crashes and Commonality between OLS and WOLS

% of Crashes and firm-year observations	3.09 std from the mean	3.20 std from the mean
OLS∩WOLS	13.61%	11.39%
Observations	22,596	18,903
OLS∉WOLS	0%	0%
Observations	0	0
WOLS ∉OLS	4.60%	4%
Observations	7,630	6,641

Notes. This table presents the common percentage of crashes between $OLS_{1,2}$ and $WOLS_{1,2}$ as well as the differences between the two ordinary least square measures.

3.5.7 Event Study

Forty crash firms under the ROB_1 and not in ordinary crash risk measures are presented in Table 3.16. More specifically, Table 3.16 offers the name of the company, the TIC name, the weekly return for the specific week that the ROB_1 detects the crash event as well as the exact crash date. The findings show that all firms present a negative weekly log-return. Some of these firms exhibit higher return (in absolute values) when compared to others.

To further analyse these firms, Table 3.17 presents the standard deviation of OLS and the corrected robust standard deviation computed by equation (3.15) for each year of these firms. In all cases the standard deviation of the robust measure is lower compared to the standard deviation of the OLS measure. This finding verifies the importance to use a technique that corrects the inflated issue on the standard deviation. The biggest difference between the standard deviations is in firm Wins Finance Holdings Inc. This firm crashed in 2017 with a -29.58% weekly return. The OLS standard deviation is σ_ε = 0.98 (un-transform OLS standard deviation) and σ_w = 0.48 (log-transform standard deviation) while in the case of the robust technique the standard deviation is σ_e = 0.22.

Furthermore, Spirit Airlines Inc (TIC: SAVE) was detected to crash in 2015 with a negative return equal -7.87%. An article mentioned that this firm was a nightmare for the investors during the year 2015. Correspondingly, shares dropped further after the announcement of an investor that the weak performance of this firm will remain in the future.[37] The robust measure detects a crash event after this announcement. Figure 3.11 shows the weekly returns for the company before and after the crash event. The figure shows the sharply dropped of the firm's returns indicating a crash event. Flexion Therapeutics Inc (TIC: FLXN) crashed in 2017 after the announcement of a $125M convertible debt offering. This led the firm's stock to drop sharply.[38] Figure 3.12 shows the weekly returns of the firm, before and after the crash event. In figure 3.12, there is a negative -11.13% return at the date that the robust measure detected the crash event. In addition, Auxilum Pharmaceuticals Inc. (TIC: AUXL) crashed on 05/08/2011 using the robust technique after the announcement of the CEO and President of the company to sell more than 10,000 shares on 03/08/11.[39] Figure 3.13 shows the returns of Auxilum firm for the period 2010-2014 showing that on 05/08/2011 Auxilum firm's return is -10.49% and the robust technique detects that at this date this firm crashed. Also, Microsoft (MSFT) crashed after the announcement of the analysts to downgrade the stock. [40]

---

[37] https://www.fool.com/investing/general/2015/10/19/spirit-airlines-plunges-another-15-falls-into-deep.aspx (accessed: April 2021).
[38] https://seekingalpha.com/news/3259667-flexion-readies-125m-convertible-debt-offering-shares-down-13-after-hours (accessed: April 2021).
[39] https://www.gurufocus.com/news/128067/auxilium-pharmaceuticals-inc-auxl-ceo--president-armando-anido-sells-10555-shares?fbclid=IwAR0-1SJs1BMkL-uks46yognTopgNRU_l98paDoA2qaxRWTDf3YknQS_26eU (accessed: April 2021).
[40] https://www.businessinsider.com/microsoft-is-crashing-after-analysts-downgrade-the-stock-thanks-to-rough-earnings-2015-1 (accessed: April 2021).

Table 3.16. Date of the Firm-Specific-Event and Return.

No.	Company Name	TIC	Weekly Return (%)	Date
1	ENERGY RECOVERY INC	ERII	-14.89	03/05/2010
2	SPDR BLMBRG BRCLYS HI YLD BD	JNK	-11.64	06/10/2008
3	MERIT MEDICAL SYSTEMS INC	MMSI	-29.57	10/04/2000
4	BOSTON SCIENTIFIC CORP	BSX	-12.05	07/04/1997
5	AVID TECHNOLOGY INC	AVID	-23.14	05/10/1998
6	GLAUKOS CORP	GKOS	-11.54	11/01/2016
7	FRP HOLDINGS INC	FRPH	-12.47	22/07/2002
8	CONSTELLIUM SE	CSTM	-12.49	06/03/2017
9	APPLIED OPTOELECTRONICS INC	AAOI	-12.28	24/09/2018
10	WINS FINANCE HOLDINGS INC	WINSF	-29.58	27/03/2017
11	COLLEGIUM PHARMACEUTICAL INC	COLL	-13.35	20/03/2017
12	TRADE DESK INC	TTD	-10.28	06/11/2017
13	LOEWS CORP	L	-6.180	29/03/1993
14	DMC GLOBAL INC	BOOM	-9.320	23/07/2018
15	TRANS-LUX CORP	TNLX	-11.54	08/10/1998
16	INSIGHT SELECT INCOME FD	INSI	-15.17	06/10/2008
17	FLEXION THERAPEUTICS INC	FLXN	-11.13	24/04/2017
18	AUXILIUM PHARMA INC	AUXL	-10.49	05/08/2011
19	HARMONIC INC	HLIT	-11.95	29/09/1997
20	BANNER CORP	BANR	-17.08	17/09/2001
21	CENTURY ALUMINUM CO	CENX	-12.23	22/07/2002
22	DATATRAK INTERNATIONAL INC	DTRK	-14.29	08/08/2005
23	TRUEBLUE INC	TBI	-17.29	30/08/1999
24	TRANSACT TECHNOLOGIES INC	TACT	-24.10	05/10/1998
25	RYMAN HOSPITALITY PPTYS INC	RHP	-11.49	17/09/2001
26	PERDOCEO EDUCATION CORP	PRDO	-13.76	05/10/1998
27	ENGLOBAL CORP	ENG	-17.22	29/09/2008
28	CREDIT SUISSE HI YIELD BD FD	DHY	-13.52	17/09/2001
29	MOBILE TELESYSTEMS PJSC	MBT	-28.55	20/10/2008
30	UNITEDHEALTH GROUP INC	UNH	-16.56	08/07/1996

Table 3.16. (Contin.)

No.	Company Name	TIC	Weekly Return (%)	Date
31	PLEXUS CORP	PLXS	-7.80	03/01/2005
32	MICROSOFT CORP	MSFT	-6.74	26/01/2015
33	PSYCHEMEDICS CORP	PMD	-15.75	17/09/2001
34	ALABAMA AIRCRAFT INDUSTRIES	AAIIQ	-28.00	10/09/2007
35	GREEN DOT CORP	GDOT	-11.71	26/01/2015
36	REPUBLIC FIRST BANCORP INC	FRBK	-11.83	21/09/2009
37	CORNERSTONE ONDEMAND INC	CSOD	-6.19	31/07/2017
38	SPIRIT AIRLINES INC	SAVE	-7.87	12/10/2015
39	APOGEE ENTERPRISES INC	APOG	-6.24	14/09/2015
40	CYBERARK SOFTWARE LTD	CYBR	-9.55	08/02/2016

Notes. This table presents the information of each crash firm under the robust measure: company name, TIC, weekly return, and the date of the weekly return.

Table 3.17. OLS Standard Deviation and Corrected Standard Deviation

No.	TIC	σ_ε	σ_w	σ_e
1	ERII	0.05838	0.05832	0.05494
2	JNK	0.03099	0.03045	0.01964
3	MMSI	0.08570	0.08564	0.07835
4	BSX	0.06626	0.06707	0.06078
5	AVID	0.10293	0.10198	0.08315
6	GKOS	0.06385	0.06257	0.05265
7	FRPH	0.06406	0.06458	0.06263
8	CSTM	0.07001	0.06993	0.06048
9	AAOI	0.08177	0.08156	0.06692
10	WINSF	0.98034	0.48353	0.22526
11	COLL	0.10204	0.09895	0.08873
12	TTD	0.07597	0.07513	0.06286
13	L	0.02515	0.02524	0.02495
14	BOOM	0.07044	0.06859	0.05553
15	TNLX	0.06290	0.06134	0.04576
16	INSI	0.06289	0.05878	0.02846
17	FLXN	0.08820	0.08630	0.07245
18	AUXL	0.05453	0.05400	0.05041
19	HLIT	0.09350	0.09098	0.07648
20	BANR	0.05893	0.05874	0.04569
21	CENX	0.07262	0.07320	0.06604
22	DTRK	0.09431	0.09204	0.07923
23	TBI	0.12327	0.12244	0.10477
24	TACT	0.10982	0.10786	0.09422
25	RHP	0.05280	0.05249	0.04909
26	PRDO	0.07591	0.07521	0.06379
27	ENG	0.08289	0.08495	0.08181
28	DHY	0.05632	0.05564	0.04807
29	MBT	0.10663	0.10300	0.08336
30	UNH	0.05002	0.05047	0.04972

Table 3.17. (Cont.)

No.	TIC	σ_ε	σ_w	σ_e
31	PLXS	0.04181	0.04181	0.03706
32	MSFT	0.02789	0.02751	0.01989
33	PMD	0.05222	0.05088	0.03199
34	AAIIQ	0.18737	0.16131	0.11348
35	GDOT	0.05814	0.05621	0.03846
36	FRBK	0.05999	0.06065	0.05589
37	CSOD	0.03793	0.03761	0.03379
38	SAVE	0.05142	0.05111	0.04533
39	APOG	0.04361	0.0437	0.03932
40	CYBR	0.05406	0.05351	0.04515

Notes. This table presents the OLS and robust standard deviations for the specific year of each crash company under the robust measure (ROB_1) and the OLS measures. The standard deviations of OLS_1 and $WOLS_1$ are represented by σ_ε and σ_w, respectively while σ_e is the standard deviation of the ROB_1 measure derived from equation (3.15).

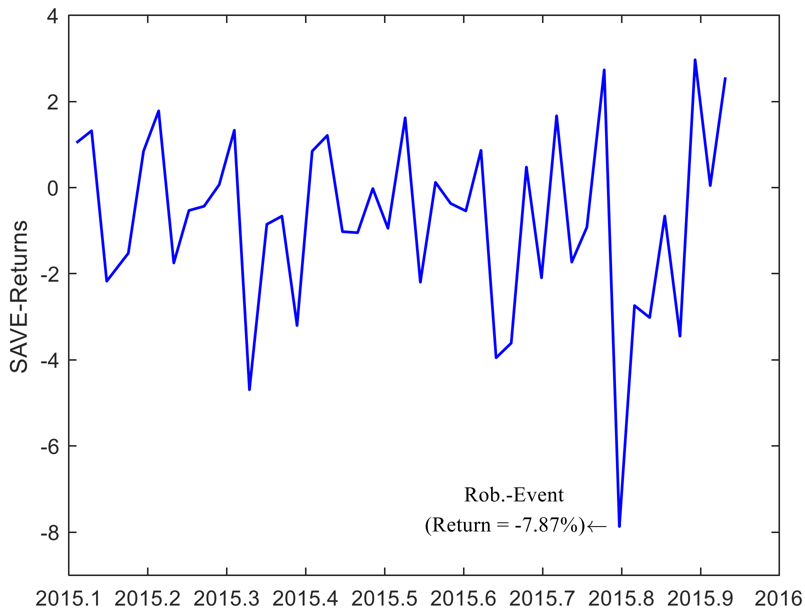


Figure 3.11. SAVE Firm-Specific Event.

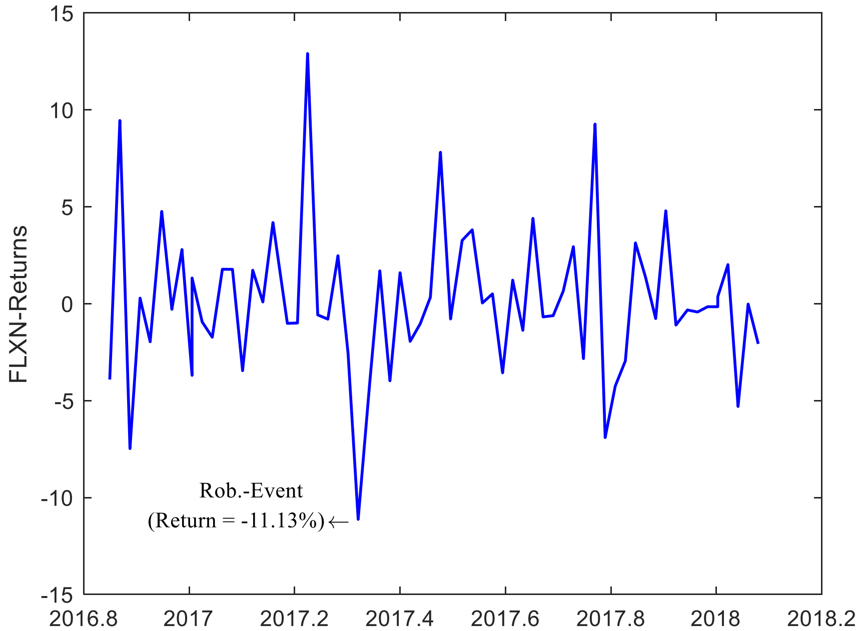


Figure 3.12. FLXN Firm-Specific Event.

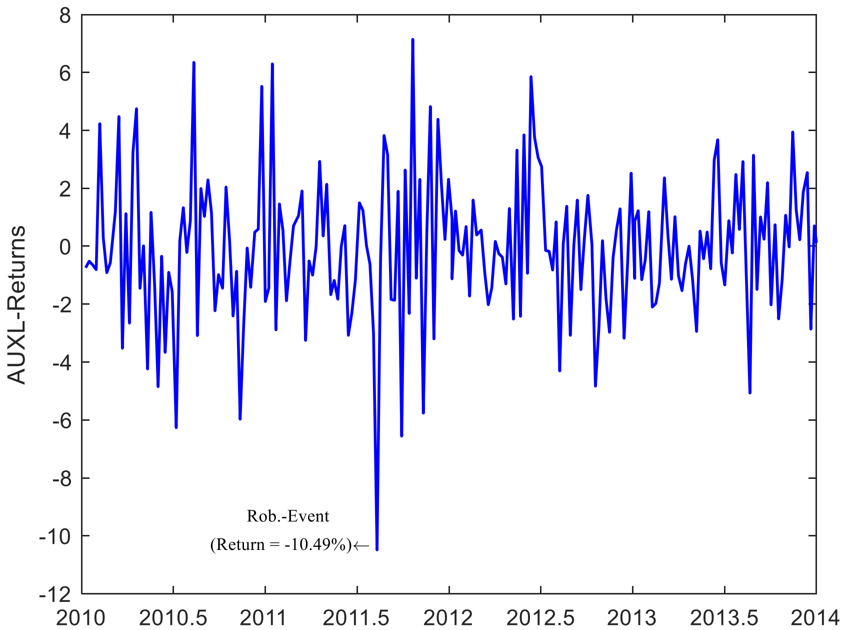


Figure 3.13. AUXL Firm-Specific Event

3.6 Summary and Conclusions

A stock price crash is an unusual sharp decrease in the stock's price caused by unexpected bad news. Recent empirical financial literature is trying to explore the stock price crash exposure in behavioural characteristics of CEOs/CFOs (age, gender, overconfident), religious beliefs, and others. The idea is that managers withhold the firms' bad news for several reasons such as career prospects (Hutton et al., 2009). When a negative firm-specific shock becomes public, there is an extreme outlier in the return distribution leading to a stock price crash.

The presence of outliers in the 52-weekly return series drives the standard deviation to be inflated. This chapter attempted to explore the validity of the percentage of crashes using the OLS and a robust framework based on a model developed by Theodossiou and Theodossiou (2019). Literature used the market model regression for each firm and year to estimate the return's residuals (Dimson, 1979). The log-transformed measure is derived by taking the logarithmic residual returns plus one. A further investigation of the stock price crashes is presented using the un-transform residual returns using the OLS and the outlier robust technique. A stock price crash is an indicator that takes the value of one when at least one firm-specific weekly returns falling 3.09/3.20 standard deviation below the mean firm-specific weekly returns and the value of zero, otherwise.

Monte – Carlo simulations show that the standard binary OLS measures detect a lower percentage of crashes compared to the robust. Likewise, the un-transform binary measure detects a lower percentage of crashes compared to the log transform measure.

The empirical findings verified the concerns of this chapter. The robust method detects higher percentage of crashes in relation to the standard methodology. The importance of the different percentage of crashes is highlighted by presenting an analysis of the common percentage of crashes as well as the differences between the measures.

More importantly, this chapter sheds light on the identification of the crash firms using a robust technique. This methodology also contributes on the existing literature using a statistical framework since a statistical theory in the crash risk literature does not exist. This is illustrated using case studies of companies in various industries. The analysis shows firms in the sample that are categorized as crash firms for a specific year using the robust technique (and non-crash using the OLS measures) by showing the negative firms returns for the specific dates.

This chapter provides the foundation for future research using robust statistics. The chapter of this dissertation focuses to explore the binary crash risk measure commonly used in the stock price crash literature. The primary objective of this chapter was to show the contamination issue on standard deviation and the impact of outliers to detect the stock price crashes. However, empirical studies also use continuous crash risk measures. Continuous measures are important, and it will be interesting to further explore the continuous crash risk measures using the robust technique. Furthermore, the detection of stock price crashes is important for investors, risk management, and researchers. Therefore, this chapter sheds light on the stock price literature to further investigate the stock price crash measures.

CONCLUSIONS

Financial data has been found in the academic literature to reject the Gaussian null hypothesis. Based on this, this dissertation focuses on the investigation of several financial puzzles using asymmetric models that account for skewness and/or kurtosis characteristics. The family of SGT distribution, as well as a robust technique, were used to explain three different topics documented in financial literature.

The SGT family of distributions nested by several well-known distributions, such as Skewed T (ST), Skewed Generalized Error Distribution (SGED), Skewed Normal (SN), Cauchy (C), Skewed Laplace (SLP), Laplace (LP), and Normal (N). The SGT is a fifth parameter distribution, where, using the log-likelihood maximum likelihood technique, it provides the estimated parameters. The parameters k and n are the two parameters that control the tails and the peakedness of the distribution. The asymmetry parameter, λ, controls the shape of the probability distribution. If the asymmetry parameter is positive, it generates a positively skewed distribution and if it is negative, it generates a negatively skewed distribution. The expected value and standard deviation are the well-known measures used in finance in many cases such as portfolio analysis. Setting $k = 2$, $n = \infty$, and $\lambda = 0$ gives the Normal distribution, $k = 1$ and $n = \infty$ the Skewed Laplace, $k = 1$, $n = \infty$, and $\lambda = 0$ the Laplace, and so on.

This dissertation focuses on the Skewed Normal (SN) and the Skewed Generalized Error Distribution (SGED) as capable distributions to incorporate in the models. The financial puzzles that this dissertation explores are related to Behavioural Finance, the Cryptocurrency market, and specifically Bitcoin’s behaviour, and the measurement of stock price crashes as an outlier event using a robust technique. An outlier-resistant method was used because the standard ordinary least square technique (OLS) underperforms in the presence of outliers.

The first chapter develops a unified probabilistic framework based on the Skewed Normal (SN) distribution to explain the perceptions of managers in the aspect of the expected value and uncertainty. The statistical framework and Monte-Carlo simulations showed that overconfident and optimist managers overestimate the expected value and underestimate the downside risk, value-at-risk, and expected shortfall. This finding leads to the conclusion that the probability distribution of overconfident and optimistic managers is skewed to the right (positively skewed probability distribution). On the other hand, underconfident and pessimist managers underestimate their expected value and overestimate the risk, downside risk, value-at-risk, and expected shortfall. Therefore, underconfident managers are characterized by a negatively skewed distribution. Also, an analytical framework is extended on the professional forecasters based on the Skewed Generalized Error Distribution (SGED). The findings show that professional forecasters are underconfident since their forecast errors are followed by a negatively skewed distribution.

The second chapter investigates the stochastic behaviour of Bitcoin compared to the common exchange rates (Euro, Japanese Yen, Canadian Dollar, and the British pound). The risk-return univariate relationship is examined using the ST-GJR-GARCH model under the Skewed Generalized Error Distribution (SGED). Adding to this, a bivariate analysis is conducted to examine the mean and volatility spillover effects from Bitcoin to exchange rates and vice versa. Lastly, the forecasting ability of the ST-GJR-GARCH-SGED is compared with the well-known GARCH and GJR models under different probability distributions.

The findings showed that Bitcoin has excess kurtosis (leptokurtosis), and extremely higher volatility compared to the other assets. The risk-return relationship showed that the skewness/kurtosis price of risk is important in all cases and especially in the case of Bitcoin. Bivariate analysis proved that there is a negligible inter-relationship between Bitcoin and exchange rates and it is a useful asset to diversify the portfolio’s risk since it behaves in a very different way compared to the other assets; Bitcoin’s behaviour is extremely leptokurtic compared to the other assets. Also, the shape distributional characteristic of Bitcoin is not affected when spillover effects are presented. Furthermore, the ST-GJR-GARCH-SGED model performs better than the rest models.

The third chapter investigates the measurement of stock price crashes using an outlier resistant method. Stock price crash is an unusual decline in the firm’s prices. Literature concluded that managers withhold bad news for several reasons and when this information becomes available in the market, it leads to a stock price crash. Also, a stock price crash is the conditional skewness of the distribution of returns. The presence of these outliers in the 52-weekly return series drives the binary measure of crashes to be mis-specified due to the standard deviation that is contaminated. In this chapter, a robust framework is developed to show this contamination issue and proposes a robust measure that accounts for the above problem. This methodology also contributes on the existing literature using a statistical framework since a statistical theory in the crash risk literature does not exist.

Monte – Carlo simulations and empirical findings show that the standard (OLS) methodology detects a lower percentage of crashes compared to the robust methodology. This is due to the outliers that affect the standard deviation of the residual returns. Additionally, the un-transformed measure detects a lower percentage of crashes compared to the log transformed measure. Therefore, the log transformed measure, that academic literature used to explain several financial puzzles, detects a higher percentage of crashes in relation to the un-transformed measure. Specifically, the empirical findings show that 6,145 firm-year observations are detected in the robust measure and not in the standard OLS method. Also, 6.59% (or 10,934 firm-year observations) higher percentage of crashes are detected using the robust un-transform measure compared to the un-transform OLS measure. The findings suggest that it is better to use a robust technique that corrects the inflation of variance driven by outliers.

Summarizing, this dissertation develops three different statistical frameworks to investigate three financial puzzles on behavioural finance and specifically on psychological biases using a probabilistic framework under a skewed distribution, the investigation of Bitcoin through an asymmetric GARCH model under a skewed distribution and the measurement of crashes using an outlier-resistant method. The models have also been tested using Monte-Carlo simulations and empirical analysis. The conclusions provide insightful findings on the importance of asymmetric models to explain several issues in finance.

REFERENCES

Alpert, M., & Raiffa, H. (1982). A progress report on the training of probability assessors. In: Kahneman, D., Slovic, P., Tversky, A. (Ed.), *Judgment under Uncertainty: Heuristics and Biases*. Cambridge University Press, Cambridge, 294-305.

An, H., & Zhang, T. (2013). Stock price synchronicity, crash risk, and institutional investors. *Journal of Corporate Finance*, *21*, 1-15.

An, Z., Li, D. & Yu, J. (2015). Firm crash risk, information environment, and speed of leverage adjustment. *Journal of Corporate Finance*, *31*, 132-151.

Andreou, C., Andreou, P. C., & Lambertides, N. (2021). Financial distress risk and stock price crashes. *Journal of Corporate Finance*, *67*, 101870.

Andreou, P.C., Antoniou, C., Horton, J., & Louca, C. (2016). Corporate governance and firm-specific stock price crashes. *European Financial Management*, *22*(5), 916-956.

Andreou, P.C., Louca, C., & Petrou, A.P. (2017). CEO age and stock price crash risk. *Review of Finance*, *21*(3), 1287-1325.

Arkes, H. R. (2001). Overconfidence in judgmental forecasting. *Principles of Forecasting*. Springer, Boston, MA, 495-515.

Baghestani, H., & Kianian, A. M. (1993). On the rationality of US macroeconomic forecasts: Evidence from a panel of professional forecasters. *Applied Economics*, *25*(7), 869-878.

Baker, M., Ruback, R. S., & Wurgler, J. (2007). Behavioral corporate finance. *Handbook of Empirical Corporate Finance*. Elsevier, 145-186.

Barndorff-Neilsen, Ole E, Silvia Kinnebrouk, & Neil Shephard. (2010). Measuring downside risk: realised semivariance. *Volatility and Time Series Econometrics: Essays in Honor of Robert F. Engle*. (Edited by T. Bollerslev, J. Russell and M. Watson). Oxford University Press, 117-136.

Barnett, V. (1978). The study of outliers: purpose and model. *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, *27*(3), 242-250.

Barnett, V. and Lewis, T. (1994). *Outliers in Statistical Data*. Wiley.

Baruník, J., Kočenda, E. & Vácha, L. (2017). Asymmetric volatility connectedness on the forex market. *Journal of International Money and Finance*, *77*, 39-56.

Baruník, J., Kocenda, E., & Vácha, L. (2015). Volatility spillovers across petroleum markets. *The Energy Journal*, *36*(3), 309-329.

Baruník, J., Kočenda, E., & Vácha, L. (2016). Asymmetric connectedness on the US stock market: Bad and good volatility spillovers. *Journal of Financial Markets*, *27*, 55-78.

Batchelor, R., and Dua, P. (1992). Conservatism and consensus-seeking among Economic forecasters. *Journal of Forecasting*, *11*(2), 169-181.

Baur, D. G., & Dimpfl, T. (2018). Asymmetric volatility in cryptocurrencies. *Economics Letters*, *173*, 148-151.

Baur, D. G., Hong, K. J., & Lee, A. D. (2016). Bitcoin–Currency or Asset? *Working Paper*.

Ben-David, I., Graham, J. R., & Harvey, C. R. (2013). Managerial miscalibration. *The Quarterly Journal of Economics*, *128*(4), 1547-1584.

Berndt, E. R., B.H. Hall, R.E. Hall & J.A. Hausman. (1974). Estimation and inference in 10 nonlinear structural models. *Annals of Economic and Social Measurement*, *3*(4), 653–665.

Block, R. A., & Harper, D. R. (1991). Overconfidence in estimation: Testing the anchoring-and-adjustment hypothesis. *Organizational Behavior and Human Decision Processes*, *49*(2), 188-207.

Bouri, E., Azzi, G., & Dyhrberg, A. H. (2017a). On the return-volatility relationship in the Bitcoin market around the price crash of 2013. *Economics-The Open-Access, Open-Assessment E-Journal*, *11*, 1-16.

Bouri, E., Das, M., Gupta, R., & Roubaud, D. (2018). Spillovers between Bitcoin and other assets during bear and bull markets. *Applied Economics*, *50*(55), 5935-5949.

Bouri, E., Molnár, P., Azzi, G., Roubaud, D., & Hagfors, L. I. (2017b). On the hedge and safe haven properties of Bitcoin: Is it really more than a diversifier? *Finance Research Letters*, *20*, 192-198.

Briere, M., Oosterlinck, K., & Szafarz, A. (2015). Virtual currency, tangible return: Portfolio diversification with bitcoin. *Journal of Asset Management*, *16*(6), 365-373.

Butler, R. J., Mcdonald, J. B., Nelson, R. D., & White, S. B. (1990). Robust and partially adaptive estimation of regression models. *The Review of Economics and Statistics*, *72*(2), 321–327.

Callen, J.L. and Fang, X. (2013). Institutional investor stability and crash risk: Monitoring versus short-termism? *Journal of Banking & Finance*, *37*(8), 3047-3063.

Callen, J.L. and Fang, X. (2015a). Religion and stock price crash risk. *Journal of Financial and Quantitative Analysis*, *50*(1-2), 169-195.

Callen, J.L. and Fang, X. (2015b). Short interest and stock price crash risk. *Journal of Banking & Finance*, *60*, 181-194.

Camerer, C. & Lovallo, D. (1999). Overconfidence and excess entry: An experimental approach. *American Economic Review, 89*(1), 306-318.

Cermak, V. (2017). Can bitcoin become a viable alternative to fiat currencies? An empirical analysis of bitcoin's volatility based on a GARCH model. An Empirical Analysis of Bitcoin's Volatility Based on a GARCH Model. *Working Paper*.

Chang, X., Chen, Y. and Zolotoy, L. (2017). Stock liquidity and stock price crash risk. *Journal of Financial and Quantitative Analysis*, *52*(4), 1605-1637.

Chen, J., Hong, H. & Stein, J.C. (2001). Forecasting crashes: Trading volume, past returns, and conditional skewness in stock prices. *Journal of Financial Economics*, *61*(3), 345-381.

Chiang, Y.M., Hirshleifer, D., Qian, Y. & Sherman, A.E. (2011). Do Investors Learn from Experience? Evidence from Frequent IPO Investors. *The Review of Financial Studies*, *24*(5), 1560-1589.

Chu, J., Chan, S., Nadarajah, S., & Osterrieder, J. (2017). GARCH modelling of cryptocurrencies. *Journal of Risk and Financial Management*, *10*(4), 17.

Corbet, S., Meegan, A., Larkin, C., Lucey, B., & Yarovaya, L. (2018). Exploring the dynamic relationships between cryptocurrencies and other financial assets. *Economics Letters*, *165*, 28-34.

Davis, H.L., Hoch, S.J. & Ragsdale, E.E. (1986). An anchoring and adjustment model of spousal predictions. *Journal of Consumer Research*, *13*(1), 25-37.

Deaves, R., Lüders, E., & Schröder, M. (2010). The dynamics of overconfidence: Evidence from stock market forecasters. *Journal of Economic Behavior & Organization*, *75*(3), 402-412.

Dimson, E. (1979). Risk measurement when shares are subject to infrequent trading. *Journal of Financial Economics*, *7*, 197-226.

Dyhrberg, A. H. (2016b). Hedging capabilities of bitcoin. Is it the virtual gold? *Finance Research Letters*, *16*, 139-144.

Dyhrberg, A.H. (2016a). Bitcoin, gold and the dollar–A GARCH volatility analysis. *Finance Research Letters*, *16*, 85-92.

Ehrbeck, T., & Waldmann, R. (1996). Why are professional forecasters biased? Agency versus behavioral explanations. *The Quarterly Journal of Economics*, *111*(1), 21-40.

Ellina, P., Mascarenhas, B., & Theodossiou, P. (2020). Clarifying managerial biases using a probabilistic framework. *Journal of Behavioral and Experimental Finance*, 100333.

Feunou B., M. R. Jahan-Parvar, & R. Tédongap (2012). Modeling market downside volatility. *Review of Finance, 17*, 443–481.

Galasso, A. & Simcoe, T.S. (2011). CEO Overconfidence and innovation. *Management Science, 57*(8), 1469-1484.

Genton, M. G., & Ronchetti, E. (2008). Robust prediction of beta. *Computational Methods in Financial Engineering, Essays in Honour of Manfred Gilli.* Springer, 147–161.

Gervais, S. & Odean, T. (2001). Learning to be overconfident. *The Review of Financial Studies, 14*(1), 1-27.

Gibran, K. (1951). *The Kahlil Gibran Reader: Inspirational Writings*. Citadel Press, New York.

Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *The Journal of Finance, 48*(5), 1779-1801.

Guesmi, K., Saadi, S., Abid, I., & Ftiti, Z. (2019). Portfolio diversification with virtual currency: Evidence from bitcoin. *International Review of Financial Analysis, 63,* 431-437.

Habib, A., Hasan, M. M., & Jiang, H. (2018). Stock price crash risk: Review of the empirical literature. *Accounting & Finance, 58*, 211-251.

Hampel, F.R., Ronchetti, E.M., Rousseeuw, P.J. & Stahel, W.A. (1986). *Robust statistics, 29-30,* New York: Wiley.

Hansen, C., McDonald, J. B., & Newey, W. K. (2010). Instrumental variables estimation with flexible distributions. *Journal of Business & Economic Statistics*, *28*(1), 13-25.

Hawkins, D.M. (1980). *Identification of Outliers* (Vol. 11). London: Chapman and Hall.

Heaton, J. B. (2002). Managerial optimism and corporate finance. *Financial Management*, *31*, 33-45.

Heidhues, P., Kőszegi, B. & Strack, P. (2018). Unrealistic expectations and misguided learning. *Econometrica, 86*(4), 1159-1214.

Holden, K., & Peel, D. A. (1990). On testing for unbiasedness and efficiency of forecasts. *The Manchester School of Economic & Social Studies, 58*(2), 120-127.

Huber, P. J. (1964). Robust estimation of a location parameter. *The Annals of*

Mathematical Statistics, *35*(1), 73–101.

Huber, P. J. (1973). Robust Regression : Asymptotics , conjectures and monte Carlo. *The Annals of Statistics*, *1*(5), 799–821.

Huber, P. J. (1981). *Robust Statistics*. John Wiley & Sons, Inc.

Hutton, A.P., Marcus, A.J. and Tehranian, H. (2009). Opaque financial reports, R2, and crash risk. *Journal of financial Economics*, *94*(1), 67-86.

Jin, L. and Myers, S.C. (2006). R2 around the world: New theory and new tests. *Journal of financial Economics*, *79*(2), 257-292.

Kahneman, D. & Tversky A. (1979). Intuitive prediction: biases and corrective procedures. *TIMS Studies in Management Science, 12*, 313-327.

Kahneman, D. (2011). *Thinking, Fast and Slow*. Macmillan.

Katsiampa, P. (2017). Volatility estimation for Bitcoin: A comparison of GARCH models. *Economics Letters*, *158*, 3-6.

Kim, J.B. and Zhang, L. (2014). Financial reporting opacity and expected crash risk: Evidence from implied volatility smirks. *Contemporary Accounting Research*, *31*(3), 851-875.

Kim, J.B. and Zhang, L. (2016). Accounting conservatism and stock price crash risk: Firm-level evidence. *Contemporary Accounting Research*, *33*(1), 412-441.

Kim, J.B., Li, Y. and Zhang, L. (2011a). CFOs versus CEOs: Equity incentives and crashes. *Journal of Financial Economics*, *101*(3), 713-730.

Kim, J.B., Li, Y. and Zhang, L. (2011b). Corporate tax avoidance and stock price crash risk: Firm-level analysis. *Journal of Financial Economics*, *100*(3), 639-662.

Kim, J.B., Wang, Z. and Zhang, L. (2016). CEO overconfidence and stock price crash risk. *Contemporary Accounting Research*, *33*(4), 1720-1749.

Kim, Y., Li, H. and Li, S. (2014). Corporate social responsibility and stock price crash risk. *Journal of Banking & Finance*, *43*, 1-13.

Kothari, S.P., Shu, S. and Wysocki, P.D. (2009). Do managers withhold bad news? *Journal of Accounting Research*, *47*(1), 241-276.

Koutmos, D. (2018). Return and volatility spillovers among cryptocurrencies. *Economics Letters*, *173*, 122-127.

Kristoufek, L. (2015). What are the main drivers of the Bitcoin price? Evidence from wavelet coherence analysis. *PloS One*, *10*(4), e0123923.

Kunda, Z. (1987). Motivated Inference: Self-serving generation and evaluation of causal theories. *Journal of Personality and Social Psychology, 53*(4), 636.

Kurka, J. (2019). Do cryptocurrencies and traditional asset classes influence each other? *Finance Research Letters, 31*, 38-46.

Kyle, A.S. & Wang, F.A. (1997). Speculation duopoly with agreement to disagree: Can overconfidence survive the market test? *The Journal of Finance, 52*(5), 2073-2090.

Langer, E. J., & Roth, J. (1975). Heads I win, tails it's chance: The illusion of control as a function of the sequence of outcomes in a purely chance task. *Journal of personality and social psychology*, *32*(6), 951.

Langer, E.J. (1975). The illusion of control. *Journal of Personality and Social Psychology, 32*(2), 311.

Lee, W., & Wang, L. (2017). Do political connections affect stock price crash risk? Firm-level evidence from China. *Review of Quantitative Finance and Accounting*, *48*(3), 643-676.

León, Á., Rubio, G., & Serna, G. (2005). Autoregresive conditional volatility, skewness and kurtosis. *The Quarterly Review of Economics and Finance*, *45*(4-5), 599-618.

Lichtenstein, S., Fischhoff, B., & Phillips, L. (1982). Calibration of probabilities: The state of the art to 1980. In: Daniel, K., Slovic, P., Tversky, A. (Eds.), *Judgment Under Uncertainty: Heuristics and Biases*. Cambridge University Press, Cambridge and New York.

Lovell, M. C. (1986). Tests of the rational expectations hypothesis. *The American Economic Review*, *76*(1), 110-124.

Lovie, P. (1985). A note on an unexpected anchoring bias in intuitive statistical inference. *Cognition, 21*(1), 69-72.

Malmendier, U. & Tate, G. (2015). Behavioral CEOs: The role of managerial overconfidence. *Journal of Economic Perspectives, 29*(4), 37-60.

Malmendier, U., Tate, G. & Yan, J. (2011). Overconfidence and early-life experiences: The effect of managerial traits on corporate financial policies. *The Journal of Finance, 66*(5), 1687-1733.

March, J.G. & Shapira, Z. (1987). Managerial perspectives on risk and risk taking. *Management Science, 33*(11), 1404-1418.

Martin, R. D., & Simin, T. T. (2003). Outlier-resistant estimates of beta. *Financial Analysts Journal*, *59*(5), 56–69.

Mazur, B., & Pipień, M. (2018). Time-varying asymmetry and tail thickness in long series of daily financial returns. *Studies in Nonlinear Dynamics & Econometrics*, *22*(5), 1-21.

McDonald, J. B., & Newey, W. K. (1988). Partially adaptive estimation of regression models via the generalized t distribution. *Econometric Theory, 4*, 428-457.

McDonald, J. B., Michelfelder, R. A., & Theodossiou, P. (2009). Robust regression estimation methods and intercept bias: A capital asset pricing model application. *Multinational Finance Journal*, *13*(3/4), 293–321.

McDonald, J.B., Xu, Y. J. (1995). A generalization of the beta distribution. *Journal of Econometrics, 66*, 133–152.

Miller, D., & M. Ross. (1975). Self-serving biases in attribution of causality: fact or fiction? *Psychological Builletin, 82*, 213-225.

Moore, D.A. and Healy, P.J. (2008). The trouble with overconfidence. *Psychological Review*, *115*(2), 502.

Morris, P. A. (1974). Decision analysis expert use. *Management Science*, *20*(9), 1233-1241.

Morris, P. A. (1977). Combining expert judgments: A Bayesian approach. *Management Science, 23*(7), 679-693.

Muth, J. F. (1961). Rational expectations and the theory of price movements. *Econometrica: Journal of the Econometric Society*, *29*, 315-335.

Nakamoto, S. (2008). Bitcoin: A peer-to-peer electronic cash system. *Working Paper.*

Neftci, S. N., & Theodossiou, P. (1991). Properties and stochastic nature of BEA's early estimates of GNP. *Journal of Economics and Business*, *43*(3), 231-239.

Oberlechner, T. & Osler, C. (2012). Survival of overconfidence in currency markets. *Journal of Financial and Quantitative Analysis*, *47*(1), 91-113.

Osterrieder, J. (2016). The statistics of bitcoin and cryptocurrencies. *Working Paper.*

Osterrieder, J., & Lorenz, J. (2017). A statistical risk assessment of Bitcoin and its extreme tail behavior. *Annals of Financial Economics*, *12*(01), 1750003.

Osterrieder, J., Strika, M. & Lorenz, J. (2017). Bitcoin and cryptocurrencies—not for the faint-hearted. *International Finance and Banking*, *4*(1), 56.

Palmon, O. and Venezia I. (2015). Strike prices of options for overconfident executives. *Handbook of Financial Econometric and Statistics,* Cheng-Few Lee and John C. Lee, (Ed.), *3*, 1491-1508.

Palmon, O. and Venezia, I. (2013). A Rationale for hiring irrationally overconfident managers. *Encyclopedia of Finance,* Cheng-Few Lee and Alice Lee, (Ed.), 803-812.

Palmon. O., Bar – Yosef, S., Chen, R. and Venezia, I. (2008). Optimal strike prices of stock options for effort averse executives. *Journal of Banking and Finance, 32*(2), 229–239

Phillip, A., Chan, J., & Peiris, S. (2018). A new look at cryptocurrencies. *Economics Letters*, *163*, 6-9.

Reboredo, J. C., Rivera-Castro, M. A., & Ugolini, A. (2016). Downside and upside risk spillovers between exchange rates and stock prices. *Journal of Banking & Finance*, *62*, 76-96.

Roll, R. (1986). The Hubris hypothesis of corporate takeovers. *Journal of Business*, *59*, 197-216.

Roll, R. (1988). R2. *Journal of Finance, 43*(3), 541–566

Russo, J. E., & Schoemaker, P. J. (1992). Managing overconfidence. *Sloan Management Review*, *33*(2), 7-17.

Savva, C. S., & Aslanidis, N. (2010). Stock market integration between new EU member states and the Euro-zone. *Empirical Economics*, *39*(2), 337-351.

Savva, C. S., & Theodossiou, P. (2018). The risk and return conundrum explained: international evidence. *Journal of Financial Econometrics*, *16*(3), 486-521.

Savva, C. S., Osborn, D. R., & Gill, L. (2009). Spillovers and correlations between US and major European stock markets: the role of the euro. *Applied Financial Economics*, *19*(19), 1595-1604.

Scheier, M. F., & Carver, C. S. (1985). Optimism, coping, and health: assessment and implications of generalized outcome expectancies. *Health Psychology*, *4*(3), 219.

Shefrin, H. (2001). Behavioral corporate finance. *Journal of Applied Corporate Finance*, *14*(3), 113-126.

Shefrin, H. (2005). *Behavioral Corporate Finance,* New York: McGraw-Hill/ Irwin.

Sorokina, N., Booth, D., & Thornton Jr, J. (2013). Robust methods in event studies: empirical evidence and theoretical implications. *Journal of Data Science*, *11*, 575–606.

Statman, M. (2017). *Finance for Normal People: How Investors and Markets Behave*. Oxford University Press.

Svenson, O. (1981). Are we all less risky and more skillful than our fellow drivers? *Acta Psychologica, 47*(2), 143-148.

Symitsi, E., & Chalvatzis, K. J. (2018). Return, volatility and shock spillovers of Bitcoin with energy and technology companies. *Economics Letters*, *170*, 127-130.

Takaishi, T. (2018). Statistical properties and multifractality of Bitcoin. *Physica A: Statistical Mechanics and its Applications*, *506*, 507-519.

Taylor, S.E. & Brown, J.D. (1988). Illusion and well-being: A social psychological perspective on mental health. *Psychological Bulletin*, *103*(2), 193.

Theodossiou P. & Theodossiou A. K. (2019). Consequences of outlier returns for event studies: A methodological investigation and treatment. *The International Journal of Accounting*.

Theodossiou, A. K., & Theodossiou, P. (2014). Stock return outliers and beta estimation: The case of U.S. pharmaceutical companies. *Journal of International Financial Markets, Institutions and Money*, *30*(1), 153–171.

Theodossiou, P. (1994). The stochastic properties of major Canadian exchange rates. *Financial Review*, *29*(2), 193-221.

Theodossiou, P. (1998). Financial data and the skewed generalized t Distribution. *Management Science*, *44*(12), 1650–1661.

Theodossiou, P. (2015). Skewed generalized error distribution of financial assets and option pricing. *Multinational Finance Journal*, *19*(4), 223-266. Initially posted on the SSRN in 2000 as a working paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=219679.

Theodossiou, P. (2021). Class of skewed distributions based on a two-piece generalized framework. *Working Paper*.

Theodossiou, P., & Lee, U. (1993). Mean and volatility spillovers across major national stock markets: Further empirical evidence. *Journal of Financial Research*, *16*(4), 337-350.

Theodossiou, P., & Savva, C. (2016). Skewness and the relation between risk and return. *Management Science*. *62*(6), 1598-1609.

Theodossiou, P., Tsouknidis, D. A., & Savva, C. S. (2020). Freight rates in downside and upside markets: pricing of own and spillover risks from other shipping segments. *Journal of the Royal Statistical Society, Series A*, *183*(3), 1–23.

Tversky, A. & Kahneman, D. (1973). Availability: A heuristic for judging frequency and Probability. *Cognitive Psychology*, *5*(2), 207-232.

Tversky, A., & Kahneman, D. (1974). Judgment under uncertainty: Heuristics and biases. *Science*, *185*(4157), 1124-1131.

Van den Steen, E. (2011). Overconfidence by bayesian - rational agents. *Management Science, 57*(5), 884-896.

Van den Steen, E. (2001). Essays on the managerial implications of differing priors. *Ph.D. Dissertation*, *Stanford University, CA.*

Van den Steen, E. (2004). Rational overoptimism (and other biases). *American Economic Review*, *94*(4), 1141-1151.

Weinstein, N. D. (1980). Unrealistic optimism about future life events. *Journal of Personality and Social Psychology, 39*(5), 806-820.

Weinstein, N.D. & Klein, W.M. (1996). Unrealistic optimism: present and future. *Journal of Social and Clinical Psychology*, *15*(1), 1-8.

Weinstein, N.D. & Lachendro, E. (1982). Egocentrism as a source of unrealistic Optimism. *Personality and Social Psychology Bulletin*, *8*(2), 195-200.

Yang, S. Y., & Doong, S. C. (2004). Price and volatility spillovers between stock prices and exchange rates: empirical evidence from the G-7 countries. *International Journal of Business and Economics*, *3*(2), 139.

Yermack, D. (2015). Is Bitcoin a real currency? An economic appraisal. *Handbook of Digital Currency*, Academic Press, 31-43.

APPENDIX I

Moment Function - Skewed Normal Distribution

Using the equation (1.4), the moment function of *x* (excess of the mode), is [41]

$$M_s = E\left(x-m\right)^s = \int_{-\infty}^{\infty}\left(x-m\right)^s dF_x = \varphi^s\left(1-\lambda\right)^{s+1}\int_{-\infty}^{0} z^s dF_z + \varphi^s\left(1+\lambda\right)^{s+1}\int_{0}^{\infty} z^s dF_z$$

dF_z is symmetric, therefore, $\int_{-\infty}^{0} z^s dF_z = \left(-1\right)^s \int_{0}^{\infty} z^s dF_z.$ The moment function of *x* can be re-written as

$$M_s = \varphi^s\left[\left(-1\right)^s\left(1-\lambda\right)^{s+1} + \left(1+\lambda\right)^{s+1}\right]\int_{0}^{\infty} z^s dF_z \, .$$

$$M_s = \frac{1}{2}\left[\left(-1\right)^s\left(1-\lambda\right)^{s+1} + \left(1+\lambda\right)^{s+1}\right]\varphi^s G_s.$$

where $G_s = E\left|z^s\right| = 2\int_0^{\infty} z^s dF_z$ is the sth moment (absolute) of z.

For *s* = 1

$$M_1 = E\left(x-m\right) = 2\lambda G_1\phi \text{ and } M_2 = \left(1+3\lambda^2\right)G_2\phi^2$$

Therefore,

$$\mu = m + 2\lambda G_1\phi$$

For *s* = 2

$$\sigma^2 = M_2 - M_1^2 = \left(\left(1+3\lambda^2\right)G_2 - 4G_1^2\lambda^2\right)\phi^2.$$

Let $t = z^2/2$, thus $z = 2^{\frac{1}{2}} t^{\frac{1}{2}},\ dz = 2^{-\frac{1}{2}} t^{-\frac{1}{2}} dt,\ z^s dz = 2^{\frac{s}{2}-\frac{1}{2}} t^{\frac{s+1}{2}-1} dt$ and

$$G_s = E\left|z^s\right| = \frac{2^{\frac{1}{2}}}{\sqrt{\pi}}\int_0^{\infty} z^s e^{-\frac{z^2}{2}} dz = \frac{1}{\sqrt{\pi}} 2^{\frac{s}{2}}\int_0^{\infty} t^{\frac{s+1}{2}-1} e^{-t} dt = \frac{1}{\sqrt{\pi}} 2^{\frac{s}{2}}\Gamma\left(\frac{s+1}{2}\right).$$

For *s* = 1 and 2

$$G_1 = \frac{2^{\frac{1}{2}}}{\sqrt{\pi}}\Gamma\left(1\right) = \sqrt{2/\pi} \text{ and } G_2 = \frac{2^{\frac{2}{2}}}{\sqrt{\pi}}\Gamma\left(\frac{3}{2}\right) = 1.$$

[41] Ellina et al. (2020).

The substitution of this result into M_s gives

$$M_s = \frac{1}{\sqrt{\pi}} 2^{\frac{s}{2}} \left[(-1)^s (1-\lambda)^{s+1} + (1+\lambda)^{s+1} \right] \Gamma\left(\frac{s+1}{2} \right) \varphi^s . \tag{A1}$$

For s =1, $M_1 = E(x-m) = \frac{1}{\sqrt{\pi}} 2^{\frac{1}{2}} \left[(-1)^1 (1-\lambda)^2 + (1+\lambda)^2 \right] \Gamma(1) \varphi = 2\sqrt{2/\pi}\lambda\varphi$, thus

$$\mu = E(x) = m + 2\sqrt{2/\pi}\lambda\varphi = m + 1.5958\lambda\varphi . \tag{A2}$$

For s = 2, $M_2 = \frac{1}{\sqrt{\pi}} 2^{\frac{2}{2}} \left[(-1)^2 (1-\lambda)^3 + (1+\lambda)^3 \right] \Gamma\left(\frac{3}{2} \right) \varphi^2 = \left(1+3\lambda^2\right)\varphi^2$, thus

$$\sigma^2 = \text{var}(x) = M_2 - M_1^2 = \left(1 + (3-8/\pi)\lambda^2\right)\varphi^2 = \left(1 + 0.4535\lambda^2\right)\varphi^2 . \tag{A3}$$

Downside Risk

The downside moment function of x (excess the mode) is

$$M_s^- \equiv E\left((x-m)^s \middle| x \le m \right) = \frac{1}{P(x \le m)} \int_{-\infty}^{m} (x-m)^s \, dF_x$$

$$M_s^- = (-1)^s (1-\lambda)^s \varphi^s 2\int_0^\infty z^s dF_z = (-1)^s (1-\lambda)^s \varphi^s G_s$$

$$= (-1)^s (1-\lambda)^s \varphi^s 2\int_0^\infty z^s dF_z = (-1)^s (1-\lambda)^s \frac{1}{\sqrt{\pi}} 2^{\frac{s}{2}} \Gamma\left(\frac{s+1}{2} \right) \varphi^s .$$

For s = 1 and 2

$$M_1^- = -(1-\lambda) G_1 \varphi \text{ and } M_2^- = (1-\lambda)^2 G_2 \varphi^2$$

and

$$\sigma^2_{x|x\le m} = \text{var}\left(x \middle| x \le m\right) = M_2^- - \left(M_1^-\right)^2 = (1-\lambda)^2 \left[G_2 - G_1^2 \right] \phi^2 .$$

For s =1 and 2

$$M_1^- = -(1-\lambda) \frac{1}{\sqrt{\pi}} 2^{\frac{1}{2}} \Gamma(1) \varphi = -(1-\lambda)\sqrt{2/\pi}\varphi ,$$

$$M_2^- = \frac{1}{\sqrt{\pi}} 2 (1-\lambda)^2 \Gamma\left(\frac{3}{2} \right) \varphi^2 = (1-\lambda)^2 \varphi^2$$

Therefore, the downside variance of x is

$$\sigma^2_{x|x\le m} = \text{var}\left(x \middle| x \le m\right) = M_2^- - \left(M_1^-\right)^2 = (1-\lambda)^2 \varphi^2 - (1-\lambda)^2 \frac{2}{\pi} \varphi^2 .$$

$$= (1-\lambda)^2 (1-2/\pi)\varphi^2 . \tag{A4}$$

Upside Risk

The upside moment function of *x* (excess of the mode) is

$$M_s^+ \equiv E\left((x-m)^s \middle| x > m\right) = \frac{1}{P(x>m)} \int_m^\infty (x-m)^s \, dF_x$$

$$= (1+\lambda)^s \varphi^s 2\int_0^\infty z^s dF_z \;= (1+\lambda)^s \frac{1}{\sqrt{\pi}} 2^{\frac{s}{2}} \Gamma\left(\frac{s+1}{2}\right)\varphi^s .$$

For *s* =1 and 2,

$$M_1^+ = E\left(x-m \middle| x>m\right) = (1+\lambda)\frac{1}{\sqrt{\pi}} 2^{\frac{1}{2}} \Gamma(1)\varphi = (1+\lambda)\sqrt{2/\pi}\varphi, \text{ thus}$$

$$M_2^+ = \frac{1}{\sqrt{\pi}} 2(1+\lambda)^2 \Gamma\left(\frac{3}{2}\right)\varphi^2 = (1+\lambda)^2 \varphi^2$$

Therefore, the upside variance of *x* is

$$\sigma^2_{x|x>m} = \mathrm{var}\left(x \middle| x>m\right) = M_2^+ - \left(M_1^+\right)^2 = (1+\lambda)^2 \varphi^2 - (1+\lambda)^2 \frac{2}{\pi}\varphi^2 \;.$$

$$= (1+\lambda)^2 (1-2/\pi)\varphi^2 . \tag{A5}$$

APPENDIX II

The random variable $Z_n = z_1 + z_2 + ... + z_n$ is the sum of n random variables.[42, 43] Therefore, the third (*SK)* and fourth (*KU*) moments of z_t are the standardized skewness and standardized kurtosis of the logarithmic return y_t. The above leads to the following: 1. The expected value of Z_n is zero, i.e., $E(Z_n) = 0$, 2. $E(z_t\ z_s) = 0$ for $t \neq s$ (under the assumption of i.i.d. returns) and 3. $Var(Z_n) = n$.

The third centered moments of Z_n is

$$M_3 = EZ_n^3 = \left(z_1 + z_2 + ... + z_n\right)^3$$

$$= \sum_t Ez_t^3 + 3\sum_{t\neq s}\sum Ez_t^2 z_s + \sum\sum_{t\neq s\neq p}\sum Ez_t z_s z_p \tag{B1}$$

The fourth centered moments of Z_n is

$$M_4 = EZ_n^4 = \left(z_1 + z_2 + ... + z_n\right)^4$$

$$= \sum_t Ez_t^4 + 4\sum_{t\neq s}\sum Ez_t^3 z_s + 3\sum_{t\neq s}\sum Ez_t^2 z_s^2$$

$$3\sum\sum_{t\neq s\neq p}\sum Ez_t^2 z_s z_p + \sum\sum\sum_{t\neq s\neq p\neq r}\sum Ez_t z_s z_p z_r \tag{B2}$$

The assumption that z_t are i.i.d. for $t \neq s \neq p \neq r,$ implies that

$$Ez_t z_s z_p = 0,\ \ Ez_t^2 z_s = 0,\ \ Ez_t z_s z_p z_r = 0,\ \ Ez_t^2 z_s^2 = Ez_t^2 Ez_s^2 = 1,\ \ Ez_t^3 z_s = 0.$$

The deviations of these measures are the result of the presence of higher order moment dependencies in the logarithmic returns.

Substitute these equations into (B1) and (B2) give

$$M_3 = EZ_n^3 = nm_3, \tag{B3}$$

and

---

[42] where $z_t = \left(y_t - \mu\right)/\sigma$, μ is the average and σ is the standard deviation of the logarithmic return y_t

[43] See also Theodossiou (2015).

$$M_4 = EZ_n^4 = nm_4 + 3n\left(n-1\right), \tag{B4}$$

where $m_3 = Ez^3$ and $m_4 = Ez^4$ are the standardized skewness and standardized kurtosis for z.

The skewness for Z_n is

$$SK_{z_n} = \frac{EZ_n^3}{\operatorname{var}\left(Z_n\right)^{3/2}} = \frac{nm_3}{n^{3/2}} = \frac{m_3}{\sqrt{n}}, \tag{B5}$$

The kurtosis for Z_n is

$$KU_{z_n} = \frac{EZ_n^4}{\operatorname{var}\left(Z_n\right)^2} = \frac{nm_4 + 3n\left(n-1\right)}{n^2} = \frac{m_4}{n} + 3\left(1 - \frac{1}{n}\right).^{44} \tag{B6}$$

Equations (B1) and (B2) are used to construct the test statistics for the higher-order dependencies in the series,

$$mz_1 = \frac{\sum z_t^2 z_{t-j}}{T} \text{ and } \operatorname{var}\left(mz_1\right) = \frac{Ez_t^4}{T}, \tag{B7}$$

$$mz_2 = \frac{\sum z_t z_{t-1} z_{t-2}}{T} \text{ and } \operatorname{var}\left(mz_2\right) = \frac{1}{T}, \tag{B8}$$

$$mz_3 = \frac{\sum z_t^3 z_{t-1}}{T} \text{ and } \operatorname{var}\left(mz_3\right) = \frac{Ez_t^6}{T}, \tag{B9}$$

$$mz_4 = \frac{\sum\left(z_t^2 z_{t-j}^2 - 1\right)}{T} \text{ and } \operatorname{var}\left(mz_4\right) = \frac{\left(Ez_t^4\right)^2}{T}, \tag{B10}$$

$$mz_5 = \frac{\sum z_t^2 z_{t-1} z_{t-2}}{T} \text{ and } \operatorname{var}\left(mz_5\right) = \frac{Ez_t^4}{T}, \tag{B11}$$

$$mz_6 = \frac{\sum z_t z_{t-1} z_{t-2} z_{t-3}}{T} \text{ and } \operatorname{var}\left(mz_6\right) = \frac{1}{T}, \tag{B12}$$

where $z = \left(y_t - \bar{y}\right)/S$, $\bar{y}$ and S are the sample average and standard deviation of logarithmic returns y_t, respectively. The sample size is denoted by T and j = 1,2, and

---

[44] As $n \to \infty$, the standardized skewness SK and kurtosis KU $\lim\limits_{n\to\infty} SK_{z_n} = \frac{m_3}{\sqrt{n}} \approx 0$ and $\lim\limits_{n\to\infty} KU_{z_n} = \frac{m_4}{n} + 3\left(1 - \frac{1}{n}\right) \approx 3.$

3.[45] The statistics have a zero mean. The sequence z_t is i.i.d random variables, therefore, the variances are computed under this hypothesis.

The conditional heteroscedasticity refers to the fact that large changes are followed by large changes, of either sign. The presence of volatility clustering in the data implies that

$$Ez_t^2 z_s^2 \neq Ez_t^2 Ez_s^2, \text{ for } t \neq s. \qquad \text{(B13)}$$

These statistics are computed using $z_t^2 z_s^2$ when $s = t - j$ where j = 1, 2 and 3.

Also, the asymmetric volatility phenomenon is the tendency for the volatility to be higher when the market downs than when the market is rising. The asymmetric volatility implies that

$$E\left(z_t^2 \middle| z_s < 0\right) > E\left(z_t^2 \middle| z_s > 0\right) \text{ or } Ez_t^2 z_s \neq 0, \text{ for } t > s. \qquad \text{(B14)}$$

These statistics are computed using the product $z_t^2 z_s$, when $s = t - j$ where j = 1, 2 and 3. A negative asymmetric volatility means that volatility is higher when the market downturns.

[45] The moments Ez_t^4 and Ez_t^6 are estimated using the equations $m_4 = \sum z_t^4 / T$ and $m_6 = \sum z_t^6 / T$.

APPENDIX III

Fama and French Industry Classification [46]

1 Agriculture
2 Food Products
3 Candy & Soda
4 Beer & Liquor
5 Tobacco Products
6 Recreation
7 Entertainment
8 Printing and Publishing
9 Consumer Goods
10 Apparel
11 Healthcare
12 Medical Equipment
13 Pharmaceutical Products
14 Chemicals
15 Rubber and Plastic Products
16 Textiles
17 Construction Materials
18 Construction
19 Steel Works Etc
20 Fabricated Products
21 Machinery
22 Electrical Equipment
23 Automobiles and Trucks
24 Aircraft
25 Shipbuilding, Railroad Equipment
26 Defense
27 Precious Metals
28 Non-Metallic and Industrial Metal Mining
29 Coal
30 Petroleum and Natural Gas
31 Utilities
32 Communication
33 Personal Services
34 Business Services
35 Computers
36 Computer Software
37 Electronic Equipment
38 Measuring and Control Equipment
39 Business Supplies
40 Shipping Containers
41 Transportation
42 Wholesale
43 Retail
44 Restaurants, Hotels, Motels
45 Banking
46 Insurance
47 Real Estate
48 Trading
49 Other Almost Nothing

[46] https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html (accessed: April 2021).