Check for updates

Composites and Advanced Materials

Review Article

Composites and Advanced Materials
Volume 30: 1–12
© The Author(s) 2021
Article reuse guidelines:
sagepub.com/journals-permissions
DOI: 10.1177/26349833211003904
journals.sagepub.com/home/acm
SAGE

Buckling and postbuckling of architectured materials: A review of methods for lattice structures and metal foams

Christina Völlmecke[1], Melanie Todt[2] and Stylianos Yiatros[3]

Abstract

Recent advances in manufacturing and material science have given rise to numerous architectured materials (archimats), which are tailored for multifunctionality and improved performance. Specifically, lattice structures and metal foams are usually lightweight optimized structural morphologies, which are prone to non-linear instability phenomena, leading to collapse or to a different stable state. This article offers an extensive review of analytical, numerical and experimental methods for investigating buckling and postbuckling in such materials. In terms of analytical modelling, linear elastic and geometrically non-linear models are presented. In numerical analysis, discrete and continuum models are presented, highlighting how numerical modelling can inform design of such materials and finally, experimental methods across different scales are reported, highlighting their merits, depending on the aim of the investigation.

Keywords

architectured materials, buckling, postbuckling, analytical model, finite element method, experiments

Introduction

Over the past decades, architectured materials (archimats) (please note that other terms are proposed in the literature, such as hybrid materials, metamaterials, multimaterials and tailored materials) have been making their way into material science and engineering. Archimats comprise both material combination and structural configuration and are thus dependent on the observation scale. Hence, an archimat combines several non-miscible materials (or one material and air) in a predefined arrangement such that a representative volume element (RVE) comprises at least one dimension that is very small in comparison with the overall dimensions of the part it composes. This includes, for example, composites, sandwiches, foams, lattices, etc.[1] Lattice materials, for example, are cellular materials with an open and periodic internal structure and have remarkable potential owing to their multifunctionality.[2,3] The periodicity of lattice materials ensures that their global material properties mainly depend on the internal architecture. Hence, the overall properties of lattice materials can be tailored towards the requirements of specific applications by designing the lattice structure accordingly. Thus, during the development and tailoring of an archimat, the underlying question on the extent of the combination of the strategies of microstructural and architectural design arises.

This aspect becomes more relevant since advanced manufacturing technologies such as additive manufacturing (AM) allow to produce highly complex and even hybrid lattice structures in customized mass production.[4–6] With the advent of AM technologies, the development of tailored materials has experienced a significant boost. A desired structure can now easily be built in three dimensions

[1] Stability and Failure of Functionally optimized Structures group (SVFS), Institute of Mechanics, School 5, Technical University Berlin, Berlin, Germany
[2] Institute of Lightweight Design and Structural Biomechanics, TU Wien, Wien, Austria
[3] Department of Civil Engineering and Geomatics, Cyprus University of Technology, Limassol, Cyprus

Date received: 31 August 2020; accepted: 07 February 2021

Corresponding author:
Christina Völlmecke, Stability and Failure of Functionally Optimized Structures Group, Institute of Mechanics, School 5, Technical University Berlin, Sekr. MS2, Einsteinufer 5, 10587 Berlin, Germany.
Email: christina.voellmecke@tu-berlin.de

Creative Commons Non Commercial CC BY-NC: This article is distributed under the terms of the Creative Commons Attribution-NonCommercial 4.0 License (https://creativecommons.org/licenses/by-nc/4.0/) which permits non-commercial use, reproduction and distribution of the work without further permission provided the original work is attributed as specified on the SAGE and Open Access pages (https://us.sagepub.com/en-us/nam/open-access-at-sage).

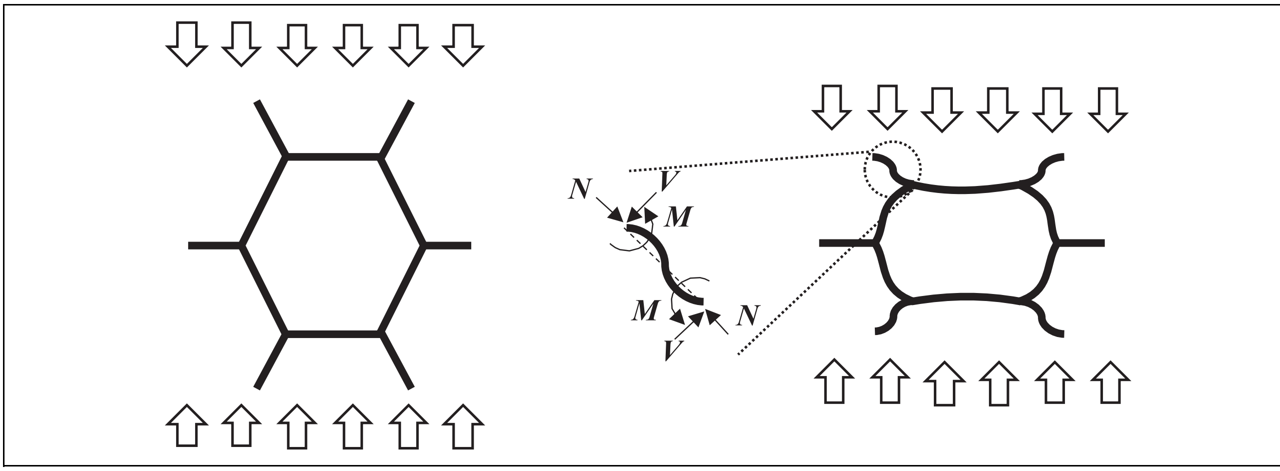

Figure 1. A hypothetical beam model to describe hexagonal lattice struts buckling under compressive load in the vertical direction.

(3D) owing to the precise deposition of printable material(s) with microscale accuracy.[1,7]

In lightweight constructions, archimats are used due to their extremely high strength-to-density ratio. The rising demand for energy-saving and, consequently, lighter construction has led to the manufacture of very slender lattice structures. Slender structures are highly prone to loss of structural stability when experiencing compressive stress states. Hence, their compressive strength is not governed by the strut material but by the buckling load of the lattice itself.[8,9] Experimental work shows that, for example, square lattice materials possess far higher compressive strengths than other lattice structures with comparable relative densities.[9–12]

The design of lattice materials previously focussed on the perfect state, that is, imperfection-free geometry and homogeneous base materials to understand the underlying deformation mechanisms (see refs.[13–16]). Yet, even small perturbations, for example, due to local manufacturing discrepancies, may result in dramatic changes in the overall structural response of an archimat and may potentially jeopardize its functionality. Thus researchers are currently accounting for imperfections via robust design architectures that minimize the impact of imperfections on the effective overall properties and, ultimately, component performance.[17,18] So despite the fact that buckling is a phenomenon that has been known for centuries, it is still commonly avoided in engineering design.

However, in 2010, Crosby already emphasized the potential of buckling and the deeper understanding of the (post)-buckling mechanisms.[19] Most recently, structural stability analysis has indeed experienced a research revival. Buckling phenomena are no longer viewed as sources of catastrophic failure but rather as novel opportunities for functionality and design as is manifested in many smart applications surrounding us, for example, sensors, switches and deployable structures,. Thus renewed interest is sparked.[20,21] For a comprehensive review on how microstructural instabilities can be exploited in archimats, the reader is referred to the article of Kochmann and Bertoldi.[22]

Lattice materials, as an example for an archimat, can be considered as special cases of open-cell foams with a distinct periodic structure. Hence, modelling strategies developed for open-cell foams can be directly applied. These modelling strategies can be categorized into analytical and computational methods. In addition, experimental investigations are mainly undertaken to gain further knowledge on the underlying mechanisms.

Analytical modelling

Herein, analytical methods comprise all methods where the problem can be stated analytically, no matter whether a closed-form solution is subsequently found or the governing equations are solved with the aid of numerical tools. Initially, linear elastic modelling approaches are presented and their limitations are discussed. Subsequently, geometrically non-linear modelling is presented with its specific application on lattice materials being demonstrated.

Linear elastic modelling

Analytical methods are often based on linear elastic beam theory and have been applied to study various aspects of open-cell foams/lattice materials in a global or homogenized manner.[23,24] In ref.,[23] relations between the elastic modulus and the relative density in the form of a power law are derived. For the overall homogenized elastic modulus $\bar{E}$, the relation $\bar{E} \approx E\overline{\rho}^{m}$ is found, where E denotes the elastic modulus of the strut material and $\overline{\rho}$ is the relative density. The parameter m accounts for the deformation mechanism active in the lattice material with respect to the loading condition and is $m = 1$ for stretch dominated and $m = 3$ for bending dominated lattices. Beam models (see Figure 1) are also used in ref.[24] to investigate the mechanical behaviour of honeycomb structures, where relations for the compressive collapse and fracture toughness are derived. Similar concepts are used in ref.[25]

In this review, the focus lies on the mechanical behaviour of archimats under compressive loading. With respect to this loading scenario, analytical approaches are limited to the evaluation of the critical behaviour, that is, the maximum peak compressive stress σ_{pk}.[10,24,25] In ref.[10], the buckling stress σ_{pk} for global elastic buckling of the lattice material is determined as follows:

$$\sigma_{pk} = \sigma'_{pk} + G \quad \text{with} \quad \sigma'_{pk} = \frac{\pi^2 E_s}{2}\left(\frac{a}{H}\right)^2 \overline{\rho} \tag{1}$$

The peak stress σ'_{pk} is the buckling stress of the struts without the influence of the horizontal bars and depends linearly on the relative density $\overline{\rho}$. The parameter G denotes the overall shear modulus of the lattice material. Similar equations exist for other lattice materials, see, for example.[9,25] Such formulations are insufficient for two reasons. First, calculating the compressive peak buckling stress offers no insight into the global postbuckling material response. Second, this approach has served, up to now, only to supplement experimental work. For efficient design and tailoring of the material behaviour, it is, however, necessary to estimate their mechanical behaviour without experimental work.

Thus, geometrically non-linear analytical models are required.

Geometrically non-linear modelling

Geometrically non-linear modelling is necessary to explore the behaviour of structures beyond their critical buckling load. The analytical procedure reviewed herein is the method of minimizing the total potential energy in the form described by Thompson and Hunt.[26] The aim of this convenient procedure is to describe the deformation behaviour of the system under the influence of load with only very few generalized coordinates or so-called degrees of freedom, q_i. The total potential energy V consists of the strain energy U of the system and the work of the external load P along the end-shortening $\mathcal{E}$:

$$V(q_i, P) = U(q_i) - P\mathcal{E}(q_i) \tag{2}$$

Once this formulation is determined for a certain structure, the stability of the system can be readily investigated. The relationship of the load P versus the displacement and hence the generalized coordinates q_i results from the equilibrium path of the system derived by the total derivative of V with respect to the generalized coordinates, thus:

$$\frac{dV}{dq_i} \overset{!}{=} 0 \tag{3}$$

The methodology allows for a large deflection formulation and the assessment of both semi-continuous and fully discretized geometric models while parameters can be varied systematically and imperfection studies can be conducted.

The above methodology has recently been successfully employed to model fibre-stayed, collinear lattice structures[27] while experimental and numerical investigations confirmed the results.[28,29] Its potential for further investigations on archimats has been accredited in ref.[30] The framework is particularly advantageous when so-called *modal nudges*, which are small, deliberate geometric alterations to the structure that were previously determined via information derived by the postbuckled configuration of the baseline idealized structure,[31] are to be introduced. By employing these nudges, the postbuckling behaviour of a structure can be altered significantly. That means that through these deliberate alterations, favourable, that is, stable, postbuckling paths can be enforced, yielding predictable and potentially exploitable responses in a structural context.

Digital fabrication and rapid prototyping (commonly known as 3D printing) techniques are currently flourishing. They are thus furthering the development of archimats using this framework since they add powerful physical insight and offer a proof-of-concept. Hence, a shift towards *buckliphilia* is made[32] focussing its efforts on demonstrating that the postbuckling regime allows for dramatic reconfigurations which can be exploited for function, thus harnessing instabilities. For example, in archimats, local bistable mechanisms triggered by small geometric variations are easily incorporated via AM leading to trapping of elastic strain energy as demonstrated experimentally and numerically in ref.[33] 3D printing is also regarded as a versatile tool for implementing nudges for tailored buckling configurations.[34] As emphasized in ref.,[30] the interplay between analytical, numerical and experimental techniques is of utmost importance, such that the way through a plethora of unstable postbuckling equilibrium can be determined. Thus, hereafter the further important building blocks are presented.

Numerical modelling

Computational methods allow gaining further insight into mechanisms observed in experiments as they are capable of providing detailed information on the local non-linear deformation and stress states within archimats.[35–38] Large deformations,[39,40] for example, due to buckling,[41–43] as well as material non-linearities, due to plastic yielding, or damage within the struts,[35,39,44–47] can be accounted for. Therefore, the complex failure behaviour of archimats involving crushing under compressive loading[35,38,42,43,48,49] or the fracture behaviour under various loading conditions[47,50,51] can be studied using numerical modelling.

Modelling approaches found in the literature for simulating of the mechanical behaviour of archimats by means of the finite element method (FEM) can be grouped into discrete and continuum modelling approaches. Both approaches will be discussed in more detail in the following subsections.

Discrete modelling

Within discrete modelling approaches, archimats are modelled in their entire complexity, that means, each lattice member is explicitly resolved. Discrete models can be divided into unit cell (UC) models and models of the whole finite-sized archimat.

In general, UC models assume archimats of infinite size and, therefore, employ periodic boundary conditions[52] allowing for the application of macroscale stress or strain fields. A UC may consist of one or several base cells of the archimat where the base cell is the smallest geometric microstructure with which the infinite (or finite) structure can be reconstructed by repetitive translation along the axes of the base cell.[53] The number of base cells within a UC depends on the mechanisms to be investigated. For determining the effective linear or non-linear elastic properties of the archimat by means of computational homogenization, one base cell is sufficient.[42,44,54] To some extent, structural irregularities can also be considered in such models.[42,50,55] For studying the onset of buckling and the postbuckling behaviour of archimats, the size of the UC, that is, the number of base cells within a UC has to be large enough to capture all relevant microscale deformation mechanisms.[48] In this case, UCs are often denoted as RVE.[40,55–57] In the literature, various studies can be found, which use UC models for studying microscale buckling within archimats.[35,40,48,53,55–58] In combination with the Bloch wave method, UC models are capable of accounting for buckling modes with wavelengths (far) larger than the UC size.[53,59–61] This can be exploited in the optimization of archimats with respect to their resistance to buckling.[62] For further details on the optimization of archimats, the reader is referred to Osanov and Guest[63] and Thomsen et al.[62] as well as the references provided therein. UC models can also provide valuable information on the yield strength[38,44] and the crushing behaviour of archimats[35,48] if the UC size is appropriately chosen.

Modelling the archimat structure in its entire size is required if the localization of deformation such as the formation of crush bands under compressive loading[38,42,43,48,55,57,64–66] or the formation of elastic shear bands in the vicinity of crack tips[51] are to be studied. Other aspects that are included in finite-sized models are boundary conditions similar to those in experiments and effects resulting from free edges. Consequently, these models allow for a direct comparison with experiments.[37,38,57,64,65,67] Furthermore, finite-sized archimat models allow considering structural irregularities in a statistical manner. Typical irregularities are a misalignment of struts and vertices,[42,43,47,48,55,66,68,69] radius variations within individual struts,[55,67] porosity of the parent material[67] and missing struts or missing clusters of struts.[43,66,68,69]

For the finite element discretization either continuum elements[36,38,55–58,62] or structural elements such as beams[40,42–44,46,49,53,55,61,67,70,71] or shells[38,66] have been used. Continuum element models provide a detailed representation of the geometry of archimats involving material aggregation at the vertices,[54] varying diameters of the individual struts[36] and manufacturing imperfections as obtained, for example, from computer tomography scans.[55] They can also account for the thickening of the struts under large deformations due to a non-zero Poisson's ratio of the parent material.[58] Furthermore, they provide highly resolved information about the stress and strain field within lattice members close to the vertices.[35,36,54,72] Continuum element models are computationally demanding and consequentially are mainly used in UC models[35,38,54,56–58,62] but, due to increasing computational power, they are getting more frequently employed for studying the mechanical behaviour of archimats of finite size.[36,56,57,72]

Using structural instead of continuum elements leads to a much higher computational efficiency, where beam elements being commonly employed. For low-density archimats, beam models have been shown to provide reliable predictions of the mechanical behaviour involving the overall (non-)linear elastic properties,[40,42,54,71] the onset of buckling,[48,53,60,61,70] the collapse and crushing behaviour,[35,42–44,64] as well as the fracture behaviour.[47,50,51] However, within beam models, a special treatment of the vertices is required to properly account for the locally increased stiffness due to material aggregation as well as the influence of the vertices on the relative density of the archimat. Concepts how these aspects can be considered are presented, for example, in Luxner et al.[54] and Smith et al.[35]

Structural and continuum element models can also be combined allowing for a detailed representation of the stress and strain states within a limited number of continuum element modelled base cells, whereas the rest of the archimat is discretized using structural elements.[37]

Continuum modelling

If a finite-sized archimat structure consists of a large number of UCs or base cells, it can become computationally expensive to resolve the archimat in its entire complexity. In this case, it is more feasible to represent the archimat by a homogeneous material showing the same effective mechanical response.

If the length scale of the macroscopic problem is far larger than the size of the UC, that is, the principle of separation of scales is satisfied, the effective behaviour of the lattice can be described using a Cauchy continuum. The corresponding effective properties of the archimat can then be derived using classical first-order homogenization schemes[52] such as asymptotic homogenization[55,73] or the periodic microfield approach.[42,54] A comparison of different homogenization schemes for predicting the effective elastic moduli of various archimats can be found in Arabnejad and Pasini.[73]

In common engineering applications of archimats, the size of the structural problem and the UC are often of the same order of magnitude leading to noticeable size effects.[74–77] Issues also arise in situations where highly heterogeneous deformations occur, for example, near sharp corners or crack tips[51] as well as during the formation of crush bands[38,48,55,64,65] as a consequence of microscale buckling. These phenomena are related to the absolute size of the microstructure and, therefore, cannot be captured using classical Cauchy continuum theory. To overcome this issue, generalized continua have been used as they introduce a material length scale into the constitutive relations. Micropolar,[78–82] strain-gradient[82–84] or micromorphic[85] continua have been employed to describe the effective behaviour of archimats. For a general review and classification of the various generalized continuum theories, the reader is referred to Noor,[86] Fatemi et al.,[87] Forest and Sievert,[88] and references therein. For simple lattice materials, the constitutive behaviour of the generalized continuum models can be derived using concepts based on beam theories[78–82,89] which may even lead to closed-form solutions for the tangent stiffness tensor.[82,89] For complex microstructures and especially if material nonlinearities of the parent material are involved, computational homogenization schemes can be employed.[85,90–92]

Other concepts falling within the scope of continuum modelling of archimats are the quasi-continuum theory proposed in Phlipot and Kochmann[93] or the non-linear constitutive models proposed by Vigliotti et al.[94,95] Alternatively, multilevel schemes can be used to study localization phenomena originating from sharp edges/cracks or local buckling in heterogeneous materials.[93,96,97] Within these schemes, the archimat is resolved in its full complexity around the localization zones, whereas far away from these ‘hot spots’, a macroscale continuum model is used. Proper coupling between the microscopic model and the continuum model has to be ensured for an accurate representation of the involved variables at different scales. Adaptive mesh refinement is employed to simulate the propagation of the localization zones through the material.[93,96,97]

The different continuum modelling concepts have been successfully applied for simulating localization phenomena observed, for example, during the indentation of archimats,[41,45,89,93] around circular holes in archimat plates under tension[95] and for other (macroscopically) inhomogeneous deformation states.[82] Other examples involve the bending of archimat beams,[77,82] the propagation of cracks within archimats[93] and studies on the complex deformation behaviour of tetra-chiral archimats.[85]

For the sake of completeness, substructuring techniques shall be mentioned. These techniques have been used to study buckling in periodic frame-like structures with low memory usage[98,99] and can also be adopted for example to treat topology optimization problems in archimats.[100]

Design of archimats

Computational methods not only provide further insight into mechanisms observed in experiments but can also be utilized for designing archimats towards target applications. Examples are the design of archimats with desired stiffness properties,[101–103] with a desired auxetic behaviour,[56,57,104,105] with tailored buckling mechanisms at the microscale[106] or with an artificially designed anisotropy.[107,108] For obtaining the desired effective properties of archimats, either the internal architecture[3] in terms of the orientation[102] and aspect ratios[106,109] of the individual structural members can be modified or a combination of different base materials can be utilized.[29,104,107] Additionally, manufacturing defects can have a strong impact on the mechanical behaviour of archimats[17,18,55] and consequently have to be considered in the design process.

In many cases, the design of archimats is based on the experience of the designers, and new archimats are, for example, developed by modifying the architecture of already existing microstructures[56,104,105,109] in terms of a trial-and-error approach. In this context, the FEM is mainly employed to provide a proof-of-concept for the new designs[56,57,109] as well as to perform parametric studies[104,108,109] to further investigate or fine-tune the effective mechanical response of the developed archimat. Although this approach often results in the desired mechanical behaviour, it does not guarantee the design to be the optimal one and it does not allow for systematic development of new archimats. Furthermore, parametric studies can become computationally expensive as the number of parameters strongly increases with the number of lattice members in the UC.

Common topology optimization allows for a more strategic design of archimats towards desired properties,[63] such as extreme bulk or shear moduli,[103,110] or a maximum buckling strength[62,100] under design constraints, such as the volume fraction,[103,110] or symmetries in the material distribution.[62] Within optimization schemes, typically the FEM is employed for discretizing the design space and for predicting the effective properties where either continuum or structural elements can be used.[63] A detailed review on the application of topology optimization schemes for the design of archimats is provided in Osanov and Guest.[63] In Pasini and Guest,[18] the consideration of manufacturing imperfections during topology optimization is discussed.

Employing topology optimization for the design of archimats comes along with some issues. First, designing archimats towards specific effective properties is an inverse homogenization problem[16] and the design space in terms of possible microstructures to be considered is infinite-dimensional.[111,112] Second, the obtained

Table 1. Experimental investigations in compression exhibiting buckling and energy absorption across different scales.

	Scale		
Type	Micro (UC)	Meso (assembly)	Macro (structural component)
Metal foams closed cell		Uniaxial stress–strain behaviour in constrained and unconstrained conditions[121–125]	Global and local buckling of sandwich plates with metal foam core[126]
Metal or metal foam hollow spheres	Mechanical behaviour of single spheres[127,128]	Compressive properties of steel foam hollow sphere assemblies[129–131]	Global buckling of sandwich struts with steel foam hollow spheres in the core[132]
APM	Mechanical behaviour of single spheres[133]	Compression testing of uniform and graded assemblies[134,135]	
Pyramidal/truss	Mechanical behaviour of single pyramid[136]	Mechanical behaviour in compression[11,137–139]	Global buckling of structural elements reinforced with lattices[140]
Open-cell lattices		Mechanical behaviour of lattices in compression[9]	Buckling and failure in cellular- and lattice-reinforced columns[6,141]
Metal foams (open cell)		Buckling and crushing in compression[142,143]	
Honeycombs and chirals		Mechanical behaviour under compression[49,144,145]	Experimental crashworthiness and buckling[146]

UC: unit cell; APM: advanced pore morphology.

optimum design is sensitive to the initial choice of the microstructure.[103,110] and third, the optimization procedure can become computationally expensive[112] when many iterations are required until the optimum design is found.

Machine learning allows to address most of the issues arising with common topology optimization schemes[111,113] and has successfully been employed in the design of archimats,[101,107,111,112] alloys,[113] composite materials[114,115] and frame structures.[116,117] Machine learning can be employed for property prediction[107,116,118] as well as for optimizing the microstructure of archimats towards target applications.[107,111] In the design of archimats, it can be used for topology optimization,[101,111] for finding the optimum composition of UCs consisting of lattice members with different materials,[107] for designing archimat families[112] as well as for multiscale system design,[112] for example, to obtain functionally graded structures showing a distinct deformation. Machine learning techniques provide a numerically efficient way in the design of archimats but rely on a sufficiently large data set for training the underlying algorithms. This training data sets consist of different realizations of the arichmat's microstructure under consideration of the design variables and the information on the respective mechanical response of each realization. The training data set is often constructed by means of finite element simulations[101,111,113,114] and consequently requires some numerical effort. However, the number of realizations to be considered within a training data set is by orders of magnitudes smaller than the number of possible realizations[107,114] of the archimats microstructure. General overviews on the application of machine learning techniques in materials design and discovery can be found, for example, in Agrawal and Choudhary[119] and Liu et al.[120]

Experimental investigations

Experimental investigations on archimats vary across scales and material type. Usually used for material property characterization, detection of imperfection and investigation of failure propagation, these experimental methods can offer rich data onto which numerical simulations can be informed and calibrated and analytical models verified. Typically archimats, as porous materials, have shown a tendency to reduce in volume progressively under compression while absorbing energy. The aforementioned characteristics have been investigated experimentally for several materials from the UC level (micro) to assembly level (meso) up to studies on the macro level where the performance of archimats in composite configurations as aggregated in Table 1.

When an archimat comprises UC patterns in a periodic or random packing setting, such, advanced pore morphology aluminium foam, steel hollow spheres, cuboidal and pyramidal lattices in 3D and chiral, honeycomb and square planar patterns, understanding the mechanical response of the UC is imperative, to create adequate analytical models and numerical simulations. Such UCs can be described by a single sphere or pyramid of a number of struts connected on an apex or even more of these elements in spatial combinations to understand the interactions of the different packing configurations. Considering the load to be compressive, the experimental investigation can be undertaken in a small testing rig under displacement control, measuring load and displacement. Owing to the size of the cells, it might prove impractical to have clip gauge extensometer or strain

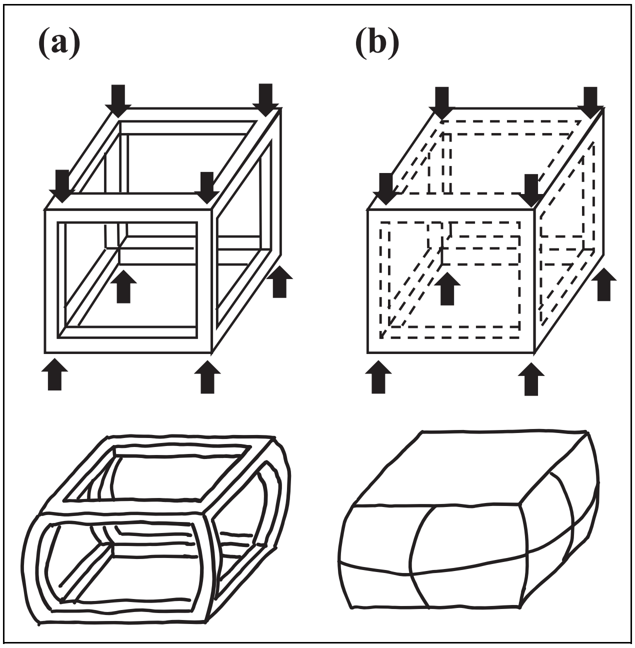

Figure 2. Hypothetical UCs under compressive loading: (a) open-cell buckling as struts buckle in in Euler type buckling and (b) closed UC, indicating cell wall buckling, which includes double curvature and thus more stable. UC: unit cell.

gauges, but non-contact X-ray tomography or surface digital image correlation (DIC) might be of use. It is expected to see some limited elastic compressive deformation locally followed by elastoplastic buckling of the ligaments or cell walls until the complete loss of strength and crushing of the UC as shown in Figure 2.

In the case of metal foams, where a single cell or a small number of them are not representative of the structure, meso (or assembly) scale investigation is needed. The same applies to adhesively bonded hollow spheres when the adhesive bonds can play a role in crushing of subsequent cells. According to the standard method for the compression of porous metals,[147] a mesoscale specimen should have at least 10 cells on average in each dimension. If the packing is regular (i.e. body-centered cubic [BCC], face-centered cubic [FCC] or hexagonal close packed [HCC]), measuring the external deformations using DIC techniques can suffice in capturing the onset of failure and its early propagation. Having said that, as ligaments or cell walls collapse, large deformations will not be captured by the DIC system, losing some of that data, which can be partially recovered from the cross head displacement monitoring.[130] For smaller specimens or when there is interest to map the internal structure of a metal foam, X-ray tomography can capture the internal 3D structure during quasi-static loading.[124] This can be done by having at least three sources and receivers in three different positions aiming at the specimen in the loading rig which is placed securely in a lead-reinforced room.

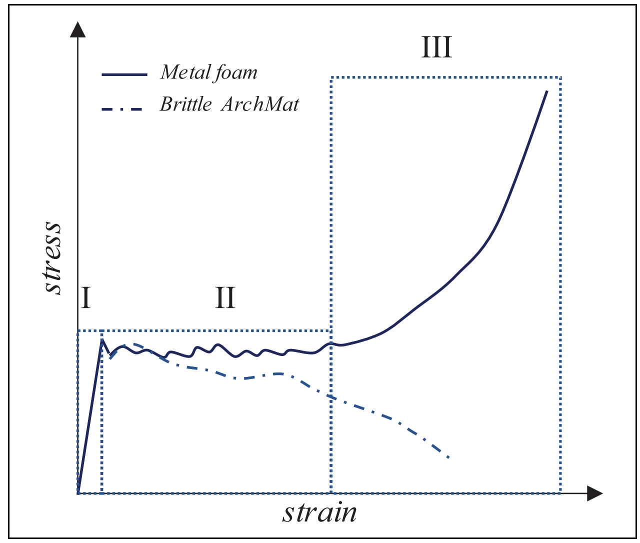

Figure 3. Typical stress–strain diagram of archimats in compression. Metal foams and metal foam hybrid assemblies will go through all three stages to densification while brittle archimats will not exhibit a plateau.[11]

The other case is having one source and receiver, spinning around the specimen under load.

The stress–strain response of porous archimats under uniaxial compression usually comprises three parts. A quasi-elastic region that is usually linear (stage I) followed by a plateau close to the first peak stress (stage II). The plateau is actually a succession of crush bands forming as deformation progresses in the assembly. According to the standard,[147] the plateau mean stress can be calculated as the average stress between 20% strain to 30 or 40% strain, depending when stage III begins. Once all the ligaments or cell walls in each crush band have collapsed, the stiffness of the specimen rises steeply as all the pores close and densification takes place. If the material is brittle and exhibits little or no cohesion, the plateau would be much smaller in extent and densification will not occur as the specimen will crumble at reduced load as depicted in Figure 3.

Buckling and postbuckling at a macroscale have been investigated when archimats are used within components of composite structural elements.[49,146] Using lattices, honeycombs or metal foams as cores in sandwich configurations or filling square tubes and cylinders with archimats, can provide the integrated structure with bending and buckling resistance, improved energy dissipation and crashworthiness. The presence of archimats can absorb strain energy upon compressive or combined loading and affect local buckling modes, by reducing the amplitude of the stress–strain snaking as the buckle pattern changes, thus increasing energy absorption. If archimats are exposed, DIC techniques can offer valuable insight with surface strain maps[6]; otherwise, the effect of archimats can be measured by comparing the strength, stiffness and deformation patterns to a control configuration, such as empty hollow sections of the same dimensions.[148]

Conclusions

In this article, an overview of analytical, numerical and experimental methods for investigating the buckling and postbuckling behaviour of archimats has been presented. Analytical methods can be split into two strategies: linear and geometrically non-linear methods. If the former is pursued, no insight into the postbuckling behaviour will be obtained. If the archimat is, however, to be investigated or even aimed to be manipulated in the postbuckling range, geometrically non-linear modelling is to be pursued. With the aid of geometrical models with few degrees of freedom, an archimats may even be tuned to follow certain desirable structural responses. AM may furthermore be useful to proof the concepts derived using geometrically non-linear models.

Numerical methods can be grouped into discrete and continuum modelling approaches. Within discrete models, the archimat is represented in its entire complexity. These models allow for a detailed investigation of the deformation and stress states at the microscale of the archimat but come with high computational requirements. Consequently, continuum models are employed for simulations of archimats within structural applications. Mainly, generalized continuum theories are used to describe the effective mechanical behaviour of the archimat at the macorscale. These theories introduce a material length scale into the constitutive relations and therefore allow to account, for example, for size effects or the buckling-induced localization of deformations observed for archimats.

In experimental methods, a scale and material type-dependent strategy is required to quantify behaviour from the UC level all the way to structural component level. When the internal structure needs to be identified, X-ray tomography methods can be used during testing; otherwise if deductions can be made from surface strains, then DICs techniques can be of use.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

ORCID iD

Christina Völlmecke https://orcid.org/0000-0003-4621-7400
Melanie Todt https://orcid.org/0000-0002-5599-6373
Stylianos Yiatros https://orcid.org/0000-0002-4803-6585

References

1. Estrin Y, Bréchet Y, Dunlop J, et al. *Architectured materials in nature and engineering. Archimats*. New York, NY: Springer, 2019.
2. Wadley HNG. Multifunctional periodic cellular metals. *Phil Trans Roy Soc A* 2006; 364(1838): 31–68.
3. Fleck NA, Deshpande VS and Ashby MF. Micro-architectured materials: past, present and future. *Proc Roy Soc A* 2010; 466(2121): 2495–2516.
4. Nezami F and Fuhr JP. Digitale Entwicklung und automatisierte CFK-Verstärkung additiv gefertigter Teile. *Lightweight Des* 2017; 10(6): 38–41.
5. Nazir A, Arshad AB and Jeng JY. Buckling and post-buckling behavior of uniform and variable-density lattice columns fabricated using additive manufacturing. *Materials* 2019; 12(21): 3539.
6. Nazir A and Jeng JY.Buckling behavior of additively manufactured cellular columns: experimental and simulation validation. *Mater Design* 2020; 186: 108349.
7. Nazir A, Abate KM, Kumar A, et al. A state-of-the-art review on types, design, optimization, and additive manufacturing of cellular structures. *Int J Adv Manuf Technol* 2019; 104(9–12): 3489–3510.
8. Bendsøe MP and Triantafyllidis N. Scale effects in the optimal design of a microstructured medium against buckling. *Int J Solids Struct* 1990; 26(7): 725–741.
9. Moongkhamklang P, Deshpande VS and Wadley HNG. The compressive and shear response of titanium matrix composite lattice structures. *Acta Mater* 2010; 58(8): 2822–2835.
10. Moongkhamklang P, Elzey DM and Wadley HNG. Titanium matrix composite lattice structures. *Compos Pt A Appl Sci Manuf* 2008; 39(2): 176–187.
11. Queheillalt DT and Wadley HNG. Cellular metal lattices with hollow trusses. *Acta Mater* 2005; 53(2): 303–313.
12. Rathbun HJ, Zok FW, Waltner SA, et al. Structural performance of metallic sandwich beams with hollow truss cores. *Acta Mater* 2006; 54(20): 5509–5518.
13. Gibson LJ and Ashby MF. *Cellular solids: structure and properties*. Cambridge: Cambridge University Press, 1999.
14. Deshpande VS, Ashby MF and Fleck NA. Foam topology: bending versus stretching dominated architectures. *Acta Mater* 2001; 49(6): 1035–1040.
15. Deshpande VS, Fleck NA and Ashby MF. Effective properties of the octet-truss lattice material. *J Mech Phys Solids* 2001; 49(8): 1747–1769.
16. Sigmund O. Materials with prescribed constitutive parameters: an inverse homogenization problem. *Int J Solids Struct* 1994; 31(17): 2313–2329.
17. Jalalpour M, Igusa T and Guest JK. Optimal design of trusses with geometric imperfections: accounting for global instability. *Int J Solids Struct* 2011; 48(21): 3011–3019.
18. Pasini D and Guest JK. Imperfect architected materials: mechanics and topology optimization. *MRS Bull* 2019; 44(10): 766–772.

19. Crosby AJ. Why should we care about buckling? *Soft Matter* 2010; 6(22): 5660–5660.
20. Hu N and Burgueño R. Buckling-induced smart applications: recent advances and trends. *Smart Mater Struct* 2015; 24(6): 063001.
21. Groh RMJ, Avitabile D and Pirrera A. Generalised path-following for well-behaved nonlinear structures. *Comput Methods Appl Mech Eng* 2018; 331: 394–426.
22. Kochmann DM and Bertoldi K. Exploiting microstructural instabilities in solids and structures: from metamaterials to structural transitions. *Appl Mech Rev* 2017; 69: 050801.
23. Grenested JL. Effective elastic behavior of some models for perfect cellular solids. *Int J Solids Struct* 1999; 36: 1471–1501.
24. Gibson LJ. Modelling the mechanical behavior of cellular materials. *Mater Sci Eng A* 1989; 110: 1–36.
25. Ashby MF. The properties of foams and lattices. *Phil Trans Roy Soc A* 2006; 364(1838): 15–30.
26. Thompson JMT and Hunt GW. *Elastic instability phenomena*. Hoboken, NJ: Wiley, 1984.
27. Zschernack C, Wadee MA and Völlmecke C. Nonlinear buckling of fibre-reinforced unit cells of lattice materials. *Compos Struct* 2016; 136: 217–228.
28. Ganzosch G, Todt M, Köllner A, et al. Experimental investigations on pre-stressed stayed columns on smaller length scales. In: *Proceedings of the eighth international conference on thin-walled structures*, Lisbon, Portugal, 24–27 July 2018, pp. 1–15.
29. Köllner A, Todt M, Ganzosch G, et al. Experimental and numerical investigation on pre-stressed lattice structures. *Thin-Walled Struct* 2019; 145: 106396.
30. Champneys AR, Dodwell TJ, Groh RM, et al. Happy catastrophe: recent progress in analysis and exploitation of elastic instability. *Front Appl Math Stat* 2019; 5: 34.
31. Cox BS, Groh RMJ, Avitabile D, et al. Modal nudging in nonlinear elasticity: tailoring the elastic post-buckling behaviour of engineering structures. *J Mech Phys Solids* 2018; 116: 135–149.
32. Reis PM. A perspective on the revival of structural (in) stability with novel opportunities for function: from buckliphobia to buckliphilia. *J Appl Mech* 2015; 82(11): 111001.
33. Shan S, Kang SH, Raney JR, et al. Multistable architected materials for trapping elastic strain energy. *Adv Mater* 2015; 27(29): 4296–4301.
34. Virgin L. Tailored buckling constrained by adjacent members. *Structures* 2018; 16: 20–26. Elsevier.
35. Smith M, Guan Z and Cantwell WJ. Finite element modelling of the compressive response of lattice structures manufactured using the selective laser melting technique. *Int J Mech Sci* 2013; 67: 28–41.
36. Tancogne-Dejean T, Spierings AB and Mohr D. Additively-manufactured metallic micro-lattice materials for high specific energy absorption under static and dynamic loading. *Acta Mater* 2016; 116: 14–28.
37. Geng X, Ma L, Liu C, et al. A FEM study on mechanical behavior of cellular lattice materials based on combined elements. *Mater Sci Eng A* 2018; 712: 188–198.
38. Li X, Lu Z, Yang Z, et al. Anisotropic in-plane mechanical behavior of square honeycombs under off-axis loading. *Mater Design* 2018; 158: 88–97.
39. Tankasala HC, Deshpande VS and Fleck NA. Tensile response of elastoplastic lattices at finite strain. *J Mech Phys Solids* 2017; 109: 307–330.
40. Demiray S, Becker W and Hohe J. Strain-energy based homogenisation of two- and three-dimensional hyperelastic solid foams. *J Mater Sci* 2005; 40: 5839–5844.
41. Pal RK, Ruzzene M and Rimoli JJ. A continuum model for nonlinear lattices under large deformations. *Int J Solids Struct* 2016; 96: 300–319.
42. Luxner MH, Stampfl J and Pettermann HE. Numerical simulations of 3D open cell structures – influence of structural irregularities on elasto-plasticity and deformation localization. *Int J Solids Struct* 2007; 44(9): 2990–3003.
43. Luxner MH, Stampfl J and Pettermann HE. Nonlinear simulations on the interaction of disorder and defects in open cell structures. *Comput Mater Sci* 2009; 47: 418–428.
44. Deshpande VS and Fleck NA. Collapse of truss core sandwich beams in 3-point bending. *Int J Solids Struct* 2001; 38(36–37): 6275–6305.
45. Forest S, Blazy JS, Chastel Y, et al. Continuum modeling of strain localization phenomena in metallic foams. *J Mater Sci* 2005; 40: 5903–5910.
46. Werner B, Todt M and Pettermann HE. Nonlinear finite element study of beams with elasto-plastic damage behavior in the post-buckling regime. *PAMM* 2019; 19: e201900248.
47. Schmidt I and Fleck NA. Ductile fracture of two-dimensional cellular structures. *Int J Fract* 2001; 111: 327–342.
48. Papka SD and Kyriakides S. In-plane compressive response and crushing of honeycomb. *J Mech Phys Solids* 1994; 42: 1499–1532.
49. Papka SD and Kyriakides S. Experiments and full-scale numerical simulations of in-plane crushing of a honeycomb. *Acta Mater* 1998; 46(8): 2765–2776.
50. Romijn NER and Fleck NA. The fracture toughness of planar lattices: imperfection sensitivity. *J Mech Phys Solids* 2007; 55: 2538–2564.
51. Alonso IQ and Fleck NA. Compressive response of a sandwich plate containing a cracked diamond-celled lattice. *J Mech Phys Solids* 2009; 57: 1545–1567.
52. Böhm HJ. *Mechanics of microstructured materials*. Wien: Springer Verlag, 2004.
53. Triantafyllidis N and Schraad MW. Onset of failure in aluminum honeycombs under general in-plane loading. *J Mech Phys Solids* 1998; 46: 1089–1124.
54. Luxner MH, Stampfl J and Pettermann HE. Finite element modeling concepts and linear analyses of 3D regular open cell structures. *J Mater Sci* 2005; 40: 5859–5866.
55. Liu L, Kamm P, García-Moreno F, et al. Elastic and failure response of imperfect three-dimensional metallic lattices: the

role of geometric defects induced by selective laser melting. *J Mech Phys Solids* 2017; 107: 160–184.
56. Ren X, Shen J, Tran P, et al. Design and characterisation of a tuneable 3D buckling-induced auxetic metamaterial. *Mater Design* 2018; 139: 336–342.
57. Bertoldi K, Reis PM, Willshaw S, et al. Negative Poisson's ratio behavior induced by an elastic instability. *Adv Mater* 2010; 22: 361–366.
58. Bluhm GL, Sigmund O, Wang F, et al. Nonlinear compressive stability of hyperelastic 2D lattices at finite volume fractions. *J Mech Phys Solids* 2020; 137: 103851.
59. Triantafyllidis N and Schnaidt WC. Comparison of microscopic and macroscopic instabilities in a class of two-dimensional periodic composites. *J Mech Phys Solids* 1993; 41: 1533–1565.
60. Gong L, Kyriakides S and Triantafyllidis N. On the stability of Kelvin cell foams under compressive loads. *J Mech Phys Solids* 2005; 53: 771–794.
61. Gong L and Kyriakides S. Compressive response of open cell foams. Part II: initiation and evolution of crushing. *Int J Solids Struct* 2005; 42: 1381–1399.
62. Thomsen CR, Wang F and Sigmund O. Buckling strength topology optimization of 2D periodic materials based on linearized bifurcation analysis. *Comput Methods Appl Mech Eng* 2018; 339: 115–136.
63. Osanov M and Guest JK. Topology optimization for architected materials design. *Ann Rev Mater Res* 2016; 46: 211–233.
64. Papka SD and Kyriakides S. In-plane biaxial crushing of honeycombs – part II: analysis. *Int J Solids Struct* 1999; 36: 4397–4423.
65. Crupi V, Kara E, Epasto G, et al. Static behavior of lattice structures produced via direct metal laser sintering technology. *Mater Design* 2017; 135: 246–256.
66. Ajdari A, Nayeb-Hashemi H and Vaziri A. Dynamic crushing and energy absorption of regular, irregular and functionally graded cellular structures. *Int J Solids Struct* 2011; 48: 506–516.
67. Campoli G, Borleffs MS, Yavari SA, et al. Mechanical properties of open-cell metallic biomaterials manufactured using additive manufacturing. *Mater Design* 2013; 49: 957–965.
68. Silva MJ and Giibson LJ. The effects of non-periodic microstructure and defects on the compressive strength of two-dimensional cellular solids. *Int J Mech Sci* 1997; 39: 549–563.
69. Ajdari A, Nayeb-Hashemi H, Canavan PK, et al. Effect of defects on elastic-plastic behavior of cellular materials. *Mater Sci Eng A* 2008; 487: 558–567.
70. Haghpanah B, Papadopoulos J, Mousanezhad D, et al. Buckling of regular, chiral and hierarchical honeycombs under a general macroscopic stress state. *Proc Roy Soc A* 2014; 470: 20130856.
71. Tekoglu C and Onck PR. Size effects in the mechanical behavior of cellular materials. *J Mater Sci* 2005; 40: 5911–5917.
72. Wang XT, Li XW and Ma L. Interlocking assembled 3D auxetic cellular structures. *Mater Design* 2016; 99: 467–476.
73. Arabnejad S and Pasini D. Mechanical properties of lattice materials via asymptotic homogenization and comparison with alternative homogenization methods. *Int J Mech Sci* 2013; 77: 249–262.
74. Lakes RS. Experimental microelasticity of two porous solids. *Int J Solids Struct* 1986; 22: 55–63.
75. Andrews EW, Gioux G, Onck P, et al. Size effects in ductile cellular solids. Part II: experimental results. *Int J Mech Sci* 2001; 43: 701–713.
76. Yoder M, Thompson L and Summers J. Size effects in lattice structures and a comparison to micropolar elasticity. *Int J Solids Struct* 2018; 143: 245–261.
77. Khakalo S and Niiranen J. Lattice structures as thermoelastic strain gradient metamaterials: evidence from full-field simulations and applications to functionally step-wise-graded beams. *Compos B Eng* 2019; 177: 107224.
78. Dos Reis F and Ganghoffer JF. Equivalent mechanical properties of auxetic lattices from discrete homogenization. *Comput Mater Sci* 2012; 51: 314–321.
79. Dos Reis F and Ganghoffer JF. Construction of micropolar continua from the asymptotic homogenization of beam lattices. *Comput Struct* 2012; 112–113: 354–363.
80. ElNady K, Goda I and Ganghoffer JF. Computation of the effective nonlinear mechanical response of lattice materials considering geometrical nonlinearities. *Comput Mech* 2016; 58: 957–979.
81. El Nady K, Dos Reis F and Ganghoffer JF. Computation of the homogenized nonlinear elastic response of 2D and 3D auxetic structures based on micropolar continuum models. *Compos Struct* 2017; 170: 271–290.
82. Glaesener RN, Lestringant C, Telgen B, et al. Continuum models for stretching- and bending-dominated periodic trusses undergoing finite deformations. *Int J Solids Struct* 2019; 171: 117–134.
83. Auffray N, Dirrenberger J and Rosi G. A complete description of bi-dimensional anisotropic strain-gradient elasticity. *Int J Solids Struct* 2015; 69–70: 195–206.
84. Lombardo M and Askes H. Higher-order gradient continuum modelling of periodic lattice materials. *Comput Mater Sci* 2012; 52(1): 204–208.
85. Biswas R, Poh L and Shedbale A. A micromorphic computational homogenization framework for auxetic tetra-chiral structures. *J Mech Phys Solids* 2020; 135: 103801.
86. Noor AK. Continuum modeling for repetitive lattice structures. *Appl Mech Rev* 1988; 41: 285–296.
87. Fatemi J, Van Keulen F and Onck PR. Generalized continuum theories: application to stress analysis in bone. *Meccanica* 2002; 37: 385–396.
88. Forest S and Sievert R. Nonlinear microstrain theories. *Int J Solids Struct* 2006; 43: 7224–7245.
89. Kumar RS and McDowell DL. Generalized continuum modeling of 2-D periodic cellular solids. *Int J Solids Struct* 2004; 41: 7399–7422.
90. Coenen EWC, Kouznetsova VG and Geers MGD. Multi-scale continuous–discontinuous framework for

computational-homogenization–localization. *J Mech Phys Solids* 2012; 60(8): 1486–1507.

91. Kouznetsova V, Geers MGD and Brekelmans WAM. Multi-scale constitutive modelling of heterogeneous materials with a gradient-enhanced computational homogenization scheme. *Int J Numer Methods Eng* 2002; 54(8): 1235–1260.
92. Kouznetsova VG, Geers MGD and Brekelmans WAM. Multi-scale second-order computational homogenization of multi-phase materials: a nested finite element solution strategy. *Comput Methods Appl Mech Eng* 2004; 193: 5525–5550.
93. Phlipot GP and Kochmann DM. A quasicontinuum theory for the nonlinear mechanical response of general periodic truss lattices. *J Mech Phys Solids* 2019; 124: 758–780.
94. Vigliotti A and Pasini D. Linear multiscale analysis and finite element validation of stretching and bending dominated lattice materials. *Mech Mater* 2012; 46: 57–68.
95. Vigliotti A, Deshpande VS and Pasini D. Non linear constitutive models for lattice materials. *J Mech Phys Solids* 2014; 64: 44–60.
96. Vernerey FJ and Kabiri M. An adaptive concurrent multiscale method for microstructured elastic solids. *Comput Methods Appl Mech Eng* 2012; 241–244: 52–64.
97. Ghosh S, Lee K and Raghavan P. A multi-level computational model for multi-scale damage analysis in composite and porous materials. *Int J Solids Struct* 2001; 38: 2335–2385.
98. Huang J and Wang TL. Buckling analysis of large and complex structures by using substructuring techniques. *Comput Struct* 1993; 46: 845–850.
99. Powell SM, Kennedy D and Williams FW. Efficient multilevel substructuring with constraints for buckling and vibration analysis of prismatic plate assemblies. *Int J Mech Sci* 1997; 39: 795–805.
100. Wu Z, Xia L, Wang S, et al. Topology optimization of hierarchical lattice structures with substructuring. *Comput Methods Appl Mech Eng* 2019; 345: 602–617.
101. Mao Y, He Q and Zhao X. Designing complex architectured materials with generative adversarial networks. *Sci Adv* 2020; 6: eaaz4169.
102. Vangelatos Z, Komvopoulos K and Grigoropoulos C. Regulating the mechanical behavior of metamaterial microlattices by tactical structure modification. *J Mech Phys Solids* 2020; 144: 104112.
103. Zhang Y, Xiao M, Li H, et al. Topology optimization of material microstructures using energy-based homogenization method under specified initial material layout. *J Mech Sci Technol* 2019; 33: 677–693.
104. Wang K, Chang YH, Chen YW, et al. Designable dual-material auxetic metamaterials using three-dimensional printing. *Mater Design* 2015; 67: 159–164.
105. Yang L, Harrysson O, West H, et al. Mechanical properties of 3D re-entrant honeycomb auxetic structures realized via additive manufacturing. *Int J Solids Struct* 2015; 69–70: 475–490.
106. Vangelatos Z, Gu GX and Grigoropoulos CP. Architected metamaterials with tailored 3D buckling mechanisms at the microscale. *Extreme Mech Lett* 2019; 33: 100580.
107. Kulagin R, Beygelzimer Y, Estrin Y, et al. Architectured lattice materials with tunable anisotropy: design and analysis of the material property space with the aid of machine learning. *Adv Eng Maters* 2020; 22: 2001069. online first.
108. Lee W, Kang DY, Song J, et al. Controlled unusual stiffness of mechanical metamaterials. *Sci Rep* 2016; 6: 20312.
109. Ziemke P, Frenzel T, Wegener M, et al. Tailoring the characteristic length scale of 3D chiral mechanical metamaterials. *Extreme Mech Lett* 2019; 32: 100553.
110. Amstutz S, Giusti SM, Novotny AA, et al. Topological derivative for multi-scale linear elasticity models applied to the synthesis of microstructures. *Int J Numer Methods Eng* 2010; 84: 733–756.
111. Kollmann HT, Abueidda DW, Koric S, et al. Deep learning for topology optimization of 2D metamaterials. *Mater Design* 2020; 196: 109098.
112. Wang L, Chan YC, Ahmed F, et al. Deep generative modeling for mechanistic-based learning and design of metamaterial systems. *Comput Methods Appl Mech Eng* 2020; 372: 113377.
113. Liu R, Kumar A, Chen Z, et al. A predictive machine learning approach for microstructure optimization and materials design. *Sci Rep* 2015; 5: 11551.
114. Gu GX, Chen CT, Richmond DJ, et al. Bioinspired hierarchical composite design using machine learning: simulation, additive manufacturing, and experiment. *Mater Horiz* 2018; 5: 939–945.
115. Xue T, Wallin TJ, Menguc Y, et al. Machine learning generative models for automatic design of multi-material 3D printed composite solids. *Extreme Mech Lett* 2020; 41: 100992.
116. Papadrakakis M, Lagaros ND and Tsompanakis Y. Structural optimization using evolution strategies and neural networks. *Comput Methods Appl Mech Eng* 1998; 156: 309–333.
117. Papadrakakis M and Lagaros ND. Reliability-based structural optimization using neural networks and Monte Carlo simulation. *Comput Methods Appl Mech Eng* 2002; 191: 3491–3507.
118. Zhang Z and Friedrich K. Artificial neural networks applied to polymer composites: a review. *Compos Sci Technol* 2003; 63: 2029–2044.
119. Agrawal A and Choudhary A. Perspective: materials informatics and big data: realization of the "fourth paradigm" of science in materials science. *APL Mater* 2016; 4: 053208.
120. Liu Y, Zhao T, Ju W, et al. Materials discovery and design using machine learning. *J Materiomics* 2017; 3: 159–177.
121. McCullough KYG, Fleck NA and Ashby MF. Uniaxial stress-strain behaviour of aluminium alloy foams. *Acta Mater* 1999; 47(8): 2323–2330.
122. Hanssen AG, Hopperstad OS, Langseth M, et al. Validation of constitutive models applicable to aluminium foams. *Int J Mech Sci* 2002; 44(2): 359–406.

123. Bastawros AF, Bart-Smith H and Evands AG. Experimental analysis of deformation mechanisms in a closed-cell aluminum alloy foam. *J Mech Phys Solids* 2000; 48(2): 301–322.
124. Mukherjee M, Kolluri M, Garcia-Moreno F, et al. Strain hardening during constrained deformation of metal foams – effect of shear displacement. *Scr Mater* 2009; 61(7): 752–755.
125. Saadatfar M, Mukherjee M, Madadi M, et al. Structure and deformation correlation of closed-cell aluminium foam subject to uniaxial compression. *Acta Mater* 2012; 60(8): 3604–3615.
126. Jasion P, Magnucka-Blandzi E, Szyc W, et al. Global and local buckling of sandwich circular and beam-rectangular plates with metal foam core. *Thin-Walled Struct* 2012; 61: 154–161.
127. Fallet A, Lhuissier P, Salvo L, et al. Mechanical behaviour of metallic hollow spheres foam. *Adv Eng Mater* 2008; 10(9): 858–862.
128. Shorter R, Smith JD, Coveney VA, et al. Axial compression of hollow elastic spheres. *J Mech Mater Struct* 2010; 5(5): 693–705.
129. Friedl O, Motz C, Peterlik H, et al. Experimental investigation of mechanical properties of metallic hollow sphere structures. *Metall Mater Trans B* 2008; 39(1): 135–146.
130. Yiatros S, Marangos O, Votsis RA, et al. Compressive properties of granular foams of adhesively bonded steel hollow sphere blocks. *Mech Res Commun* 2018; 94: 13–20.
131. Smith BH, Szyniszewski S, Hajjar JF, et al. Characterization of steel foams for structural components. *Metals* 2012; 2(4): 399–410.
132. Yiatros S, Marangos O and Brennan FP. Characterisation of thin walled sandwich structures comprising steel hollow spheres for the core. In: Duarte I, Vesenjak M and Ren Z (eds) *InCell 2019: book of abstracts of the international conference on multifunctional cellular materials*. Aveiro: UA Editora Universidade de Aveiro, 2019, p. 60.
133. Stöbener K, Lehmhus D, Avalle M, et al. Aluminum foam-polymer hybrid structures (APM aluminum foam) in compression testing. *Int J Solids Struct* 2008; 45(21): 5627–5641.
134. Hohe J, Hardenacke V, Fascio V, et al. Numerical and experimental design of graded cellular sandwich cores for multi-functional aerospace applications. *Mater Design* 2012; 39: 20–32.
135. Skarlatos D and Yiatros S. Deformation monitoring of materials under stress in laboratory experiments. *ISPRS Ann Photogramm Remote Sens Spatial Inf Sci* 2016; 3(5): 35–41.
136. Yin S, Wu L, Ma L, et al. Pyramidal lattice sandwich structures with hollow composite trusses. *Compos Struct* 2011; 93(12): 3104–3111.
137. Lim JH and Kang KJ. Mechanical behavior of sandwich panels with tetrahedral and Kagome truss cores fabricated from wires. *Int J Solids Struct* 2006; 43(17): 5228–5246.
138. Li M, Wu L, Ma L, et al. Mechanical response of all-composite pyramidal lattice truss core sandwich structures. *J Mater Sci Technol* 2011; 27(6): 570–576.
139. Dong L, Deshpande V and Wadley H. Mechanical response of Ti–6Al–4V octet-truss lattice structures. *Int J Solids Struct* 2015; 60–61: 107–124.
140. Chen L, Fan H, Sun F, et al. Improved manufacturing method and mechanical performances of carbon fiber reinforced lattice-core sandwich cylinder. *Thin-Walled Struct* 2013; 68: 75–84.
141. Sun F, Lai C and Fan H. Failure mode maps for composite anisogrid lattice sandwich cylinders under fundamental loads. *Compos Sci Technol* 2017; 152: 149–158.
142. Gaitanaros S and Kyriakides S. On the effect of relative density on the crushing and energy absorption of open-cell foams under impact. *Int J Impact Eng* 2015; 82: 3–13.
143. Gong L and Kyriakides S. On the crushing stress of open cell foams. *J Appl Mech* 2006; 73: 807–814.
144. Wilbert A, Jang WY, Kyriakides S, et al. Buckling and progressive crushing of laterally loaded honeycomb. *Int J Solids Struct* 2011; 48(5): 803–816.
145. Shim J, Shan S, Košmrlj A, et al. Harnessing instabilities for design of soft reconfigurable auxetic/chiral materials. *Soft Matter* 2013; 9(34): 8198–8202.
146. Sun F, Lai C, Fan H, et al. Crushing mechanism of hierarchical lattice structure. *Mech Mater* 2016; 97: 164–183.
147. Standard I. ISO 13314: 2011 (E) 2011 Mechanical testing of metals-ductility testing-compression test for porous and cellular metals. *Ref number ISO*; 13314(13314): 1–7.
148. Sun G, Li S, Liu Q, et al. Experimental study on crashworthiness of empty/aluminum foam/honeycomb-filled CFRP tubes. *Compos Struct* 2016; 152: 969–993.