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# Quantile Risk–Return Trade-Off

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**Abstract:** We investigate the risk–return trade-off on the US and European stock markets. We investigate the non-linear risk–return trade-off with a special eye to the tails of the stock returns using quantile regressions. We first consider the US stock market portfolio. We find that the risk–return trade-off is significantly positive at the upper tail (0.9 quantile), where the upper tail is large positive excess returns. The positive trade-off is as expected from asset pricing models. For the lower tail (0.1 quantile), that is for large negative stock returns, the trade-off is significantly negative. Additionally, for the median (0.5 quantile), the risk–return trade-off is insignificant. These results are recovered for the US industry portfolios and for Eurozone stock market portfolios.

**Keywords:** risk–return trade-off; quantile regressions; VIX; stock markets

**JEL Classification:** C22; G12; G15



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## 1. Introduction

In this paper we investigate the risk–return trade-off on the US and European stock markets. The contribution of this paper is to investigate the non-linear risk–return trade-off with a special eye to the tails of the stock returns using quantile regressions.

The previous literature has analyzed the risk–return trade-off using the volatility index from the Chicago Board of Options Exchange (VIX) to measure the risk of the US stock market, cf. Adrian et al. (2019a, 2019b); Bansal et al. (2020).

Adrian et al. (2019b) consider the non-linear risk–return trade-off for 11 US industry portfolios and bonds. Their measure if risk is the VIX volatility index. Adrian et al. (2019a) consider the non-linear risk–return trade-off for 30 intentional stock markets and bond. The risk measure is the VIX volatility index. They also consider the cross-sectional relation between the VIX risk premiums and macroeconomic variables. Bansal et al. (2020) create stock portfolios based on their VIX betas. The long-minus-short portfolios is regressed on a dummy variable for large levels of the VIX index (above the 0.8 quantile).

Kanas (2013) detects a significantly positive risk–return relation for the S&P 500 market index when the squared implied volatility index (VIX) is allowed for as an exogenous variable in the generalized autoregressive conditional heteroskedasticity (GARCH(1,1)) conditional variance equation. Allen et al. (2013) also find a significant relationship between the S&P 500 market index and the VIX using nonparametric methods. Adrangi et al. (2019) investigate the relationship between four large stock market indices and VIX. Empirical findings on the risk–return trade-off vary greatly in terms of the sign and significance across markets and time span. See for example the work of Baillie and De Gennaro (1990); Nelson (1991); Chan et al. (1992); Campbell and Hentschel (1992), Glosten et al. (1993); Corhay and Rad (1994); Theodossiou and Lee (1995); Bali (2008); Guo and Neely (2008); Bali et al. (2009); Bali and Zhou (2016), among others.

We build on the previous literature and also consider a non-linear risk–return trade-off. However, we use quantile regressions to analyze how the current stock excess returns depend on the lagged VIX volatility index. This allows us to investigate if the trade-off differs across quantiles, and we look closer at the differences between the lower tail, the median, and the upper tail. Our hypothesis is that the risk–return trade-off differs between the lower tail, the median, and the upper tail. We expect the investors to react differently to risk when the returns are low and when they are high, which is similar to differences in the risk–return trade-off in expansions and recessions.

The empirical analysis first investigates the risk–return trade-off for the US market portfolio. For the upper tail (0.9 quantile), that is the large positive excess returns, the risk–return trade-off is significantly positive. Here the empirical findings are in accordance with our expectations from asset pricing models such as the capital asset pricing model (CAPM). In contrast, the empirical risk–return trade-off is significantly negative for the lower tail (0.1 quantile). Additionally, for the median (0.5 quantile) the risk–return trade-off does not exist, which is similar to the findings using linear OLS. The qualitative same results also apply for US industry portfolios and for nine Eurozone stock markets.

The remaining part of the paper is structured as follows. Section 2 describes the data, while Section 3 contains the econometric method. The empirical results are provided in Section 4, while Section 5 concludes.

## 2. Data

Our data set was based on daily observations and the sample period begins on 1 January 1990 for the US and on 1 January 1999 for Europe and ends on 30 September 2020.

In Section 2.1 we describe the stock return data and in Section 2.2 the risk measures.

### *2.1. Stock Returns*

We used stock excess returns above the risk free interest rate.

For the US we analyzed the market portfolio and ten industry portfolios; consumer nondurables (NoDur), consumer durables (Durbl), manufacturing (Manuf), oil, gas, and coal extraction and products (Enrgy), business equipment (HiTec), telephone and television transmission (Telcm), wholesale, retail, and some services (Shops), healthcare, medical equipment, and drugs (Hlth), utilities (Utils), and other (Other). The US risk free interest rate is the 1-month Treasury bill rate.[1]

We considered the market indices from nine Eurozone countries: Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, the Netherlands, and Spain.[2] We used the six month euro London inter-bank offered rate (LIBOR) interest rate as the risk free interest rate.

Table 1 shows the summary statistics for the excess stock returns. The average excess stock returns were very small and highly variable. The excess returns were close to symmetric (small, but negative skewness) while they had fat tails (large kurtosis).

### *2.2. Risk Measures*

For the US stock indices, we used the VIX volatility index as the main risk variable. The VIX is based on option volatility with the S&P 500 as the underlying.[3]

For the Eurozone stock markets, we used the European volatility index, VSTOXX, which is based on option volatility with the EURO STOXX 50 as the underlying. The EURO STOXX 50 covered 50 stocks from the nine Eurozone countries under investigation.

Further, we used the conditional skewness and kurtosis for the US market index, estimated using the GARCH-M model with the skewed generalized *t* (SGT) distribution developed by Savva and Theodossiou (2018).

Table 1 shows summary statistics for the risk measures. The VIX and VSTOXX had similar characteristics, yet the VIX had fatter tails. They were also highly correlated, with a correlation coefficient of 0.91 for their common sample period. The VIX and VSTOXX were both stationary.

**Table 1.** Descriptive statistics.

| | Mean | Median | Maximum | Minimum | Std. Dev. | Skewness | Kurtosis |
|---|---|---|---|---|---|---|---|
| US MKT | 0.03 | 0.07 | 11.35 | −12 | 1.15 | −0.26 | 13.63 |
| NoDur | 0.04 | 0.06 | 10.24 | −9.87 | 0.95 | −0.19 | 13.49 |
| Durbl | 0.05 | 0.06 | 15.03 | −14.43 | 1.56 | −0.12 | 10.5 |
| Manuf | 0.05 | 0.08 | 10.83 | −11.11 | 1.18 | −0.29 | 12.76 |
| Enrgy | 0.04 | 0.03 | 19.33 | −19.73 | 1.59 | −0.21 | 17.94 |
| HiTec | 0.06 | 0.12 | 16.04 | −13.18 | 1.61 | 0.16 | 9.61 |
| Telcm | 0.04 | 0.06 | 14.5 | −9.67 | 1.3 | 0.08 | 12.5 |
| Shops | 0.05 | 0.08 | 10.99 | −10.61 | 1.17 | −0.07 | 9.68 |
| Hlth | 0.05 | 0.07 | 11.1 | −9.74 | 1.16 | −0.19 | 8.7 |
| Utils | 0.04 | 0.07 | 14.43 | −11.61 | 1.07 | 0.13 | 22.09 |
| Other | 0.04 | 0.07 | 12.24 | −13.38 | 1.38 | −0.15 | 14.98 |
| Belgium | 0 | 0.01 | 8.23 | −14.39 | 1.17 | −0.6 | 12.62 |
| Finland | 0 | 0 | 15.33 | −18.26 | 1.77 | −0.42 | 11.77 |
| France | 0 | 0.03 | 9.91 | −12.28 | 1.28 | −0.29 | 9.43 |
| Germany | 0 | 0.05 | 16.04 | −9.83 | 1.23 | −0.05 | 13.1 |
| Ireland | 0 | 0.01 | 9.09 | −13.38 | 1.32 | −0.76 | 11.18 |
| Italy | −0.01 | 0.01 | 10.47 | −17.43 | 1.36 | −0.69 | 13.19 |
| Luxemburg | −0.01 | 0 | 10.1 | −6.84 | 1.1 | −0.27 | 8.7 |
| Netherland | 0 | 0.03 | 9.29 | −10.66 | 1.25 | −0.42 | 9.83 |
| Spain | −0.01 | 0.03 | 11.74 | −14.2 | 1.31 | −0.38 | 10.9 |
| VIX | 19.42 | 17.38 | 82.69 | 9.14 | 8.12 | 2.22 | 11.37 |
| Skewness | −0.33 | −0.34 | 0.36 | −1.13 | 0.16 | −0.02 | 3.87 |
| Kurtosis | 4.34 | 4.32 | 5.3 | 4.23 | 0.1 | 2.18 | 13.4 |
| VSTOXX | 23.94 | 22.04 | 87.51 | 10.68 | 9.75 | 1.7 | 7.11 |

Notes: The table shows summary statistics for the daily excess returns for the US market, the US industry portfolios, the Eurozone stock markets, and for the risk measures, VIX, skewness, kurtosis, and VSTOXX. The sample period is 1990–2020 for the US and 1999–2020 for the Eurozone.

## 3. Econometric Method

For the linear risk–return trade-off the current stock excess returns depends on the lagged risk measure, here the volatility index. The linear model implies that the influence of risk is identical for all levels of the excess stock returns. Instead, we are interested in the non-linear risk–return trade-off, where the trade-off is allowed to vary when the stock excess returns are large (positive, upper tail) and small (negative, lower tail). The quantile regression model is especially suited to investigate this particular non-linear risk–return format. The advantage of the quantile regression is that it allows us to investigate if and how the risk–return trade-off differs between the lower tail, the median, and the upper tail.

We consider the quantile risk–return trade-off between excess returns and lagged risk and we include lagged excess returns to account for any autocorrelation.[4]

$$R_{i,t} = c_{i,0} + c^{\tau}_{i,1} R^{\tau}_{i,\,t-1} + c^{\tau}_{i,2} VI^{\tau}_{t-1} + \varepsilon^{\tau}_{t} \quad (1)$$

where $R_{it}$ is the return on stock $i$ at time $t$, $VI^{\tau}_{t-1}$ is the relevant volatility index at time $t - 1$ for the $\tau$-quantile, and $\varepsilon^{\tau}_{t}$ is the error term. Like in Adrian et al. (2019a), we use the lagged risk measure because we investigate the relationship between future returns and current risk. We consider nine quantiles, namely $\tau = \{0.1,\ 0.2, \cdots, 0.9\}$ and pay special attention to the lower tail ($\tau = 0.1$), the upper tail ($\tau = 0.9$), and the median ($\tau = 0.5$). We use bootstrapped standard errors.

## 4. Empirical Analysis

In Section 4.1 we discuss the risk–return trade-off for the US market portfolio, while Section 4.2 is concerned with the industry portfolios. Section 4.3 analyzes the Eurozone countries. Various extensions are contained in Section 4.4 (skewness and kurtosis effects), and Section 4.5 (subsample analysis).

### *4.1. Market Portfolio*

In Table 2, we considered the risk–return trade-off for the US market portfolio. First, for comparison we show the linear ordinary least squares (OLS) results. Here, we see that the VIX had no bearing on the market portfolio return. The VIX coefficient was close to zero and insignificant. The lagged market return had a negative and significant coefficient. Still, the R-squared was close to zero. This is similar to the findings in Adrian et al. (2019b).

**Table 2.** US market portfolio.

| Quantile | OLS | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|---|---|---|---|---|---|---|---|---|---|---|
| cons | −0.03 | 0.44 *** | 0.32 *** | 0.19 *** | 0.09 | 0.00 | −0.08 * | −0.16 *** | −0.25 *** | −0.31 *** |
| Mkt(−1) | −0.06 ** | 0.07 ** | 0.05 ** | 0.01 | −0.01 | −0.04 ** | −0.05 *** | −0.06 *** | −0.05 *** | −0.07 *** |
| VIX(−1) | 0.00 | −0.09 *** | −0.05 *** | −0.03 *** | −0.01 *** | 0.00 * | 0.02 *** | 0.04 *** | 0.06 *** | 0.08 *** |
| Pseudo R2 | 0.00 | 0.12 | 0.05 | 0.02 | 0.00 | 0.00 | 0.01 | 0.03 | 0.07 | 0.16 |
| Slope equality test | | | | | | 1067.3 *** | | | | |

Notes: The table shows the estimation results from the OLS regression using Newey and West (1987) standard errors and the quantile regressions using bootstrapped standard errors. The LHS variable is the US market excess return and the RHS variables are its lag and the lagged VIX. The slope equality test shows the chi-square test statistic for identical slope coefficients at the 0.1, 0.5, and 0.9 quantiles. */**/*** indicates significance at the 10%/5%/1% level.

The results were quite different, when we consider the quantile regressions. At the lower quantiles, the VIX coefficient was negative implying a negative risk–return trade-off. On the other hand, at the upper quantiles, the VIX coefficient is positive, implying a positive risk–return trade-off. The slope equality test shows that the coefficients were significantly different for the 0.1, median, and 0.9 quantiles. The coefficient to the lagged market return itself also varied across quantiles, going from positive for lower quantiles to negative at upper quantiles.

Further, the explanatory power varies across the quantiles. The pseudo *R*-squared was high for the lowest and the highest quintiles and decreased towards the median, where it was around zero. The pseudo R-squared for the 0.1 quantile was 0.12, while it was 0.16 for the 0.9 quantile.

The results for the median resemble those from the linear model. Both the linear model and the median quantile regression results resembled an average across the other quantiles. Adrian et al. (2019b) also found that the risk–return trade-off is non-linear, but not using the quantile regression method.

Figure 1 shows the coefficients across the quantiles together with the 95% confidence bands. The coefficient for the lagged return was monotonically decreasing across the quantiles, while the coefficient for the lagged VIX was monotonically increasing across the quantiles. The confidence band around the VIX coefficient was very tight while it was wider for the lagged return coefficient.

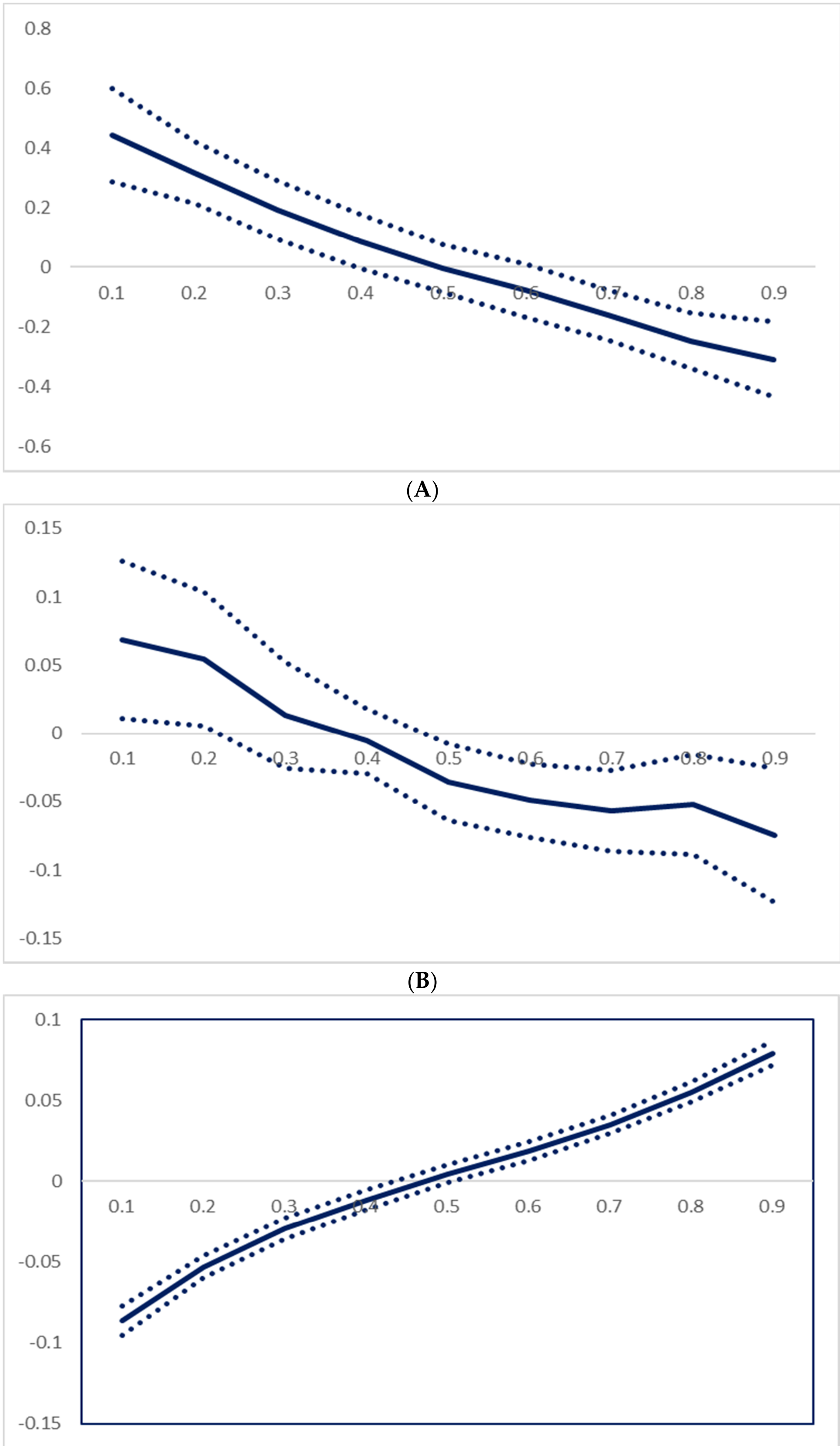



**Figure 1.** Quantile coefficients for the US market portfolio, (**A**): constant; (**B**): lagged return; (**C**): lagged VIX; notes: the figure shows the estimated coefficients from Table 2.

In Table 3, we considered the risk–return trade-off without the lagged excess return, that is for $c_{i,1}^{\tau} = 0$. We include this analysis in order to investigate if the differences across quantiles for the volatility index were caused by the changes of opposite direction for the lagged return. The risk–return trade-off results hardly changed, so that the coefficient to the volatility index were about the same as before and the pseudo R-squared remained the same. For this reason, we continued the following analysis with the lagged return included in the quantile regressions.

**Table 3.** US market portfolio.

| Quantile | OLS | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|---|---|---|---|---|---|---|---|---|---|---|
| cons | −0.07 | 0.40 *** | 0.35 *** | 0.21 *** | 0.11 ** | 0.00 | −0.08 * | −0.21 *** | −0.34 *** | −0.39 *** |
| VIX(−1) | 0 | −0.09 *** | −0.06 *** | −0.03 *** | −0.01 *** | 0.00 | 0.02 *** | 0.04 *** | 0.06 *** | 0.09 *** |
| Pseudo R2 | 0 | 0.13 | 0.06 | 0.02 | 0 | 0.00 | 0.01 | 0.03 | 0.08 | 0.17 |
| Slope equality test | | | | | | 832.1 *** | | | | |

Notes: The table shows the estimation results from the OLS regression using Newey and West (1987) standard errors and the quantile regressions using bootstrapped standard errors. The LHS variable is the US market excess return and the RHS variable is the lagged VIX. The slope equality test shows the chi-square test statistic for identical slope coefficients at the 0.1, 0.5, and 0.9 quantiles. */**/*** indicates significance at the 10%/5%/1% level.

The results imply that when the excess market returns are large and positive, then the larger the risk as measured by the VIX volatility index is, then the larger will the future excess return be. This is the positive risk–return relationship that we would expect from the asset pricing models such as CAPM. However, when the excess return is small and negative, then the risk–return trade-off is negative, the opposite to what we would expect from asset pricing models. The two very different results for the upper and lower tails explain the results for the median and the liner model, namely that the two effects cancel out and leave an insignificant effect from the risk, i.e., from the volatility index.

### *4.2. Industry Portfolios*

Now, we examine if the results for the overall US market portfolio also hold for the 10 industry portfolios. Table 4 shows that the results are qualitatively identical across the industry portfolios to the overall market portfolio. Additionally, the industry portfolios had qualitative similar risk–return behavior.

**Table 4.** US industry portfolios.

| Industry | Quantile | 0.1 | 0.5 | 0.9 | Equality Test |
|---|---|---|---|---|---|
| NoDur | cons | 0.09 * | −0.02 | 0.04 | |
| | R(−1) | 0.04 | −0.03 ** | −0.05 ** | |
| | VIX(−1) | −0.06 *** | 0.00 ** | 0.05 *** | |
| | Pseudo R2 | 0.08 | 0.00 | 0.10 | 694.1 ** |
| Durbl | cons | 0.05 | 0.03 | −0.10 | |
| | R(−1) | 0.07 *** | 0.02 | 0.03 | |
| | VIX(−1) | −0.09 *** | 0.00 | 0.10 *** | |
| | Pseudo R2 | 0.09 | 0.00 | 0.11 | 630.5 *** |
| Manuf | cons | 0.32 *** | 0.04 | −0.23 *** | |
| | R(−1) | 0.07 ** | −0.01 | −0.02 | |
| | VIX(−1) | −0.08 *** | 0.00 | 0.08 *** | |
| | Pseudo R2 | 0.10 | 0.00 | 0.14 | 1005.7 *** |
| Enrgy | cons | −0.11 | −0.04 | 0.16 * | |
| | R(−1) | 0.04 | −0.03 * | −0.06 ** | |
| | VIX(−1) | −0.08 *** | 0.00 | 0.08 *** | |
| | Pseudo R2 | 0.06 | 0.00 | 0.08 | 488.0 *** |

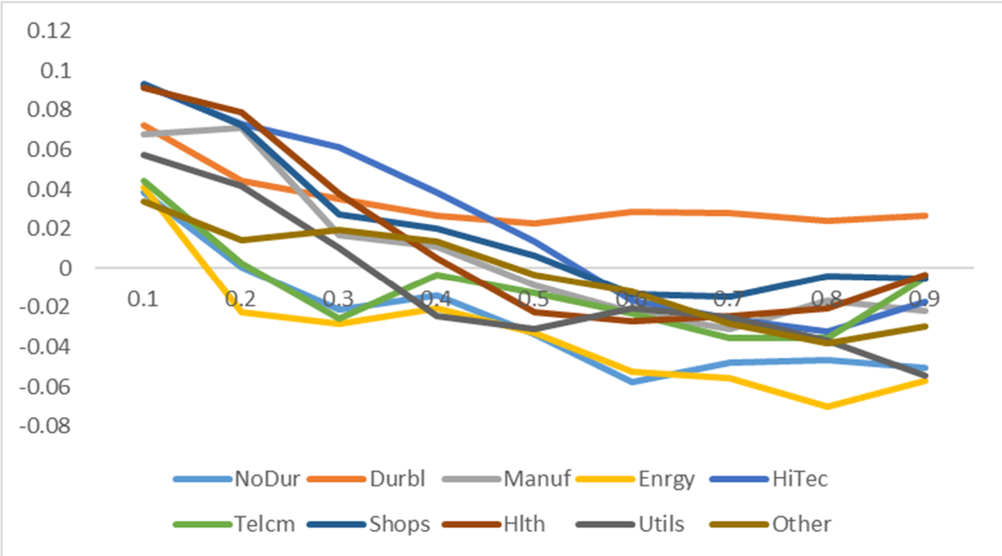
**Table 4.** *Cont.*

| Industry | Quantile | 0.1 | 0.5 | 0.9 | Equality Test |
|---|---|---|---|---|---|
| HiTec | cons | 0.11 | 0.06 | −0.23 *** | |
| | R(−1) | 0.09 *** | 0.01 | −0.02 | |
| | VIX(−1) | −0.10 *** | 0.00 | 0.10 *** | |
| | Pseudo R2 | 0.09 | 0.00 | 0.12 | 962.7 *** |
| Telcm | cons | 0.28 *** | −0.04 | −0.27 *** | |
| | R(−1) | 0.04 ** | −0.01 | 0.00 | |
| | VIX(−1) | −0.08 *** | 0.01 * | 0.08 *** | |
| | Pseudo R2 | 0.11 | 0.00 | 0.13 | 957.1 *** |
| Shops | cons | 0.07 | −0.02 | −0.14 ** | |
| | R(−1) | 0.09 *** | 0.01 | −0.01 | |
| | VIX(−1) | −0.07 *** | 0.01 ** | 0.07 *** | |
| | Pseudo R2 | 0.08 | 0.00 | 0.12 | 920.4 *** |
| Hlth | cons | −0.17 ** | −0.02 | 0.17 ** | |
| | R(−1) | 0.09 *** | −0.02 * | 0.00 | |
| | VIX(−1) | −0.06 *** | 0.01 ** | 0.06 *** | |
| | Pseudo R2 | 0.05 | 0.00 | 0.08 | 348.4 *** |
| Utils | cons | 0.08 | 0.04 | 0.06 | |
| | R(−1) | 0.06 * | −0.03 * | −0.05 *** | |
| | VIX(−1) | −0.06 *** | 0.00 | 0.05 *** | |
| | Pseudo R2 | 0.08 | 0.00 | 0.09 | 598.3 *** |
| Other | cons | 0.54 *** | 0.04 | −0.46 *** | |
| | R(−1) | 0.03 | 0.00 | −0.03 | |
| | VIX(−1) | −0.10 *** | 0.00 | 0.10 *** | |
| | Pseudo R2 | 0.13 | 0.00 | 0.16 | 874.9 *** |

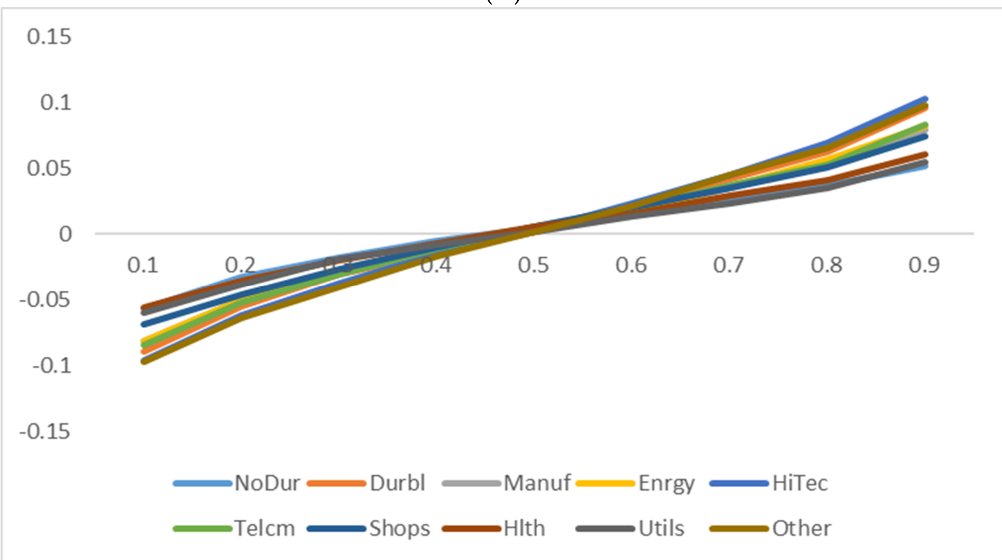
Notes: The table shows the estimation results from the quantile regressions using bootstrapped standard errors. The LHS variables are the US industry excess return and the RHS variables are their lag and the lagged VIX. The slope equality test shows the chi-square test statistic for identical slope coefficients at the 0.1, 0.5, and 0.9 quantiles. */**/*** indicates significance at the 10%/5%/1% level.

The coefficient to the VIX volatility index was significantly negative for the lower tail and significantly positive for the upper tail for all ten industries. For the median, the VIX coefficient was around zero for all industries and it was insignificant except for NoDur, Shops, and Hlth. The coefficient to the lagged industry portfolio was either insignificant and when it was significant it had the same sign as for the overall market portfolio. The pseudo *R*-squared values also resembled those for the market portfolio and the slope equality test gave rise to the same conclusion, namely that the coefficients were significantly different across the upper, lower, and median quantiles.

To investigate the differences across the industries further, Figure 2 shows the slope coefficients across the quantiles for each of the industries. For the coefficient to the lagged return, we see some variation across industries, implying that the autocorrelation varied across industries. For the coefficient to the lagged VIX, there was only little variation across industries, implying that the risk–return trade-off was very similar across the ten industries. The variation across quantiles was highest for industries such as Durbl, Enrgy, HiTec, and Other.




(**A**)




(**B**)

**Figure 2.** Quantile coefficients for US industry portfolios. (**A**): lagged return; (**B**): lagged VIX. Notes: The figure shows the estimated coefficients from Table 4.

### 4.3. European Market Portfolios

In Table 5 we considered the empirical risk–return trade-off for the nine European stock markets.

The qualitative results for the European stock markets were identical to those for the US stock market. For the lower tail, there was a negative risk–return trade-off, while it was positive at the upper tail. The effect from the lagged stock return was also similar to the US stock market. The slope equality test shows that there were significant differences between the tails. The pseudo $R$-squared values were about the same size as for the US stock market.

**Table 5.** European portfolios.

| Country | Quantile | 0.1 | 0.5 | 0.9 | Equality Test |
|---|---|---|---|---|---|
| Belgium | cons | 0.05 | 0.02 | −0.13 * | |
| | R(−1) | 0.14 *** | 0.02 | 0.03 | |
| | VSTOXX(−1) | −0.06 *** | 0.00 | 0.06 *** | |
| | Pseudo R2 | 0.10 | 0.00 | 0.10 | 642.5 *** |
| Finland | cons | 0.22 | −0.01 | −0.37 *** | |
| | R(−1) | 0.08 ** | 0.01 | −0.03 | |
| | VSTOXX(−1) | −0.09 *** | 0.00 | 0.09 *** | |
| | Pseudo R2 | 0.09 | 0.00 | 0.12 | 821.7 *** |
| France | cons | 0.33 *** | 0.02 | −0.28 *** | |
| | R(−1) | 0.09 *** | −0.01 | −0.03 | |
| | VSTOXX(−1) | −0.07 *** | 0.00 | 0.07 *** | |
| | Pseudo R2 | 0.11 | 0.00 | 0.13 | 813.5 *** |
| Germany | cons | 0.13 | 0.04 | −0.24 ** | |
| | R(−1) | 0.17 *** | 0.01 | 0.00 | |
| | VSTOXX(−1) | −0.07 *** | 0.00 | 0.07 *** | |
| | Pseudo R2 | 0.10 | 0.00 | 0.12 | 675.9 *** |
| Ireland | cons | −0.13 | 0.01 | 0.20 * | |
| | R(−1) | 0.11 *** | 0.03 ** | −0.01 | |
| | VSTOXX(−1) | −0.05 *** | 0.00 | 0.05 *** | |
| | Pseudo R2 | 0.06 | 0.00 | 0.06 | 254.5 *** |
| Italy | cons | −0.05 | 0.02 | −0.13 ** | |
| | R(−1) | 0.13 *** | −0.02 | −0.10 *** | |
| | VSTOXX(−1) | −0.06 *** | 0.00 | 0.07 *** | |
| | Pseudo R2 | 0.07 | 0.00 | 0.11 | 616.9 *** |
| Luxemburg | cons | −0.46 *** | 0.05 * | 0.59 *** | |
| | R(−1) | 0.02 | −0.02 * | −0.07 ** | |
| | VSTOXX(−1) | −0.03 *** | 0.00 * | 0.02 *** | |
| | Pseudo R2 | 0.03 | 0.00 | 0.02 | 105.4 *** |
| Netherland | cons | 0.32 *** | 0.04 | −0.37 *** | |
| | R(−1) | 0.12 *** | 0.04 ** | −0.04 | |
| | VSTOXX(−1) | −0.07 *** | 0.00 | 0.07 *** | |
| | Pseudo R2 | 0.12 | 0.00 | 0.14 | 754.8 *** |
| Spain | cons | −0.13 | −0.01 | −0.19 ** | |
| | R(−1) | 0.10 *** | 0.01 | −0.01 | |
| | VSTOXX(−1) | −0.06 *** | 0.00 | 0.07 *** | |
| | Pseudo R2 | 0.08 | 0.00 | 0.10 | 399.4 *** |

Notes: The table shows the estimation results from the quantile regressions using bootstrapped standard errors. The LHS variables are the European excess return and the RHS variables are their lag and the lagged VSTOXX. The slope equality test shows the chi-square test statistic for identical slope coefficients at the 0.1, 0.5, and 0.9 quantiles. */**/*** indicates significance at the 10%/5%/1% level.

Figure 3 shows the variation in the slope coefficients across the European countries. As for the US industry portfolios, the variation across countries in the lagged stock return coefficients was fairly wide, while the variation in the coefficient to the VSTOXX risk measure was fairly narrow. The variation across quantiles was largest for Finland and the Netherlands.

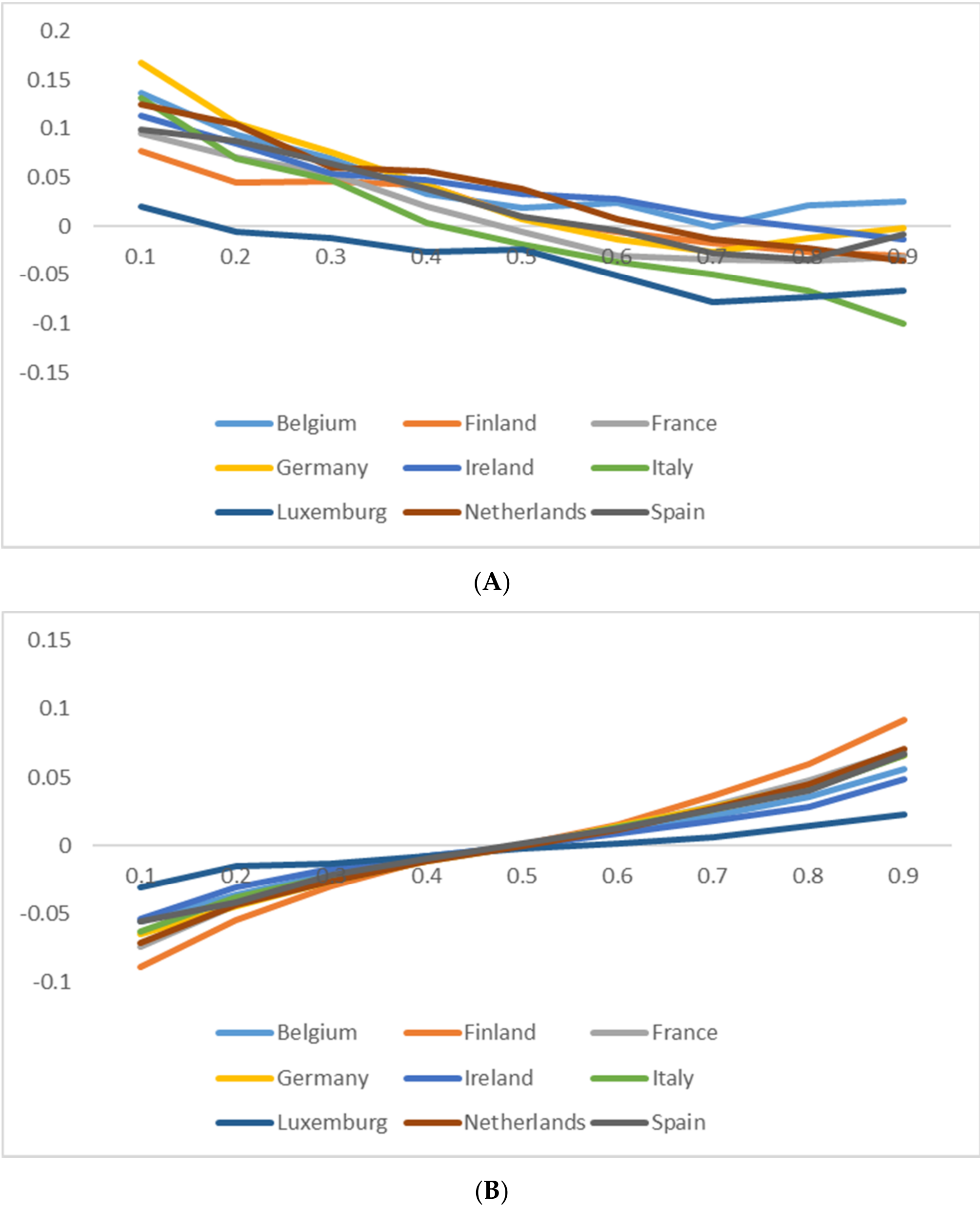



**Figure 3.** Quantile coefficients for European portfolios. (**A**) Lagged return; (**B**) lagged VSTOXX. Notes: The figure shows the estimated coefficients from Table 5.

### *4.4. Skewness and Kurtosis Effects*

In Table 6 we show the results from adding the skewness and kurtosis to the risk–return trade-off for the US market portfolio.

The skewness only entered significantly at the upper quantile with a negative coefficient, which implies a negative effect from higher skewness. The kurtosis was not significant at any of the quantiles.

**Table 6.** US market portfolio, skewness, and kurtosis.

| Quantile | OLS | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|---|---|---|---|---|---|---|---|---|---|---|
| cons | 1.01 | 4.17 * | 2.02 | 0.78 | 0.73 | 0.92 | 0.22 | −0.87 | 0.13 | 1.14 |
| Mkt(−1) | −0.06 * | 0.07 ** | 0.05 * | 0.01 | −0.01 | −0.04 ** | −0.05 *** | −0.06 *** | −0.05 *** | −0.06 ** |
| VIX(−1) | 0.00 | −0.08 *** | −0.05 *** | −0.03 *** | −0.01 *** | 0.00 * | 0.02 *** | 0.03 *** | 0.05 *** | 0.08 *** |
| Skewness(−1) | −0.17 | 0.12 | 0.07 | −0.02 | −0.08 | −0.23 * | −0.19 | −0.22 | −0.42 * | −0.59 *** |
| Kurtosis(−1) | −0.25 | −0.86 | −0.39 | −0.14 | −0.16 | −0.23 | −0.08 | 0.15 | −0.11 | −0.37 |
| Pseudo R2 | 0.00 | 0.12 | 0.06 | 0.02 | 0.00 | 0.00 | 0.01 | 0.03 | 0.08 | 0.16 |
| Slope equality test | | | | | | 988.0 *** | | | | |

Notes: The table shows the estimation results from the OLS regression using Newey and West (1987) standard errors and the quantile regressions using bootstrapped standard errors. The LHS variable is the US market excess return and the RHS variables are its lag, the lagged VIX, the lagged skewness, and the lagged kurtosis. The slope equality test shows the chi-square test statistic for identical slope coefficients at the 0.1, 0.5, and 0.9 quantiles. */**/*** indicates significance at the 10%/5%/1% level.

In Table 7 we show the same quantile regressions for the ten industry portfolios. The skewness and kurtosis were never significant at the lower tail and at the median. At the upper tail, the skewness was significantly different for four industries (HiTec, Telcm, Shops, and Utils), while it was insignificant for the other six industries. The kurtosis was only significant for the Utils industry where the effect was negative.

Overall, the skewness and kurtosis are not very important for the risk–return trade-off.

**Table 7.** Ten US industry portfolios, skewness, and kurtosis.

| Industry | Quantile | 0.1 | 0.5 | 0.9 | Equality Test |
|---|---|---|---|---|---|
| NoDur | cons | 2.84 | −0.38 | 0.63 | |
| | R(−1) | 0.05 * | −0.03 ** | −0.05 *** | |
| | VIX(−1) | −0.06 *** | 0.00 ** | 0.05 *** | |
| | Skewness(−1) | 0.01 | 0.03 | −0.29 | |
| | Kurtosis(−1) | −0.64 | 0.09 | −0.15 | |
| | Pseudo R2 | 0.08 | 0.00 | 0.11 | 920.1 *** |
| Durbl | cons | −2.19 | −2.99 | 1.98 | |
| | R(−1) | 0.07 ** | 0.02 | 0.02 | |
| | VIX(−1) | −0.09 *** | 0.00 | 0.10 *** | |
| | Skewness(−1) | 0.76 * | 0.16 | −0.38 | |
| | Kurtosis(−1) | 0.58 | 0.71 | −0.51 | |
| | Pseudo R2 | 0.09 | 0.00 | 0.11 | 1045.6 *** |
| Manuf | cons | 4.96 * | 1.42 | 0.00 | |
| | R(−1) | 0.07 *** | −0.01 | −0.03 | |
| | VIX(−1) | −0.08 *** | 0.00 | 0.08 *** | |
| | Skewness(−1) | −0.17 | −0.29 | −0.36 * | |
| | Kurtosis(−1) | −1.09 | −0.34 | −0.07 | |
| | Pseudo R2 | 0.11 | 0.00 | 0.14 | 1093.9 *** |
| Enrgy | cons | 4.91 | 2.51 | 0.20 | |
| | R(−1) | 0.05 * | −0.03 ** | −0.05 ** | |
| | VIX(−1) | −0.08 *** | 0.00 | 0.08 *** | |
| | Skewness(−1) | −0.29 | −0.35 | −0.32 | |
| | Kurtosis(−1) | −1.19 | −0.62 | −0.04 | |
| | Pseudo R2 | 0.06 | 0.00 | 0.08 | 491.1 *** |
| HiTec | cons | 5.92 | −1.63 | 0.81 | |
| | R(−1) | 0.09 *** | 0.01 | −0.02 | |
| | VIX(−1) | −0.09 *** | 0.00 | 0.10 *** | |
| | Skewness(−1) | 0.06 | 0.16 | −0.85 *** | |
| | Kurtosis(−1) | −1.36 | 0.40 | −0.29 | |
| | Pseudo R2 | 0.09 | 0.00 | 0.12 | 1099.3 *** |
| Telcm | cons | 4.80 * | −0.67 | 1.36 | |
| | R(−1) | 0.05 ** | −0.01 | 0.00 | |
| | VIX(−1) | −0.08 *** | 0.01 * | 0.08 *** | |
| | Skewness(−1) | −0.16 | 0.05 | −0.52 ** | |
| | Kurtosis(−1) | −1.06 | 0.15 | −0.41 | |
| | Pseudo R2 | 0.11 | 0.00 | 0.13 | 849.1 *** |

**Table 7.** *Cont.*

| Industry | Quantile | 0.1 | 0.5 | 0.9 | Equality Test |
|---|---|---|---|---|---|
| Shops | cons | 2.87 | −0.01 | 0.98 | |
| | R(−1) | 0.10 *** | 0.00 | −0.01 | |
| | VIX(−1) | −0.07 *** | 0.01 ** | 0.07 *** | |
| | Skewness(−1) | 0.10 | −0.05 | −0.75 *** | |
| | Kurtosis(−1) | −0.65 | 0.00 | −0.32 | |
| | Pseudo R2 | 0.08 | 0.00 | 0.13 | 1095.0 *** |
| Hlth | cons | 1.88 | −0.01 | −3.06 | |
| | R(−1) | 0.10 *** | −0.03 ** | 0.00 | |
| | VIX(−1) | −0.05 *** | 0.01 ** | 0.06 *** | |
| | Skewness(−1) | 0.14 | −0.10 | −0.03 | |
| | Kurtosis(−1) | −0.47 | −0.01 | 0.74 | |
| | Pseudo R2 | 0.06 | 0.00 | 0.09 | 543.1 *** |
| Utils | cons | 1.82 | 1.78 | 5.30 *** | |
| | R(−1) | 0.07 ** | −0.03 ** | −0.06 ** | |
| | VIX(−1) | −0.06 *** | 0.00 | 0.05 *** | |
| | Skewness(−1) | 0.02 | −0.32 ** | −0.66 *** | |
| | Kurtosis(−1) | −0.41 | −0.43 | −1.26 *** | |
| | Pseudo R2 | 0.08 | 0.00 | 0.09 | 648.1 *** |
| Other | cons | 5.24 * | −0.85 | −0.77 | |
| | R(−1) | 0.05 | 0.00 | −0.03 | |
| | VIX(−1) | −0.09 *** | 0.00 | 0.10 *** | |
| | Skewness(−1) | −0.26 | 0.02 | −0.21 | |
| | Kurtosis(−1) | −1.12 * | 0.21 | 0.06 | |
| | Pseudo R2 | 0.13 | 0.00 | 0.16 | 1173.4 *** |

Notes: The table shows the estimation results from the quantile regressions using bootstrapped standard errors. The LHS variables are the US industry excess return and the RHS variables are their lag, the lagged VIX, the lagged skewness, and the lagged kurtosis. The slope equality test shows the chi-square test statistic for identical slope coefficients at the 0.1, 0.5, and 0.9 quantiles. */**/*** indicates significance at the 10%/5%/1% level.

### 4.5. Subsample Analysis

We investigated the potential effects from the recent financial crisis on the risk–return trade-off for the US market portfolio. For this reason, we considered two subsamples, namely before the financial crisis (1990–2006) and during and after the financial crisis (2007–2020), cf. Table 8. We obtained similar results in the two subsamples and for the entire sample period, both when we included and excluded the lagged market return. The explanatory power was higher in the most recent subsample than the entire period, which was again higher than for the period before the financial crisis.

**Table 8.** US market portfolio, subsamples.

| Period | Quantile | 0.1 *** | 0.5 | 0.9 | Equality Test |
|---|---|---|---|---|---|
| 1990–2006 | cons | 0.29 *** | 0.01 | −0.20 ** | |
| | Mkt(−1) | 0.14 *** | 0.02 | 0.01 | |
| | VIX(−1) | −0.07 *** | 0.00 | 0.07 *** | |
| | Pseudo R2 | 0.10 | 0.00 | 0.10 | 380.8 *** |
| 1990–2006 | cons | 0.30 *** | 0.01 | −0.19 ** | |
| | VIX(−1) | −0.07 *** | 0.00 | 0.07 *** | |
| | Pseudo R2 | 0.10 | 0.00 | 0.10 | 468.0 *** |
| 2007–2020 | cons | 0.51 *** | −0.03 | −0.40 *** | |
| | Mkt(−1) | 0.01 | −0.09 *** | −0.14 *** | |
| | VIX(−1) | −0.10 *** | 0.01** | 0.09 *** | |
| | Pseudo R2 | 0.14 | 0.01 | 0.22 | 524.5 *** |

**Table 8.** *Cont.*

| Period | Quantile | 0.1 *** | 0.5 | 0.9 | Equality Test |
|---|---|---|---|---|---|
| 2007–2020 | cons | 0.53 *** | −0.04 | −0.50 *** | |
| | VIX(−1) | −0.10 *** | 0.01 * | 0.09 *** | |
| | Pseudo R2 | 0.14 | 0.00 | 0.21 | 553.7 *** |

Notes: The table shows the estimation results from the quantile regressions using bootstrapped standard errors for three sample periods. The LHS variable is the US market excess return and the RHS variables are its lag and the lagged VIX. The slope equality test shows the chi-square test statistic for identical slope coefficients at the 0.1, 0.5, and 0.9 quantiles. */**/*** indicates significance at the 10%/5%/1% level.

## 5. Conclusions

We investigated the risk–return trade-off on the US and European stock markets. We investigated the non-linear risk–return trade-off focusing on the tails of the stock returns using quantile regressions. The upper tail (0.9 quantile) contained large positive excess returns, while the lower tail (0.1 quantile) contained large negative excess returns. For the US stock market portfolio, the risk–return trade-off was significantly positive at the upper tail, while it was significantly negative at the lower tail. Therefore, only the upper tail risk–return trade-off results were in accordance with our expectations from asset pricing models such as the CAPM. The results for the median (0.5 quantile) were similar to the linear risk–return trade-off results, namely that the trade-off was not significant. We found similar empirical risk–return trade-off results for US industry portfolios and for Eurozone stock market portfolios.

Our empirical results document that we cannot simply rely on linear models when describing the relationship between returns and risk. This implies that linear pricing models are not adequate for asset pricing and risk management.

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**Conflicts of Interest:** The authors declare no conflict of interest.

## Notes

1. The US stock returns and risk free interest rate are gratefully available from Kenneth French's data library. The market portfolio is the value-weight return of all CRSP firms listed on NYSE, AMEX, or NASDAQ, cf. Fama and French (1993).
2. The Eurozone stock returns and euro LIBOR rate are available from DataStream.
3. The VIX data are available from CBOE's webpage with VIX historical price data.
4. The quantile regression estimation follows Aslanidis and Christiansen (2014).

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