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ORIGINAL PAPER

Empirical Relationships Between the Elastic Settlement of Rigid Rectangular Foundations and the Settlement of the Respective Flexible Foundations

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Abstract The problem of settlement of shallow foundations is among the most important ones in classical soil mechanics. And while for the settlement of flexible foundations elastic solutions are widely used, for rigid rectangular foundations where the actual contact pressure distribution is still unknown, the problem is approximated either analytically assuming a contact pressure distribution or semi-empirically combining the theory of elasticity with experimental and/or numerical results. A third and often attractive choice is the use of simple empirical relationships or relevant tabulated values relating the elastic settlement of rigid foundations (ρ_R) with the settlement of the respective flexible foundations (e.g. at the center, ρ_{Ce}). Reviewing the relathionships of this third approach, the author revealed serious lack of consesous between the various sources; for example, according to the literature, ρ_R ranges between 68 and 125% of ρ_{Ce}, the time when it is well-known that $\rho_R < \rho_{Ce}$. In this paper, comparison of the settlement of 210 rigid foundation cases derived from 3D elastic finite element analysis, with the settlement of the respective flexible foundations derived from the theory of elasticity, led to simple empirical relationships between ρ_R and ρ_{Ce} as well as between ρ_R and ρ_{Av} (ρ_{Av} = average settlement of the flexible foundation) with coefficient of determination (R^2) almost unity. The analysis showed that these relationships are largely independent of the aspect ratio of foundations and the thickness and Poisson's ratio (v) of the compressible medium, although separate relationships are given for $v = 0.5$, slightly increasing R^2. Finally, a correction factor for foundation rigidity is given exploting the known linear relationship that exists between the relative stiffness factor of foundations and settlement.

Keywords Elastic settlement analysis · Flexible foundations · Rigid foundations · Empirical correlations · Foundation rigidity

1 Introduction

The problem of settlement of shallow foundations is among the most important ones in classical soil mechanics. The settlement of flexible foundations is usually calculated based on the widely acccepted theory of Boussinesq (1885) and the constitutive relationship of Hooke's law. For rigid rectangular foundations, the exact solution is still not available, since the contact pressure distribution is unknown. In this respect, analytical approximations based on assumed contact pressure distribution have been

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provided by Borodachev (1976), Borodachev and Galin (1974), Brothers et al. (1977), Butterfield and Banerjee (1971), Dempsey and Li (1989), Fabrikant (1986), Gorbunov-Possadov et al. (1984), Mullan et al. (1980), Noble (1960), Panek and Kalker (1977) and Sovinc (1961). Semi-empirical approaches combining the theory of elasticity with experimental and/or numerical results have been proposed, among others, by Foye et al. (2008), Lee and Salgado (2002), Mayne and Poulos (1999), Schmertmann (1970) and Schmertmann et al. (1978). Among these, one of the most popular approaches is Schmertmann et al.'s (1978) method, which is included in various design codes and public agencies' reference manuals, such as, AASHTO (2010), Eurocode 7 (EN 1997-2 2007) and FHWA NHI-06-088/089 (Samtani and Nowatzki 2006). An in-depth review of Schmertmann's method has been offered by the author in Pantelidis (2020a); as it has been shown, Schmertmann's method has been proposed without proper and adequate documentation, whilst it presents serious weaknesses related to its calibration and the values adopted for various factors used. In addition, comparison between measured and calculated settlement values of structures or full-size test footings indicates that Schmertmann's method is a poor prediction tool. An attractive approach to this rather confusing problem, especially to practitioners who often seek for handy solutions, is the adoption of an empirical relationship between the settlement of rigid foundation and the settlement of the respective flexible foundation; the latter may be the average, the maximum (at the centre) or the minimum (at the corner) settlement value (or an algevraic combination). In the present paper the validity of probably the most commonly met empirical relationships in literature is examined. New relationships are then proposed comparing the settlement values of rigid foundations derived from 3D elastic finite element analysis with the respective ones of flexible foundations.

2 Literature Review

The relationships between the settlement of a rigid foundation and the settlement of the respective flexible foundation can generally be distinguished into three categories.

The relationships belonging in the first category ignore the influence of all the factors affecting the magnitude of settlement, such as, the aspect ratio of foundation and the thickness of the compressible stratum. These are summarized in Table 1. According to these relationships, the settlement of rigid foundations (ρ_R) ranges between 75 and 93.1% of the maximum settlement (i.e. at the center, ρ_{Ce}) or between 92.6 and 100% of the average settlement (ρ_{Av}) of the respective flexible foundations. According to Davis and Taylor (1962), on the other hand, the settlement of rigid foundation, ρ_R, is equal to a weighted average between the maximum and minimum settlement of the flexible foundation., i.e. $\rho_R \approx (2\rho_{Ce} + \rho_{Co})/3$; ρ_{Co} is the settlement at the corner of the foundation.

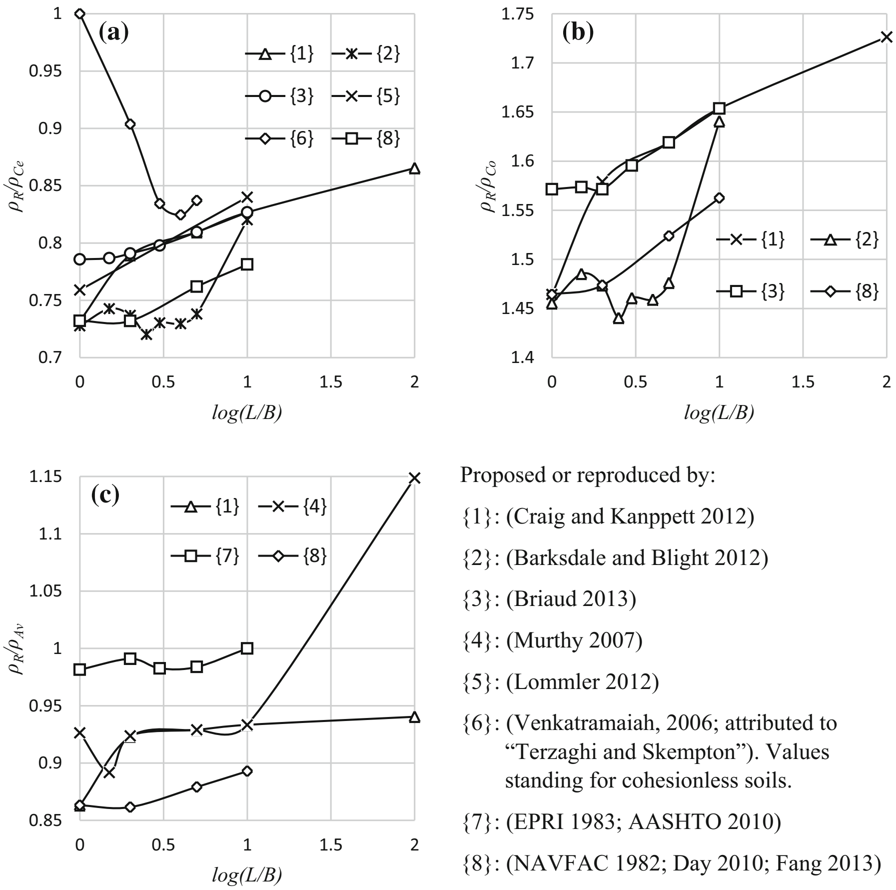
The relationships of the second category take into account the aspect ratio of foundations. Such relationships are presented in chart form in Fig. 1. In addition to the fact that there is not a consensus between the different sources, the great fluctuation of the ρ_R/ρ_{Ce} (or ρ_R/ρ_{Co} or ρ_R/ρ_{Av}) values as the *L*/*B* ratio increases is highlighted. It is moreover mentioned that some of these relationships present minimum for *L*/*B* approximately equal to 3 and also some extreme or rather not consistent values.

The relationships of the third category consider, in addition to the *L*/*B* ratio, the effect of the thickness of the compressible medium on the ρ_R/ρ_{Ce} (or ρ_R/ρ_{Co} or ρ_R/ρ_{Av}) value. Such ρ_R/ρ_{Ce} values derived from Fraser and Wardle's (1976) work are given in Table 2 (see also Barnes 2016); these ρ_R/ρ_{Ce} values range from 0.68 to 0.8 depending on *H*/*B* and *L*/*B*. ρ_R/ρ_{Ce} values depending on both *L*/*B* and *H*/*B* have also been given by Derski et al. (2012); these are presented in chart form in Fig. 2. Derski et al. considered both the case of smooth and rigid substratum (meaning the stratum below the compressible layer). It is interesting that in the case of smooth substratum, these values present minimum for *H*/*B* about equal to unity. Also, it seems that the settlement is greatly affected by the roughness of the substratum, even if the substratum lies below the influence depth of foundation, z_I; the latter was given by Terzaghi et al. (1996) as $2B(1 + \log(L/B))$. It is even more interesting that, according to Derski et al., for small *H*/*B* values, the settlement of the rigid foundation appears to be greater or much greater than the settlement at the centre of the respective flexible foundation.

Table 1 Empirical relationships of the first category

Empirical relationships	Introduced or reproduced by[a]
$\rho_R \approx 0.75\rho_{Ce}$	Wesley (2010), Briaud (2013)
$\rho_R \approx \frac{\pi}{4}\rho_{Ce} = 0.785\rho_{Ce}$	Mayne and Poulos (1999, it refers to the their I_F factor), Aysen (2002)
$\rho_R \approx 0.8\rho_{Ce}$	IS8009 (1976), Kaniraj (1988), Charles (2005), Varghese (2005), Murthy (2007), Barnes (2016)
$\rho_R \approx 0.833\rho_{Ce}$	Ishibashi and Hazarika (2015)
$\rho_R \approx 0.85\rho_{Ce}$	Coduto (2001)
$\rho_R \approx 0.93\rho_{Ce}$	Bowles (1996), Baban (2016), Das (2019)
$\rho_R \approx (2\rho_{Ce} + \rho_{Co})/3$	Davis and Taylor (1962), Kaniraj (1988), Poulos and Davis (1991)
$\rho_R \approx 0.9\rho_{Av}$	Das (2019)
$\rho_R \approx 0.926\rho_{Av}$	Aysen (2002)
$\rho_R \approx 0.93\rho_{Av}$	Schleicher (1926), Kaniraj (1988)
$\rho_R \approx \rho_{Av}$	Fox (1948), Kaniraj (1988)

[a] An older or a sole reference does not necessarily indicate the researcher(s) who introduced the respective relationship. Also, some of these relationships derived by the author from tabulated values appeared in the indicated source



Proposed or reproduced by:

{1}: (Craig and Kanppett 2012)

{2}: (Barksdale and Blight 2012)

{3}: (Briaud 2013)

{4}: (Murthy 2007)

{5}: (Lommler 2012)

{6}: (Venkatramaiah, 2006; attributed to "Terzaghi and Skempton"). Values standing for cohesionless soils.

{7}: (EPRI 1983; AASHTO 2010)

{8}: (NAVFAC 1982; Day 2010; Fang 2013)

Fig. 1 Empirical relationships of the second category

Fig. 2 Empirical relationships of the third category (Derski et al. 2012)

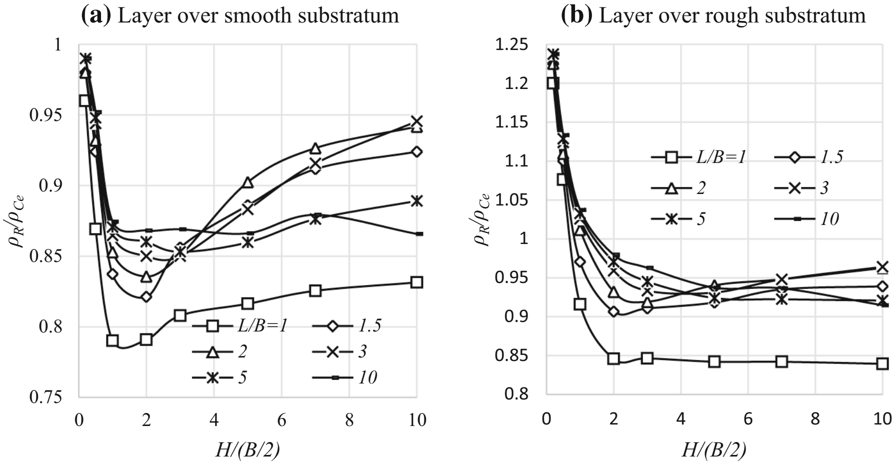

(a) Layer over smooth substratum **(b)** Layer over rough substratum

Table 2 Empirical relationship of the third category (see Barnes 2016; Fraser and Wardle 1976): Table giving the ρ_R/ρ_{Ce} value for various *H/B* and *L/B* ratio values

H/B	*L/B* = 1	*L/B* = 2	*L/B* = 3, 4, 5
1	0.68	0.72	0.79
∞	0.77	0.78	0.8

Two additional observations can be made. First, all the empirical relationships appear to be independent of the Poisson's ratio (v) value of soil. Second, some of these relationships relate the settlement of the rigid foundation with the average settlement of the respective flexible foundation. This, in essence, cancels the most important advantage of the empirical relationships, which is the ease of use. In this respect, an effective calculation of the average settlement requires integration or the calculation of a great number of settlement values over the plan-view of the flexible foundation. According to the best knowledge of the author, such solutions, giving the average settlement value, have been provided by Schleicher (1926) and Janbu et al. (1956). The first one, however, refers exclusively to compressible layers of infinite depth, while the second one, which has been offered in chart form, stands only for v equal to 0.5 (an improved chart has been published by Christian and Carrier 1978).

3 Derivation of the Proposed Empirical Relationships

The problem of elastic settlement of a $B \times L$ rectangular flexible foundation resting on the surface of a homogenous stratum of thickness H has been solved analytically by Steinbrenner (1934). The solution, which is based on Boussinesq's (1885) theory and Hooke's law, has as follows:

$$\rho = a_{cc} q B \frac{1 - v^2}{E} I_s \tag{1}$$

where $a_{cc} = 1$ or 4 for the corner or the center of the foundation respectively, E and v are the modulus and the Poisson's ratio of the compressible layer, B is the width of the foundation, q is the uniform surcharge of foundation (in the same units with E) and I_s is a dimensionless factor taking into account the shape of foundation and the thickness of the compressible stratum. The foundation is supposed to rest on the surface of the compressible medium. More specifically,

$$I_s = F_1 + \frac{1 - 2v}{1 - v} F_2 \tag{2}$$

with

$$F_1 = \frac{1}{\pi}(A_0 + A_1) \tag{3}$$

$$F_2 = \frac{n}{2\pi} \tan^{-1} A_2 \tag{4}$$

$$A_0 = m \ln \frac{\left(1+\sqrt{m^2+1}\right)\sqrt{m^2+n^2}}{m\left(1+\sqrt{m^2+n^2+1}\right)} \quad (5)$$

$$A_1 = \ln \frac{\left(m+\sqrt{m^2+1}\right)\sqrt{1+n^2}}{m+\sqrt{m^2+n^2+1}} \quad (6)$$

$$A_2 = \frac{m}{n\sqrt{m^2+n^2+1}} \quad (7)$$

and $m = L/B$ and $n = 2H/B$ or H/B for the center and the corner of the foundation respectively.

Working with this basic solution for the corner of the foundation and using the principle of superposition (Barnes 2016), the settlement at any point on the plan-view of the foundation can be calculated. The average settlement of the foundation can be obtained from the same method with settlements averaged over the rectangular area. All settlement values in the case of flexible foundation have been calculated using Wolfram Mathematica.

Any settlement value of a $B \times L$ foundation can conveniently be rewritten as follows:

$$\rho_i = \frac{qB}{E} A_i \quad (8)$$

where $i = Ce$ and Co for the settlement at the centre and the corner of the foundation respectively, while $i = Av$ for the average settlement of the foundation (the case of rigid foundations is indicated by the subscript R, i.e. ρ_R and A_R). A_i is the chart area bounded by the strain influence factor (I_z) curve and the normalized depth (z/B) axis over the influence depth of the foundation (Pantelidis 2020b, c) and it is dimensionless.

The settlement of rigid rectangular footings is calculated, herein, using the finite element method. In this respect, the rsetl3d program developed by Professors D.V. Griffiths and G. Fenton has been used (the rsetl3d program is freely available at http://random.engmath.dal.ca/rfem/). Although the program in question performs three-dimensional probabilistic shallow foundation settlement analysis (Fenton and Griffiths 2005), it also returns the deterministic elastic settlement value of the problem considered. Various cases were considered, which are discretized by the aspect ratio of foundation as well as by the thickness and the Poisson's ratio of the compressible medium. These are shown in Table 3, while the Poisson's ratio values considered were ν = 0, 0.1, 0.2, 0.3, 0.4, 0.45 and 0.499. To avoid indeterminate behavior in the finite element analysis of the undrained condition, a ν value equal to 0.499 was used instead of 0.5. According to Potts and Zdravković (2001), if a value of Poisson's ratio ν = 0.499 is used, the results would be very similar to that for ν = 0.5; a ν value equal to 0.49 results to a modest error. Homogenous conditions were considered for the compressible medium. The foundation width B was kept constant and equal to 1.0 m for all 210 cases analyzed. The relative dimensions of the finite element mesh considered in each case are also given in Table 3 (B' and L' are the dimensions of mesh along the B and L side of foundation). The finite element mesh for the case having L/B = 1 and H/B = 2 is indicatively shown in Fig. 3. Eight-noded cubic elements of edge 0.1 m were used in all cases.

The results are presented in tabular form in Table 4, where A_R, A_R/A_{Ce}, A_R/A_{Av} and A_R/A_{Co} values are given for various L/B, H/B and ν values. Based on these results, several very strong relationships ($R^2 > 0.98$) have been drawn in Fig. 4. The author noticed that the ν = 0 and ν = 0.5 are special cases. The ν = 0 is rather unrealistic and thus, it has not been processed further. The ν = 0.5 case was processed separately from the other ν values, because it produces its own cloud with points on the relevant charts. This very peculiar behavior was further investigated in the present paper. As shown in the A_R (also A_{Ce}, A_{Co} and A_{Av}) versus ν example chart of Fig. 5, in the close vicinity of the ν = 0.5 value an abrupt reduction in the A_R value is observed. Such a behaviour is not observed for the respective A_{Ce}, A_{Co} and A_{Av} values.

Finally, the proposed relationships are summarized below for: the settlement of rigid footings under drained soil conditions (ν = 0.1–0.45)

$$\rho_R \approx \begin{cases} 0.898\rho_{Av} & (R^2 = 0.9985) \\ 0.902(1.515\rho_{Ce} + 0.485\rho_{Co})/2 & (R^2 = 0.9961) \\ 0.907(1.5\rho_{Ce} + 0.5\rho_{Co})/2 & (R^2 \text{ also } 0.9961) \\ 0.761\rho_{Ce} & (R^2 = 0.9968) \end{cases} \quad (9)$$

and the settlement of rigid footings under undrained soil conditions (ν = 0.5)

Table 3 Geometric data for the finite element models considered. The dimension of each element was 0.1 m, while the foundation width, B, considered in all cases was 1.0 m (meaning 10 elements along the small side of foundation)

L/B	log (L/B)	H/B	B'/B	L'/B	L'/B	log (L/B)	H/B	B'/B	L'/B
1	0	1	6.4	6.4	5.6	0.75	1	9.6	17.6
		1.5	6.4	6.4			1.5	9.6	17.6
		2	6.4	6.4			2	9.6	17.6
		4	6.4	6.4			2.5	9.6	17.6
1.8	0.25	1	9.6	11.0			3	9.6	17.6
		1.5	9.6	11.0			3.5	9.6	17.6
		2	9.6	11.0			7	9.6	17.6
		2.5	9.6	11.0	10	1	1	9.6	25.0
		5	9.6	11.0			1.5	9.6	25.0
3.2	0.5	1	9.6	13.4			2	9.6	25.0
		1.5	9.6	13.4			2.5	9.6	25.0
		2	9.6	13.4			3	9.6	25.0
		2.5	9.6	13.4			3.5	9.6	25.0
		3	9.6	13.4			4	9.6	25.0
		6	9.6	13.4			8	9.6	25.0

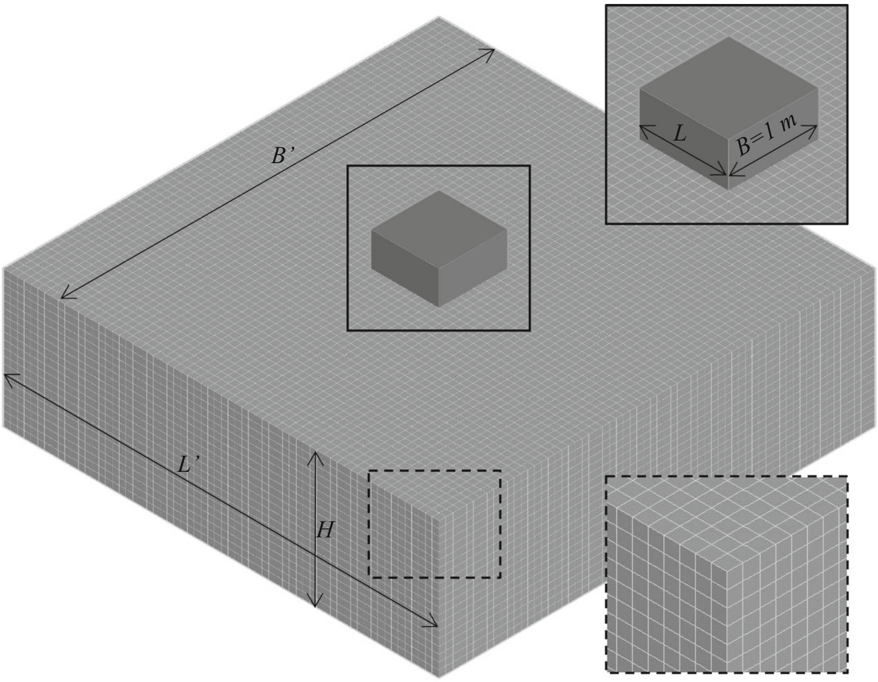


Fig. 3 3D finite element mesh referring to the case of rigid footing with B = 1.0 m, L/B = 1 and H/B = 2. Eight-noded cubic elements of edge 0.1 m were used in all models

$$\rho_R \approx \begin{cases} 0.713(qB/E)(A_{Av})^{1.515} & (R^2 = 0.9818) \\ 0.637(1.3\rho_{Ce} + 0.7\rho_{Co})/2 & (R^2 = 0.9903) \\ 0.863(qB/E)(A_{Ce})^{1.545} & (R^2 = 0.9804) \end{cases} \tag{10}$$

The application of the relationships of Eq. 10 which rely on the strain influence area (A_{Ce} or A_{Av}) requires either homogenous medium or the use of the equivalent elastic modulus value (Pantelidis 2019, 2020d, 2021).

Table 4 A_R, A_R/A_{Ce}, A_R/A_{Av} and A_R/A_{Co} values with respect to *L/B*, *H/B* and *v*

$\frac{L}{B}$	$\frac{H}{B}$	v	A_R	$\frac{A_R}{A_{Ce}}$	$\frac{A_R}{A_{Av}}$	$\frac{A_R}{A_{Co}}$	$\frac{L}{B}$	$\frac{H}{B}$	v	A_R	$\frac{A_R}{A_{Ce}}$	$\frac{A_R}{A_{Av}}$	$\frac{A_R}{A_{Co}}$
1	4.0	0	0.659	0.657	0.786	1.479	1	2.0	0	0.580	0.650	0.792	1.660
		0.1	0.667	0.674	0.808	1.526			0.1	0.586	0.670	0.817	1.729
		0.2	0.657	0.688	0.826	1.569			0.2	0.577	0.689	0.842	1.803
		0.3	0.633	0.705	0.847	1.620			0.3	0.550	0.703	0.861	1.877
		0.4	0.580	0.707	0.850	1.642			0.4	0.497	0.704	0.865	1.932
		0.45	0.536	0.692	0.834	1.621			0.45	0.455	0.650	0.804	2.014
		0.5	0.342	0.472	0.570	1.117			0.5	0.272	0.444	0.548	1.270
1.0	1.5	0	0.533	0.648	0.796	1.784	1	1.0	0	0.454	0.650	0.804	2.014
		0.1	0.537	0.669	0.823	1.869			0.1	0.456	0.674	0.835	2.135
		0.2	0.527	0.689	0.849	1.962			0.2	0.445	0.695	0.864	2.267
		0.3	0.449	0.703	0.869	2.058			0.3	0.415	0.709	0.883	2.408
		0.4	0.446	0.700	0.870	2.133			0.4	0.360	0.699	0.874	2.526
		0.45	0.403	0.680	0.846	2.127			0.45	0.316	0.667	0.836	2.520
		0.5	0.220	0.404	0.505	1.311			0.5	0.132	0.308	0.388	1.240
1.8	5.0	0	0.870	0.671	0.801	1.528	1.8	2.5	0	0.759	0.667	0.810	1.731
		0.1	0.881	0.689	0.824	1.579			0.1	0.766	0.687	0.835	1.805
		0.2	0.870	0.706	0.845	1.630			0.2	0.753	0.706	0.860	1.883
		0.3	0.832	0.718	0.861	1.674			0.3	0.716	0.720	0.880	1.963
		0.4	0.759	0.717	0.861	1.693			0.4	0.646	0.720	0.883	2.022
		0.45	0.698	0.700	0.841	1.665			0.45	0.591	0.705	0.866	2.019
		0.5	0.449	0.483	0.581	1.160			0.5	0.376	0.485	0.598	1.426
1.8	2.0	0	0.710	0.666	0.813	1.831	1.8	1.5	0	0.637	0.667	0.819	1.986
		0.1	0.715	0.687	0.840	1.917			0.1	0.641	0.691	0.849	2.094
		0.2	0.702	0.707	0.866	2.011			0.2	0.626	0.712	0.877	2.213
		0.3	0.664	0.722	0.887	2.107			0.3	0.587	0.727	0.897	2.339
		0.4	0.593	0.720	0.888	2.186			0.4	0.515	0.721	0.893	2.446
		0.45	0.538	0.701	0.867	2.185			0.45	0.459	0.696	0.864	2.450
		0.5	0.323	0.459	0.569	1.480			0.5	0.247	0.412	0.513	1.525
1.8	1.0	0	0.526	0.678	0.830	2.251	3.2	6.0	0	1.093	0.695	0.823	1.618
		0.1	0.526	0.705	0.864	2.399			0.1	1.104	0.713	0.845	1.671
		0.2	0.510	0.731	0.895	2.567			0.2	1.088	0.729	0.866	1.723
		0.3	0.470	0.747	0.916	2.749			0.3	1.036	0.739	0.879	1.767
		0.4	0.398	0.733	0.900	2.909			0.4	0.935	0.732	0.873	1.776
		0.45	0.341	0.692	0.851	2.911			0.45	0.856	0.712	0.850	1.745
		0.5	0.138	0.315	0.388	1.441			0.5	0.553	0.494	0.590	1.225
3.2	3.0	0	0.936	0.693	0.832	1.838	3.2	2.5	0	0.883	0.694	0.835	1.918
		0.1	0.943	0.714	0.858	1.917			0.1	0.889	0.716	0.862	2.008
		0.2	0.926	0.734	0.883	2.003			0.2	0.870	0.736	0.888	2.104
		0.3	0.877	0.748	0.902	2.087			0.3	0.821	0.751	0.908	2.202
		0.4	0.786	0.747	0.903	2.149			0.4	0.731	0.749	0.908	2.279
		0.45	0.716	0.729	0.883	2.144			0.45	0.661	0.730	0.886	2.278
		0.5	0.467	0.518	0.629	1.565			0.5	0.415	0.501	0.609	1.617
3.2	2.0	0	0.710	0.666	0.813	1.831	3.2	1.5	0	0.715	0.702	0.844	2.189
		0.1	0.817	0.720	0.868	2.132			0.1	0.717	0.729	0.877	2.316

Table 4 continued

$\frac{L}{B}$	$\frac{H}{B}$	v	A_R	$\frac{A_R}{A_{Ce}}$	$\frac{A_R}{A_{Av}}$	$\frac{A_R}{A_{Co}}$	$\frac{L}{B}$	$\frac{H}{B}$	v	A_R	$\frac{A_R}{A_{Ce}}$	$\frac{A_R}{A_{Av}}$	$\frac{A_R}{A_{Co}}$
		0.2	0.797	0.742	0.896	2.246			0.2	0.697	0.754	0.907	2.457
		0.3	0.748	0.758	0.916	2.363			0.3	0.647	0.770	0.928	2.606
		0.4	0.657	0.754	0.913	2.459			0.4	0.557	0.762	0.919	2.730
		0.45	0.588	0.731	0.886	2.462			0.45	0.488	0.731	0.882	2.731
		0.5	0.348	0.476	0.579	1.673			0.5	0.254	0.425	0.513	1.681
3.2	1.0	0	0.575	0.717	0.855	2.446	5.6	7.0	0	1.314	0.726	0.850	1.735
		0.1	0.574	0.750	0.893	2.614			0.1	1.324	0.744	0.873	1.793
		0.2	0.553	0.780	0.927	2.804			0.2	1.299	0.760	0.892	1.848
		0.3	0.505	0.799	0.949	3.010			0.3	1.228	0.767	0.901	1.888
		0.4	0.418	0.782	0.926	3.181			0.4	1.094	0.752	0.885	1.884
		0.45	0.350	0.733	0.867	3.167			0.45	0.990	0.724	0.854	1.835
		0.5	0.135	0.326	0.384	1.539			0.5	0.625	0.493	0.581	1.266
5.6	3.5	0	1.097	0.725	0.853	1.948	5.6	3.0	0	1.042	0.727	0.855	2.010
		0.1	1.104	0.747	0.880	2.032			0.1	1.046	0.750	0.883	2.100
		0.2	1.079	0.767	0.905	2.119			0.2	1.023	0.772	0.910	2.198
		0.3	1.018	0.782	0.923	2.206			0.3	0.961	0.788	0.929	2.294
		0.4	0.905	0.779	0.920	2.263			0.4	0.850	0.784	0.925	2.362
		0.45	0.818	0.758	0.896	2.248			0.45	0.764	0.762	0.900	2.350
		0.5	0.518	0.524	0.621	1.598			0.5	0.488	0.534	0.631	1.697
5.6	2.5	0	0.972	0.730	0.858	2.087	5.6	2.0	0	0.884	0.734	0.861	2.191
		0.1	0.976	0.755	0.888	2.189			0.1	0.886	0.761	0.893	2.306
		0.2	0.951	0.778	0.915	2.297			0.2	0.861	0.786	0.922	2.432
		0.3	0.891	0.794	0.935	2.406			0.3	0.801	0.803	0.943	2.559
		0.4	0.780	0.790	0.930	2.485			0.4	0.693	0.797	0.935	2.654
		0.45	0.697	0.766	0.901	2.474			0.45	0.612	0.769	0.902	2.643
		0.5	0.427	0.518	0.610	1.735			0.5	0.353	0.495	0.580	1.776
5.6	1.5	0	0.767	0.740	0.866	2.340	5.6	1.0	0	0.606	0.752	0.875	2.577
		0.1	0.767	0.770	0.901	2.478			0.1	0.605	0.787	0.915	2.755
		0.2	0.743	0.798	0.933	2.630			0.2	0.581	0.820	0.953	2.956
		0.3	0.685	0.817	0.955	2.787			0.3	0.527	0.841	0.975	3.167
		0.4	0.581	0.807	0.942	2.907			0.4	0.429	0.821	0.950	3.327
		0.45	0.501	0.770	0.899	2.891			0.45	0.354	0.765	0.882	3.287
		0.5	0.252	0.439	0.511	1.744			0.5	0.131	0.329	0.379	1.548
10	8.0	0	1.531	0.765	0.882	1.877	10	4.0	0	1.239	0.759	0.872	2.044
		0.1	1.539	0.784	0.904	1.938			0.1	1.245	0.784	0.900	2.131
		0.2	1.492	0.794	0.916	1.980			0.2	1.214	0.806	0.925	2.218
		0.3	1.407	0.802	0.926	2.027			0.3	1.137	0.819	0.940	2.296
		0.4	1.230	0.776	0.896	1.996			0.4	0.999	0.811	0.930	2.333
		0.45	1.096	0.738	0.853	1.923			0.45	0.896	0.786	0.901	2.303
		0.5	0.639	0.465	0.538	1.231			0.5	0.563	0.542	0.622	1.626
10	3.5	0	1.182	0.761	0.874	2.089	10	3.0	0	1.114	0.763	0.875	2.143
		0.1	1.187	0.787	0.903	2.181			0.1	1.118	0.790	0.905	2.243
		0.2	1.156	0.809	0.928	2.275			0.2	1.088	0.814	0.933	2.346
		0.3	1.083	0.826	0.946	2.365			0.3	1.016	0.831	0.951	2.444
		0.4	0.948	0.818	0.937	2.409			0.4	0.886	0.824	0.943	2.500

Table 4 continued

$\frac{L}{B}$	$\frac{H}{B}$	v	A_R	$\frac{A_R}{A_{Ce}}$	$\frac{A_R}{A_{Av}}$	$\frac{A_R}{A_{Co}}$	$\frac{L}{B}$	$\frac{H}{B}$	v	A_R	$\frac{A_R}{A_{Ce}}$	$\frac{A_R}{A_{Av}}$	$\frac{A_R}{A_{Co}}$
		0.45	0.847	0.792	0.907	2.379			0.45	0.787	0.797	0.911	2.469
		0.5	0.530	0.546	0.625	1.683			0.5	0.493	0.553	0.631	1.763
10	2.5	0	1.033	0.766	0.877	2.214	10	2.0	0	0.931	0.768	0.879	2.306
		0.1	1.035	0.794	0.909	2.322			0.1	0.932	0.798	0.913	0.429
		0.2	1.005	0.819	0.937	2.435			0.2	0.903	0.825	0.944	2.559
		0.3	0.935	0.837	0.957	2.545			0.3	0.836	0.844	0.965	2.688
		0.4	0.809	0.829	0.948	2.611			0.4	0.715	0.835	0.954	2.769
		0.45	0.714	0.800	0.914	2.583			0.45	0.623	0.801	0.915	2.737
		0.5	0.427	0.533	0.608	1.780			0.5	0.351	0.507	0.579	1.806
10	1.5	0	0.801	0.770	0.883	2.442	10	1.0	0	0.626	0.775	0.889	2.660
		0.1	0.800	0.802	0.919	2.585			0.1	0.624	0.813	0.932	2.845
		0.2	0.772	0.832	0.954	2.742			0.2	0.598	0.847	0.971	3.050
		0.3	0.709	0.853	0.977	2.901			0.3	0.540	0.868	0.995	3.259
		0.4	0.594	0.838	0.960	3.002			0.4	0.436	0.844	0.967	3.403
		0.45	0.508	0.797	0.913	2.969			0.45	0.356	0.782	0.896	3.339
		0.5	0.249	0.445	0.510	1.455			0.5	0.128	0.329	0.377	1.536

A useful relationship between the average and the maximum settlement of flexible footings for v = 0–0.5 is also given (see also Fig. 6):

$$\rho_{Av} \approx 0.848\rho_{Ce} \quad (R^2 = 0.9994) \tag{11}$$

4 Correction Factor for Foundation Rigidity

According to ACI Committee 336 (Ulrich et al. 1988), DIN 4018 (1974) and IS 2950-1 (1981), whether a foundation behaves as a rigid or a flexible structure depends on the relative stiffness of the structure and the foundation soil. The relative stiffness factor, K_r, is given as follows:

$$K_r = \frac{E_b I}{E L^3 B} \tag{12}$$

where $E_b I$ is the flexural rigidity of the superstructure and foundation per unit length at right angles to B and E_b is the modulus of concrete. An approximate value of $E_b I$ per unit width of building can be determined by summing the rigidity of foundation, $E_b I_F$, the rigidity of beams, $E_b I_b$ and the rigidity of shearwalls,$E_b t_w h_w^3/12$:

$$E_b I = E_b\left(I_F + \sum I_b + t_w h_w^3/12\right) \tag{13}$$

where t_w and h_w are the thickness and height of the shearwalls respectively (e.g. see Ulrich et al. 1988; DIN 4018 1974; IS 2950-1 1981; Varghese 2005). Ignoring the effect of superstructure, the relative stiffness factor simplifies to:

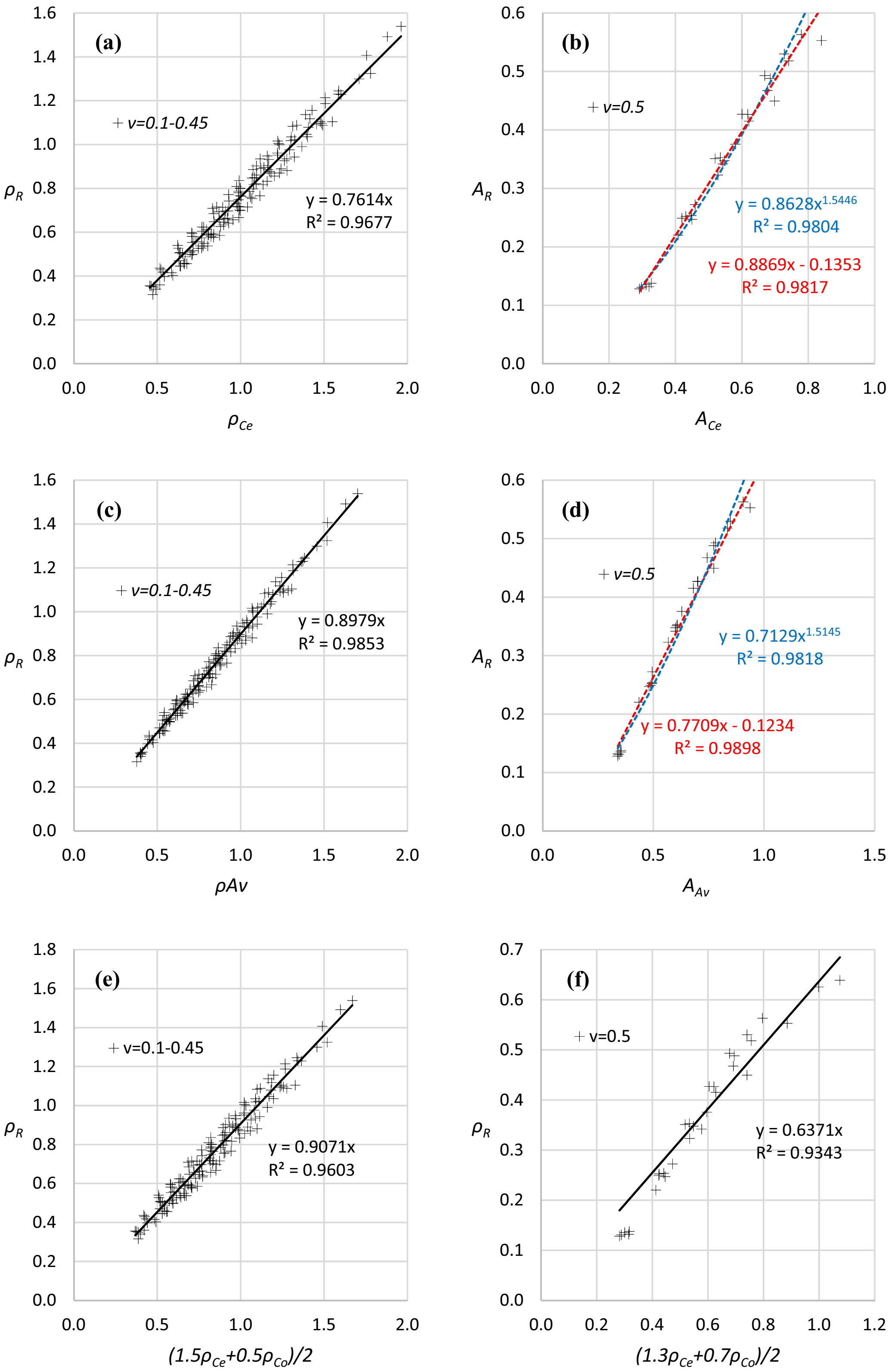
$$K_r = \frac{E_b}{12E}\left(\frac{d}{L}\right)^3 \tag{14}$$

because, $I = Bd^3/12$, where d is the thickness of foundation.

Alternatively, the expression including the Poisson's ratio values can be used (e.g. see Milovic 1992):

$$K_r = \frac{E_b(1-v^2)}{12E(1-v_b^2)}\left(\frac{d}{L}\right)^3 \tag{15}$$

Generally, the foundation is considered to be rigid when $K_r > 0.5$ and flexible when $K_r < 0.5$. Based on the finite element analysis carried out by Brown (1969; see also Mayne and Poulos 1999), however, a foundation can be regarded as flexible if $K_r < 0.05$, rigid if $K_r > 5$ and of intermediate rigidity if $0.05 \le K_r \le 5$. Indeed, between the limits defining the intermediate condition, a linear relationship exists between K_r and settlement. In this respect, the


(a)
+ v=0.1-0.45
y = 0.7614x
R² = 0.9677
ρR
ρCe
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
(b)
+ v=0.5
y = 0.8628x^1.5446
R² = 0.9804
y = 0.8869x - 0.1353
R² = 0.9817
AR
ACe
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0
0.2
0.4
0.6
0.8
1.0
(c)
+ v=0.1-0.45
y = 0.8979x
R² = 0.9853
ρR
ρAv
(d)
+ v=0.5
y = 0.7129x^1.5145
R² = 0.9818
y = 0.7709x - 0.1234
R² = 0.9898
AR
AAv
0.0
0.5
1.0
1.5
(e)
+ v=0.1-0.45
y = 0.9071x
R² = 0.9603
ρR
(1.5ρCe+0.5ρCo)/2
1.8
(f)
+ v=0.5
y = 0.6371x
R² = 0.9343
ρR
(1.3ρCe+0.7ρCo)/2
0.7
1.2

◀**Fig. 4** Charts indicating the proposed relationships

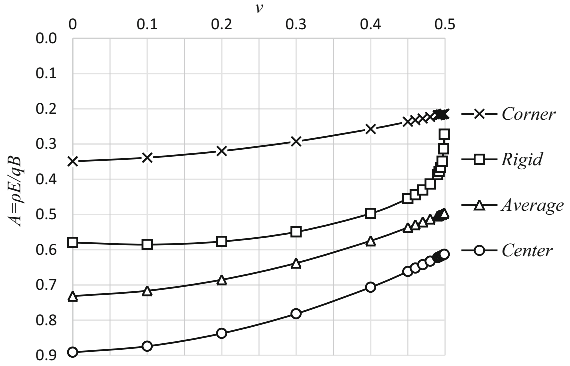


Fig. 5 *A* (i.e. A_R, A_{Ce}, A_{Co} and A_{Av}) versus ν chart for *L*/*B* = 1 and *H*/*B* = 2 (the maximum *ν* value for the rigid footing considered was 0.499)

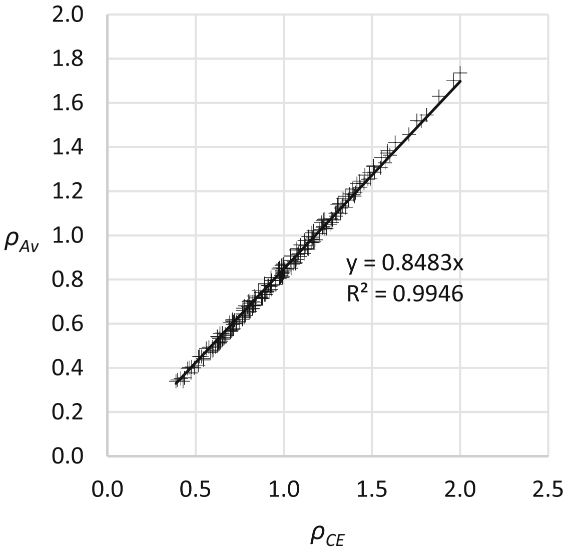


Fig. 6 The derived relationship between the maximum and average settlement of flexible rectangular foundations

following linear interpolation for calculating the settlement of footings of intermediate rigidity is suggested:

$$\rho_{Int} = \rho_R I_F \tag{16}$$

where

$$I_F = 1 + \frac{5 - K_r}{4.95}\left(\frac{\rho_{Ce}}{\rho_R} - 1\right) \tag{17}$$

ρ_{Ce} is calculated using the theory of elasticity (Steinbrenner 1934; Harr 1966) or based on Table 4. The author narrowed the $K_r = 0.01$ and 10 limits, initially proposed by Mayne and Poulos (1999), to exploit the linear relationship that exists for $0.05 \leq K_r \leq 5$. This does not affect the results because these limits are, in essence, values indicating the two starting points (on the "flexible" and "rigid" side) of the asymptotic behavior.

5 Summary and Conclusions

The problem of settlement of shallow foundations is among the most important ones in classical soil mechanics. For rigid rectangular foundations as the actual contact pressure distribution is unknown, the problem is often approximated analytically assuming a contact pressure distribution or semi-empirically combining the theory of elasticity with experimental and/or numerical results. A third and often attractive choice is the use of simple empirical relationships or relevant tabulated values relating the elastic settlement of rigid footings (ρ_R) with the settlement of the respective flexible footings (ρ_{Ce}). According to the literature, the settlement of rigid footings (ρ_R) ranges between 68 and 125% of the maximum settlement of the respective flexible footings (ρ_{Ce}), between 86 and 115% of the average settlement of the respective flexible footings (ρ_{Av}) and between 145 and 172% of the minimum settlement of the respective flexible footings (ρ_{Co}). These values indicate that, not only there is not a consensus in the literature but also some rather unrealistic relationships exist.

The problem of approximating the settlement of rigid footings through simple empirical relationship(s) has been revisited in the present paper comparing the settlement of 210 rigid foundation cases derived from 3D elastic finite element analysis with the settlement of the respective flexible footings. In this respect a number of relationships with coefficient of determination (R^2) almost unity have been proposed. The analysis shows that these relationships are rather insensitive to the aspect ratio of foundation as well as to the thickness and the Poisson's ratio of the compressible layer, although for undrained conditions (ν = 0.5) a separate relationship gives slightly better approximation. Under drained conditions the author suggests that $\rho_R \approx 0.9\rho_{Av}$ or $\rho_R \approx 0.76\rho_{Ce}$, while under undrained conditions $\rho_R \approx 0.64(1.3\rho_{Ce} + 0.7\rho_{Co})/2$. For the average settlement of flexible footings, the author suggests that $\rho_{Av} \approx 0.85\rho_{Ce}$. Finally, because often footings are neither flexible nor rigid, but of intermediate rigidity, a interpolation correction factor for foundation rigidity is given, exploiting the known linear relationship that exists between the relative stiffness factor of foundation and its settlement.

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Code availability Code freelly available at http://random.engmath.dal.ca/rfem.

Compliance with Ethical Standards

Conflict of interest The author declares that they have no conflict of interest.

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