Journal of Financial Econometrics, 2008, Vol. 6, No. 3, 307–325

Periodic Dynamic Conditional Correlations between Stock Markets in Europe and the US

DENISE R. OSBORN
University of Manchester

CHRISTOS S. SAVVA
Economics Research Centre, University of Cyprus

LEN GILL
University of Manchester

ABSTRACT

This study extends the dynamic conditional correlation model of Engle (2002, *Journal of Business and Economic Statistics 20*, 339–350) to allow periodic (day-specific) conditional correlations of shocks across international stock markets. The properties of the resulting periodic dynamic conditional correlation (PDCC) model are examined, focusing particularly on stationarity and the implications for unconditional shock correlations. When applied to the intraweek interactions between six developed European stock markets and the United States over 1993–2005, we find very strong evidence of periodic conditional correlations for the shocks. The highest correlations are generally observed on Thursdays, with these sometimes being twice those on Monday or Tuesday. In addition to these PDCC effects, strong day-of-the-week effects are found in mean returns for the French, Italian, and Spanish stock markets, while periodic effects are also present in volatility for all stock markets except Italy. (*JEL*: G10, G12, G22)

KEYWORDS: Day-of-the-week-effect, dynamic conditional correlations, periodic models, volatility

The authors would like to thank Ralf Becker, Mardi Dungey, Dick van Dijk, together with two anonymous referees and the editor of the journal, for their constructive comments on this work. We are also grateful to Richard Paap for providing us with Gauss routines, which were modified and used for the estimations of the periodic AR-EGARCH model. An earlier version of this paper constituted a chapter of the second author's PhD thesis at the University of Manchester, and he would like to acknowledge financial support from the University of Manchester. The second author would also like to thank Alexandra Petridou. Address correspondence to Denise Osborn, Economics, School of Social Sciences, University of Manchester, UK. Tel.: +44-161-275-4861; e-mail: denise.osborn@manchester.ac.uk

doi: 10.1093/jjfinec/nbn005
Advance Access publication May 8, 2008

An important feature of stock market prices is the presence of so-called calendar effects, by which predictable patterns associated with the month of the year, the day of the week, or the hour of the day exist in mean returns and/or their volatility. Although the existence of such patterns does not necessarily imply that the market is irrational, nevertheless they provide information about the functioning of the markets and the nature of the shocks to which they are subject. Indeed, explanations proposed for the existence of intraweek patterns include settlement procedures (Gibbons and Hess, 1981; Lakonishok and Levi, 1982), trading volumes (Kiymaz and Berument, 2003), the timing, origin and source of information (Penman, 1987; Gau and Hau, 2004; Brusa, Pu, and Schulman, 2005), and dependence on economic factors and macroeconomic news (Steely, 2001; Arshanapalli et al., 2006).

The study of calendar patterns in stock returns has been almost entirely confined to single markets. However, the existence of intraweek patterns in the univariate distributions of returns, combined with recent research pointing to the time-variation in cross-market correlations (including Longin and Solnik, 2001; Kim, Moshirian, and Wu, 2005; Cappiello, Engle, and Sheppard, 2006), suggests that models of daily returns correlations should consider the possibility that these exhibit periodic effects associated with the day of the week. To date, it appears that only Chandra (2006) has examined whether cross-market correlations exhibit intraweek patterns. Even in this case, however, Chandra (2006) considers only deterministic shift effects, whereas in the context of volatility equations it has been shown that day-specific effects also permeate the dynamics (Bollerslev and Ghysels, 1996; Franses and Paap 2000; Fantazzini and Rossi, 2005; Bubak and Zikes, 2006). Consequently, the existence and nature of patterns in daily returns cross-correlations should be examined in the context of a richer specification.

To this end, the present paper extends the dynamic conditional correlation (DCC) model of Engle (2002) to allow for day-specific effects. Adopting the usual terminology that refers to models in which parameters change systematically with the calendar as being periodic, we refer to our model as a periodic dynamic conditional correlation, or PDCC, specification. Properties of the PDCC model are examined, before it is applied to daily closing prices for seven developed stock markets (US, UK, Germany, France, Spain, Italy, and Switzerland). While our results generally confirm previous studies in finding periodic effects in the mean and (more especially) the volatility equations, the evidence of day-specific parameter shifts in the conditional correlations across markets is generally much stronger, with the shock correlations exhibiting intraweek patterns that are persistent over time.

The remainder of this paper is structured as follows. Our PDCC model is described in Section 1, where its properties are also examined. Section 2 then outlines the methodology we use in our empirical application, while Section 3 presents the data and describes its properties in relation to intraweek patterns. Empirical results are presented in Section 4, which includes the results of hypothesis tests that examine the nature of periodic effects. Finally, Section 5 concludes.

1 THE PERIODIC DYNAMIC CONDITIONAL CORRELATION MODEL

After specifying the PDCC model in Section 1.1, its key properties are discussed in Section 1.2.

1.1 Periodic Conditional Correlations

Consider the multivariate $n \times 1$ stochastic process $\{\varepsilon_t\}$ such that

$$\varepsilon_t | \Omega_{t-1} \sim (0, \Sigma_t), \tag{1}$$

where $\varepsilon_t = [\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{nt}]'$ and Ω_{t-1} denotes all information available at time $t-1$, so that ε_{it} is the shock relating to market i for day t. The time-varying matrix of conditional covariances in Equation (1) can be written as

$$\Sigma_t = S_t R_t S_t, \tag{2}$$

where $S_t = diag(\sqrt{h_{1t}}, \sqrt{h_{2t}}, \ldots, \sqrt{h_{nt}})$ is the vector of conditional standard deviations, so that $h_{it} = E[\varepsilon_{it}^2 | \Omega_{t-1}]$ and R_t is the conditional correlation matrix.

Generalizing the DCC model of Engle (2002), the PDCC model for daily data describes the dynamics of the conditional correlations using

$$Q_t = \sum_{s=1}^{5} [C_s + A_s \upsilon_{t-1} \upsilon'_{t-1} A_s + B_s Q_{t-1} B_s] D_{s,t}, \tag{3}$$

in which $\upsilon_{it} = \varepsilon_{it}/\sqrt{h_{it}}$ is the standardized innovation for market i at time t and $\upsilon_t = (\upsilon_{1t}, \upsilon_{2t}, \ldots, \upsilon_{nt})$; C_s is an $n \times n$ symmetric matrix of constants, while A_s and B_s are $n \times n$ diagonal matrices of non-negative constants; the scalar $D_{s,t}$ is a dummy variable for day s, which is unity when t falls on day s ($s = 1, 2, 3, 4, 5$) and zero otherwise. The conditional correlation matrix R_t, with unit diagonal elements, is recovered from Equation (3) using

$$R_t = (Q_t^*)^{-1} Q_t (Q_t^*)^{-1} \tag{4}$$

in which Q_t^* is diagonal with $q_{ii,t}^* = \sqrt{q_{ii,t}}$ and lower case letters indicate the appropriate elements of the corresponding matrices. As in Cappiello, Engle, and Sheppard (2006), Q_t, and hence R_t, is positive-definite with probability 1 if C_s ($s = 1, \ldots, 5$) is positive-definite.

Although the relationship between R_t and Q_t in Equation (4) is nonlinear, Engle (2002, p. 341) parameterizes the nonperiodic DCC model to ensure that the unconditional means of these matrices are equal, that is, $E(R_t) = E(Q_t)$. In the PDCC case, however, the cross-market unconditional shock correlation is day-specific. To capture this, we follow the usual convention of periodic models by working in vector notation (see Tiao and Grupe, 1980; or Osborn, 1991) and define the 5×1 vector of shocks[1] for market i in week w ($w = 1, 2, \ldots, T/5$ for a sample

[1]For simplicity of notation, we assume that the first sample observation relates to the first day of the week (Monday). Also for notational simplicity, $T/5$ is assumed to be an integer.

$t = 1, 2, \ldots, T$) as $V_w^i = (\upsilon_{1w}^i, \upsilon_{2w}^i, \upsilon_{3w}^i, \upsilon_{4w}^i, \upsilon_{5w}^i)'$, where $\upsilon_{sw}^i = \upsilon_{it}$ for $t = 5(w-1) + s$ and $s = 1, 2, 3, 4, 5$. Then the corresponding vector of day-specific periodic unconditional correlations between markets i and j is

$$P_{ij} = \left[E\left(\upsilon_{1w}^i \upsilon_{1w}^j\right),\ E\left(\upsilon_{2w}^i \upsilon_{2w}^j\right),\ E\left(\upsilon_{3w}^i \upsilon_{3w}^j\right),\ E\left(\upsilon_{4w}^i \upsilon_{4w}^j\right),\ E\left(\upsilon_{5w}^i \upsilon_{5w}^j\right)\right]'. \tag{5}$$

Using the same approach, the vector Q_w^{ij} contains the elements $q_{ij,t}$ for week w and, generalizing the restriction of Engle (2002), we impose $E(Q_w^{ij}) = P_{ij}$.

It should be noted that our approach in Equations (1) to (4) is of a multivariate GARCH type, which assumes that Σ_t in Equation (1) is measurable with respect to the information set Ω_{t-1}. We prefer this to a multivariate stochastic volatility form, because parameter estimation is more difficult to achieve in the latter case (see Asai, McAleer, and Yu, 2006). This is an especially important consideration in our case due to increased parameterization arising from the periodic specification.

1.2 Properties of the PDCC Model

Using the periodic vector notation, and noting that the coefficient matrices A_s, B_s are diagonal, the PDCC model of Equation (3) for the conditional correlation between markets i and j can be written as

$$B_0^{ij} Q_w^{ij} = C^{ij} + B_1^{ij} Q_{w-1}^{ij} + A_0^{ij}\left(V_w^i \circ V_w^j\right) + A_1^{ij}\left(V_{w-1}^i \circ V_{w-1}^j\right), \tag{6}$$

where $C_{ij} = (c_{ij,1}, c_{ij,2}, c_{ij,3}, c_{ij,4}, c_{ij,5})'$ is a 5×1 vector of constants and $c_{ij,s}$ is the i,jth element of C_s, $\circ$ denotes the Hadamand product obtained by element-by-element multiplication, and

$$B_0^{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -b_{ii,2}b_{jj,2} & 1 & 0 & 0 & 0 \\ 0 & -b_{ii,3}b_{jj,3} & 1 & 0 & 0 \\ 0 & 0 & -b_{ii,4}b_{jj,4} & 1 & 0 \\ 0 & 0 & 0 & -b_{ii,5}b_{jj,5} & 1 \end{pmatrix},$$

$$A_0^{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ a_{ii,2}a_{jj,2} & 0 & 0 & 0 & 0 \\ 0 & a_{ii,3}a_{jj,3} & 0 & 0 & 0 \\ 0 & 0 & a_{ii,4}a_{jj,4} & 0 & 0 \\ 0 & 0 & 0 & a_{ii,5}a_{jj,5} & 0 \end{pmatrix},$$

while the 5×5 matrices B_1^{ij}, A_1^{ij} have all elements zero, except for the (1,5) element, which is given by $b_{ii,1}, b_{jj,1}$ and $a_{ii,5}a_{jj,5}$, respectively. Further defining $N_w^{ij} = (V_w^i \circ V_w^j) - Q_w^{ij}$, Equation (6) can also be written in the vector ARMA form

$$\left(B_0^{ij} - A_0^{ij}\right)\left(V_w^i \circ V_w^j\right) = C^{ij} + \left(B_1^{ij} + A_1^{ij}\right)\left(V_{w-1}^i \circ V_{w-1}^j\right) + B_0^{ij} N_w^{ij} - B_1^{ij} N_{w-1}^{ij}, \tag{7}$$

where it should be noted that $E(Q_w^{ij}) = P_{ij}$ implies $E(N_w^{ij}) = 0$.

From this representation, it is clear that the existence of the stationary vector $P_{ij} = E(V_w^i \circ V_w^j)$ requires the roots of the characteristic equation of (7) to lie outside the unit circle, namely that

$$\prod_{s=1}^{5}(a_{ii}a_{jj,s} + b_{ii,s}b_{jj,s}) < 1. \tag{8}$$

Clearly, the existence of stationary $E(V_w^i \circ V_w^j)$ for all market pairs requires Equation (8) to be satisfied for all $i, j = 1, \ldots, n$.

Assuming that this stationarity condition is satisfied, taking expectations in Equation (6) yields

$$C^{ij} = \left(B_0^{ij} - A_0^{ij} - B_1^{ij} - A_1^{ij}\right)P_{ij}. \tag{9}$$

From the definitions of B_k^{ij}, A_k^{ij} $(k = 0, 1)$, Equation (9) can be written in the notation of Equation (3) as

$$C_s = \bar{R}_s - A_s\bar{R}_{s-1}A_s - B_s\bar{R}_{s-1}B_s, \tag{10}$$

where the (i, j)th element of the $n \times n$ matrix $\bar{R}_s$ is the sth element of P_{ij}. Therefore, Equation (10) provides the link between the unconditional correlations and the parameters of Equation (3).

2 EMPIRICAL METHODOLOGY

This outline of our empirical methodology discusses the mean and volatility equations we employ, followed by issues related to hypothesis testing and estimation of the PDCC model.

2.1 Mean and Volatility Equations

To ensure unmodeled periodic mean and volatility effects do not distort inferences relating to the PDCC specification, a periodic autoregressive (PAR) specification allows the responses of returns to vary with the day of the week, while a periodic EGARCH (PEGARCH) specification does the same for the volatility. Denoting an individual stock index at time t by P_t, the PAR(p)–PEGARCH(1,1) model for the continuously compounded stock returns $y_t = 100^*[\ln(P_t) - \ln(P_{t-1})]$, is given by

$$y_t = \sum_{s=1}^{5}\left[\beta_s + \sum\nolimits_{i=1}^{p}\phi_{is}y_{t-i}\right]D_{s,t} + \varepsilon_t \tag{11}$$

$$\varepsilon_t = \upsilon_t\sqrt{h_t}, \quad \upsilon_t \sim iid(0, 1) \tag{12}$$

$$h_t = \sum_{s=1}^{5}[\exp\{\omega_s + \gamma_s\upsilon_{t-1} + \theta_s(|\upsilon_{t-1}| - E\,|\upsilon_{t-1}|) + \delta_s \ln h_{t-1}\}]D_{s,t}. \tag{13}$$

Note that all parameters in Equations (11) and (13), namely β_s, ϕ_{is}, ω_s, γ_s, θ_s, and δ_s, are allowed to be day-specific.[2] In practice, a PAR(1) is sufficient in Equation (11).[3] The stationarity conditions and unconditional mean vectors for PAR processes are discussed by, among others, Tiao and Grupe (1980), Osborn (1991), Franses and Paap (2000), and Ghysels and Osborn (2001, pp. 144–147).

The EGARCH volatility specification of Nelson (1991) is adopted to ensure that all implied volatilities are positive while also allowing the possibility of asymmetry. First-order dynamics in Equation (13) are sufficient to adequately capture dynamics, with day-specific volatility persistence measured by δ_s and the magnitude effect captured by θ_s, while volatility asymmetry typically implies negative γ_s. Our specification differs from previous studies that employ periodic GARCH models (including Bollerslev and Ghysels, 1996; Franses and Paap, 2000; Fantazzini and Rossi, 2005) in that we allow δ to vary over the days of the week and employ an EGARCH specification to ensure positive implied variances.

Employing the vector approach in order to generalize the nonperiodic analysis of Nelson (1991), the PEGARCH model of Equation (13) can be written as

$$\Delta_0 H_w = \omega + \Delta_1 H_{w-1} + U_w, \tag{14}$$

where $Hw = [\ln(h_{5(w-1)+1}), \ln(h_{5(w-1)+2}), \ldots, \ln(h_{5(w-1)+5})]$ is the vector of log conditional volatility for week w, $\omega = (\omega_1, \omega_2, \ldots, \omega_5)'$, the 5×5 matrices Δ_0 and Δ_1 are defined analogously to B_0^{ij}, B_1^{ij} above with δ_s replacing $b_{ii,s}b_{jj,s}$, and the sth element of the 5×1 vector U_w is given by $\gamma_s \upsilon_{5(w-1)+s-1} + \theta_s(|\upsilon_{5(w-1)+s-1}| - E|\upsilon_{5(w-1)+s-1}|)$. From Equation (14) it is clear that the PEGARCH process has a constant mean if $|\delta_1\delta_2\delta_3\delta_4\delta_5| < 1$ and is integrated if this product is unity.[4] With $|\delta_1\delta_2\delta_3\delta_4\delta_5| < 1$, taking expectations in Equation (14) yields

$$E[H_w] = (\Delta_0 - \Delta_1)^{-1}\omega, \tag{15}$$

where the day-specific log volatility in Equation (15) arises through periodic variation in the intercepts ω_s and/or the persistence coefficients δ_s of Equation (13).

2.2 Hypotheses of Interest

Previous literature finds evidence of periodic effects in daily stock market returns and their volatility (Keim and Stambaug, 1984; Bessembinde and Herzel, 1993; Bollerslev and Ghysels, 1996; Franses and Paap, 2000; Tsiakis, 2006, among others). To investigate whether these apply in our case, the following hypotheses are

[2]In principle, the model of Equation (11) could include cross-market lagged periodic effects in the returns. However, we found this to be infeasible for the present application, due to problems of dimensionality and convergence in estimation.

[3]Models with AR lags 1 and 5 were also investigated to capture any systematic weekly patterns in the data. However, the fifth-order lag coefficients were insignificant, with AR(1) models also preferred by both AIC and SIC.

[4]The properties of unconditional volatility are complex in an EGARCH model; see, for example, the expressions of Karanasos and Kim (2003) for the moments of ε_t^2 in a nonperiodic EGARCH specification. Since our focus is the PDCC model, we do not explore the properties of unconditional volatility.

examined for the parameters of Equations (11)–(13):

$$H_1: \phi_{is} = \phi_i \quad i = 1, \ldots, p;\ s = 1, 2, 3, 4, 5$$
$$H_2: \phi_{is} = \phi_i,\ \beta_s = \beta \quad i = 1, \ldots, p;\ s = 1, 2, 3, 4, 5$$
$$H_3: \delta_s = \delta \quad s = 1, 2, 3, 4, 5$$
$$H_4: \omega_s = \omega,\ \theta_s = \theta,\ \gamma_s = \gamma,\ \delta_s = \delta \quad s = 1, 2, 3, 4, 5.$$

In addition to the general tests H_2 and H_4, these hypotheses are designed to shed light on the nature of any periodic variation.

For our principal interest, namely the PDCC model, we employ four hypothesis tests, namely

$$H_5: A_s = A \quad s = 1, 2, 3, 4, 5$$
$$H_6: B_s = B \quad s = 1, 2, 3, 4, 5$$
$$H_7: A_s = A,\ B_s = B \quad s = 1, 2, 3, 4, 5$$
$$H_8: a_{jj,s} = a_s,\ b_{jj,s} = b_s \quad j = 1, \ldots, n;\ s = 1, 2, 3, 4, 5.$$

Thus, in addition to the overall test of H_7, H_5 and H_6 examine whether any periodicity is confined to the parameters capturing short-term or long-term persistency, respectively. Finally, H_8 examines whether the PDCC coefficients vary over countries, or whether common day-specific effects apply.

2.3 Estimation and Inference

Engle and Sheppard (2001) show that the log-likelihood function for a DCC model can be written as the sum of a returns/volatility part and a correlation part. Denoting the parameters of Equations (11) and (13) by the vector ξ and the parameters of the PDCC model by ζ, this implies for our case that

$$L(\xi, \zeta) = L_v(\xi) + L_c(\xi, \zeta) \tag{16}$$

with

$$L_v(\xi) = -\frac{1}{2}\sum_{t=1}^{T}(n\log(2\pi) + 2\log|S_t| + \varepsilon_t' S_t^{-1} S_t^{-1}\varepsilon_t, \tag{17}$$

$$L_c(\xi, \zeta) = -\frac{1}{2}\sum_{t=1}^{T}\left(-v_t' v_t + \log|R_t| + v_t' R_t^{-1} v_t\right). \tag{18}$$

Following Engle (2002), consistent parameter estimates are obtained by first maximizing the volatility likelihood (17) separately for each market, which involves simultaneous estimation of the parameters of Equations (11)–(13) for each observed returns series, to find $\hat{\xi} = \arg\max\{L_v(\xi)\}$. The standardized residuals from this first-stage estimation are then used to estimate the DCC parameters ζ of Equation (3) as $\hat{\zeta} = \arg\max\{L_c(\hat{\xi}, \zeta)\}$. This second-stage estimation is conducted jointly across all market pairs, with the standard errors calculated using the method described by Engle and Sheppard (2001) to take account of the first-step estimation.[5]

In this second stage, in an analogous way to the nonperiodic approach of Engle (2002), the constant matrix C_s of Equation (3) is replaced by Equation (10) with unknown $\bar{R}_s$ estimated from the corresponding day-specific sample correlation matrix of the standardized residuals. It was noted in Section 1 above that the conditional correlation matrix R_t of Equation (3)/(4) is positive-definite, with probability 1, provided C_s is positive-definite. Although Equation (10) does not apparently guarantee positive-definite C_s, this is checked after estimation and no violation of positive-definiteness occurs.

Inference related to periodic effects in the PAR–PEGARCH model, captured through H_1 to H_4, is conducted via Wald tests applied in Equations (11)–(13), while the PDCC tests of H_5 to H_8 are conducted using likelihood ratio statistics.[6] All test statistics are compared to the asymptotic χ^2 distribution, with degrees of freedom given by the number of restrictions.

3 DATA

Our stock market data consist of the closing daily prices of S&P500 (USA), DAX-30 (Germany), FTSE-100 (UK), CAC-40 (France), IBEX-35 (Spain), and the total indices of the Italian and Swiss markets.[7,8] These account for more than 80% of the total stock market capitalization in Europe, and four of the countries (Germany, France, Italy, and Spain) have adopted the common euro currency. The US, UK, and German stock markets are the leading world markets, while Switzerland attracts international investment due to its political and economic stability and the traditionally high quality of services provided. The sample period is from January 1, 1993 to April 30, 2005.[9]

[5]Cappiello, Engle, and Sheppard (2006) point out that this approach assumes that the univariate first-stage models are correctly specified. However, many univariate models imply similar volatility patterns, and hence standardized residuals with similar characteristics. Therefore, it is anticipated that the correlation patterns are not unduly dependent on the specific univariate model employed.

[6]Estimation of the PDCC model is computationally burdensome, and likelihood ratio tests are used for these joint hypotheses due to their computational convenience.

[7]Closing prices are nonsynchronous across countries (especially between US and European stock markets), which may lead to underestimation of correlations (see Martens and Poon, 2001). Although the results reported are based on closing data, as we believe these best represent daily returns and volatilities for each market, models were also estimated using synchronous data (pseudo closing prices, recorded at 16:00 London time), with qualitatively similar results.

[8]DAX-30 is a price-weighted index of the 30 most heavily traded stocks in the German market, while FTSE-100 consists of the largest 100 UK companies by full market value. CAC-40 is calculated on the basis of 40 largest French stocks based on market capitalization on the Paris Bourse. IBEX-35 is composed of the 35 securities quoted on the Joint Stock Exchange System of the four Spanish Stock Exchanges, while S&P500 is a value-weighted index representing approximately 75% of the total US market capitalization.

[9]Since the data come from different countries, different holidays apply across markets. To ensure complete samples for all countries, a missing value is replaced by the closing price on the day before the holiday.

Descriptive information on the patterns of cross-market correlations of returns is provided by the sample correlations shown in Table 1 for all days and separately for each day of the week. These indicate that all markets tend to be most strongly correlated with the United States toward the end of the week and least correlated around mid-week. Indeed, the US/UK correlation on Wednesday (0.26) is half the corresponding correlation for Friday (0.51). However, although correlations across European markets are generally higher than those with the United States, systematic intraweek correlation patterns are less evident across European pairs. Nevertheless, this descriptive information cannot indicate the source(s) and significance of differences across days, which is the focus of the estimated models.

4 EMPIRICAL RESULTS

Empirical results for the mean and volatility discussed in the first subsection, followed by the PDCC results in Section 4.2.

4.1 Mean and Volatility Equations

Panel A of Table 2 presents estimated values of the parameters of Equations (11)–(13) and Panel B gives results relating to the hypotheses H_1 to H_4 outlined in Section 2.2. Before considering detailed results, it should be noted that the diagnostic statistics indicate that these models adequately describe the variation in the conditional mean and variance of stock returns.[10]

Table 2 (Panel B) shows strong evidence (at 1% significance for H_1 or H_2, or both) of periodic effects in the mean equations for Italy, Spain, and France, and the corresponding estimates in Panel A are characterized by positive AR coefficients on Mondays and Fridays that are statistically significant at 5%, and smaller coefficients for other days that are typically insignificant and sometimes negative. The UK also displays some evidence of a PAR mean equation. Although, overall, a nonperiodic specification is adequate for the remaining markets, all Monday AR coefficients are positive (and significant at 10%), which is compatible with findings of Franses and Paap (2000), Herwartz (2000), and Bubak and Zikes (2006).

However, the general tests present stronger evidence of periodic effects in volatility than in the mean, with all countries except Italy rejecting the overall hypothesis of nonperiodic volatility (H_4) at 10% or less, with this periodic volatility confirming results of Bollerslev and Ghysels (1996), Franses and Paap (2000), Fantazzini and Rossi (2005), and Bubak and Zikes (2006). Although the relatively weak evidence in Table 2 of periodic volatility for the United States differs from the findings of Franses and Paap (2000) and Bollerslev and Ghysels (1996), these studies use a GARCH form and hence do not account for the asymmetry evident in the estimated γ_s for the United States in Table 2. Nevertheless, the general finding of stronger periodic effects in voatility than in returns themselves accords with results of Tsiakis (2006), who adopts a periodic stochastic volatility model for the United States.

[10] These results are available upon request.

Table 1 Unconditional correlations for daily returns.

	UK	Germany	France	Italy	Spain	Switzerland	USA
Overall							
UK	1						
Germany	0.691	1					
France	0.782	0.765	1				
Italy	0.505	0.560	0.551	1			
Spain	0.686	0.691	0.775	0.538	1		
Switzerland	0.715	0.710	0.735	0.519	0.679	1	
USA	0.410	0.466	0.429	0.242	0.386	0.378	1
Monday							
UK	1						
Germany	0.747	1					
France	0.779	0.821	1				
Italy	0.554	0.594	0.579	1			
Spain	0.696	0.750	0.784	0.591	1		
Switzerland	0.756	0.788	0.781	0.579	0.726	1	
USA	0.435	0.510	0.463	0.305	0.434	0.425	1
Tuesday							
UK	1						
Germany	0.713	1					
France	0.800	0.784	1				
Italy	0.491	0.560	0.531	1			
Spain	0.674	0.667	0.772	0.482	1		
Switzerland	0.704	0.733	0.762	0.503	0.677	1	
USA	0.382	0.353	0.399	0.175	0.320	0.275	1
Wednesday							
UK	1						
Germany	0.688	1					
France	0.765	0.760	1				
Italy	0.449	0.533	0.532	1			
Spain	0.687	0.686	0.776	0.514	1		
Switzerland	0.675	0.692	0.709	0.473	0.674	1	
USA	0.257	0.402	0.296	0.134	0.280	0.280	1
Thursday							
UK	1						
Germany	0.649	1					
France	0.816	0.747	1				
Italy	0.531	0.576	0.588	1			
Spain	0.716	0.699	0.805	0.572	1		
Switzerland	0.737	0.671	0.726	0.542	0.687	1	
USA	0.453	0.551	0.484	0.336	0.441	0.462	1

Table 1 (*continued*)

	UK	Germany	France	Italy	Spain	Switzerland	USA
Friday							
UK	1						
Germany	0.651	1					
France	0.746	0.705	1				
Italy	0.491	0.528	0.523	1			
Spain	0.655	0.645	0.731	0.525	1		
Switzerland	0.693	0.650	0.684	0.483	0.626	1	
USA	0.513	0.511	0.498	0.254	0.454	0.438	1

Note that the rejection of H_3 (at the 10% level) for all countries implies that the nonperiodic persistence term used in previous studies is inappropriate. For the major international markets of the US, UK, Germany, and France, $\hat{\delta}_s$ is largest on Thursday, so that volatility effects are transferred more strongly from Wednesday to Thursday than between other days. Also note that, although some individual $\hat{\delta}_s$ in Table 2 exceed unity, our analysis above shows that the periodic integration hypothesis depends on the product of these coefficients. The estimated model clearly satisfies the condition $|\delta_1\delta_2\delta_3\delta_4\delta_5| < 1$ and, although the detailed results are not shown to conserve space, the integration EGARCH hypothesis is strongly rejected for all markets.

4.2 Conditional Correlations

Turning to the PDCC results of Table 3[11], notice first, that the marginal significance level for H_5 in Panel B is around 5%, so that short-run persistency, measured by the matrix coefficients A_s in Equation (3), does not have strong periodic variation. In contrast, hypotheses relating to nonperiodicity of the overall conditional correlation model and the long-run persistency coefficients (H_7 and H_6, respectively) are rejected at less than 0.1% significance. This not only emphasizes the inadequacy of a nonperiodic DCC model, but also emphasizes the persistency over time in the periodic effects. Further, the strong rejection of common PDCC coefficients across markets (H_8) indicates that the nature of this periodicity depends on the origin of the shock. However, it should be noted that the estimates satisfy the stationarity condition of Equation (8), with the null hypothesis of an integrated PDCC model decisively rejected for all market pairs.[12]

The estimated PDCC coefficients in Table 3 exhibit some clear patterns. For instance, European markets have their highest long-run persistence coefficients at the end of the week (Friday in all cases, except for Thursday in Germany),

[11] In addition, we also estimated scalar versions of the PDCC model, in which the coefficients $a_{jj,s}$ and $b_{jj,s}$ are restricted to be nonperiodic and constant across countries ($a_{jj,s} = a, b_{jj,s} = b, j = 1, \ldots, 7, s = 1, \ldots, 5$). However, these restrictions are always rejected at a low marginal significance level and hence the results are not reported.

[12] The results of these integration tests are available on request.

Table 2 PAR–PEGARCH results.

Panel A: Estimated PAR–PEGARCH models

Day	Coefficient	UK	Germany	France	Italy	Spain	Switzerland	USA
Monday	β_1	−0.001	0.059	0.016	0.016	−0.008	0.052	0.072**
	ϕ_{11}	0.134***	0.076*	0.087**	0.324***	0.199***	0.109**	0.056*
	ω_1	−0.053	0.026	0.043	−0.044	0.076	−0.086	−0.155**
	γ_1	−0.026	−0.029	−0.059***	−0.036	−0.017	−0.066**	−0.101***
	θ_1	0.101***	0.118***	0.140***	0.210***	0.167***	0.140***	0.148***
	δ_1	0.763***	1.010***	0.872***	1.098***	0.923***	0.812***	0.986***
Tuesday	β_2	0.027	0.049	0.046	0.029**	0.073*	0.023	0.010
	ϕ_{12}	−0.046	−0.044	−0.063*	0.081**	−0.045	−0.052	−0.041
	ω_2	0.041	−0.024	0.090	−0.007	0.036	0.165	0.078
	γ_2	−0.095***	−0.034	−0.035*	−0.038	−0.049*	−0.026	−0.079***
	θ_2	0.114***	0.159***	0.106***	0.143***	0.153***	0.184***	0.083***
	δ_2	1.256***	1.066***	1.066***	0.888***	1.001***	1.176***	0.911***
Wednesday	β_3	0.008	0.034	−0.040	0.010	−0.026	0.030	0.047*
	ϕ_{13}	0.028	−0.051	0.001	0.143***	0.070	0.015	0.001
	ω_3	0.042	0.170**	−0.020	−0.053	0.036	−0.167	0.027
	γ_3	−0.051***	−0.055***	−0.059***	−0.038	−0.088***	−0.097***	−0.098***
	θ_3	0.124***	0.226***	0.127***	0.218***	0.180***	0.149***	0.130***
	δ_3	0.791***	0.790***	0.775***	1.081***	0.845***	0.930***	0.953***
Thursday	β_4	0.021	−0.015	0.018	0.052	0.116***	0.040	−0.021
	ϕ_{14}	−0.032	−0.027	−0.055	0.084**	0.033	0.008	0.048
	ω_4	−0.090	0.070	−0.073	0.299***	0.127	0.318***	0.040
	γ_4	−0.101***	−0.058**	−0.048*	−0.041	−0.049**	−0.077**	−0.092***
	θ_4	0.141***	0.132***	0.055	0.185***	0.032	0.133***	0.070***
	δ_4	1.329***	1.118***	1.386***	0.923***	1.030***	0.925***	1.151***

Table 2 (*continued*).

Friday	β_5	0.033	0.038	0.050	0.026	0.116***	0.059*	0.015
	ϕ_{15}	0.022	0.014	0.098**	0.244***	0.091**	0.071*	0.037
	ω_5	0.050	−0.214*	−0.018	−0.164	−0.274	−0.222**	0.007
	γ_5	−0.064***	−0.100***	−0.068***	−0.040	−0.036	−0.147***	−0.084***
	θ_5	−0.008	0.079**	0.026	0.138**	0.075	0.130***	0.108***
	δ_5	0.933***	0.974***	0.951***	0.951***	1.146***	1.069***	0.941***

Panel B: Hypothesis tests for PAR–PEGARCH models

Hypothesis	*DoF*	UK	Germany	France	Italy	Spain	Switzerland	USA
Mean equation								
H_1 Nonperiodic AR	4	12.87**	6.02	16.15***	21.53***	20.33***	8.21*	3.14
H_2 Nonperiodic Intercept-AR	8	13.18	7.59	19.15**	26.01***	29.94***	10.92	8.82
Volatility equation								
H_3 Nonperiodic persistence	4	11.32**	28.87***	9.32*	7.88*	43.08***	10.34**	8.84*
H_4 Nonperiodic EGARCH	16	28.06**	32.92***	25.96*	19.92	28.74***	26.47**	25.01*

The estimated model is given by Equations (11)–(13).
H_1 and H_2 examine periodicity in Equation (11), while H_3 and H_4 relate to Equation (13). All statistics are computed as Wald tests and are compared to an asymptotic χ^2 distribution, with degrees of freedom indicated by *DoF*.
*** denotes significance at 1% level; ** denotes significance at 5% level; * denotes significance at 10% level.

Table 3 Periodic dynamic conditional correlation results.

Panel A: Estimated periodic DCC model

	UK	Germany	France	Italy	Spain	Switzerland	USA
Short-run persistence ($a_{jj,s}$)							
Monday	0.153***	0.066***	0.117***	0.102***	0.104***	0.155***	0.048***
Tuesday	0.167***	0.102***	0.068***	0.048***	0.094***	0.104***	0.065***
Wednesday	0.140***	0.117***	0.148***	0.102***	0.148***	0.105***	0.056***
Thursday	0.107***	0.101***	0.106***	0.089***	0.125***	0.078***	0.058***
Friday	0.093***	0.076***	0.069***	0.090***	0.122***	0.080***	0.036**
Long-run persistence ($b_{jj,s}$)							
Monday	1.023***	0.825***	1.075***	1.206***	0.785***	1.205***	1.025***
Tuesday	1.027***	0.950***	0.803***	0.870***	1.031***	0.876***	1.205***
Wednesday	0.867***	1.026***	1.067***	0.883***	1.165***	0.888***	0.993***
Thursday	0.902***	1.231***	0.805***	0.858***	0.818***	0.821***	0.985***
Friday	1.135***	0.988***	1.305***	1.218***	1.239***	1.225***	0.841***

Panel B: Hypothesis tests for periodic DCC model

Hypothesis		*DoF*	Statistic
H_5	Nonperiodic a_{jj}	28	41.56**
H_6	Nonperiodic b_{jj}	28	116.78***
H_7	Nonperiodic DCC	56	169.56***
H_8	Simple PDCC	60	218.26***

H_5 *to* H_8 examine periodicity in Equation (3) using likelihood ratio test statistics. All statistics are computed using Wald tests and compared to an asymptotic χ^2 distribution with degrees of freedom indicated by *DoF*.
*** denotes significance at 1% level; ** denotes significance at 5% level.

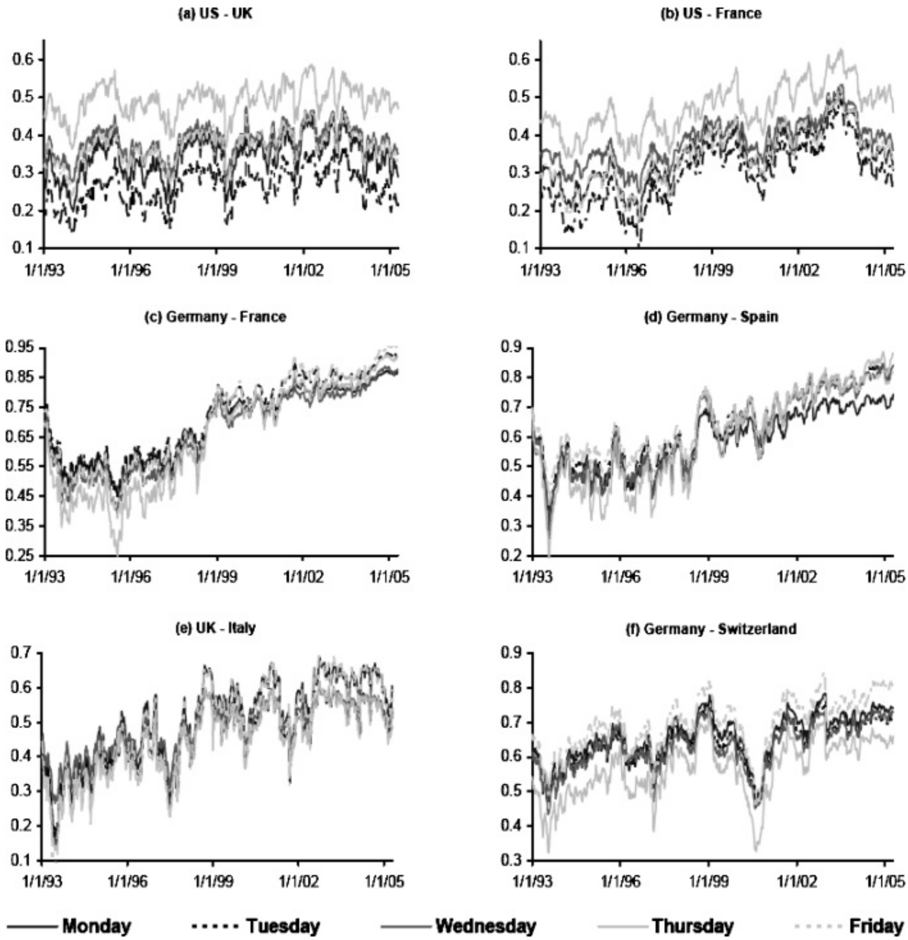

Figure 1 Daily conditional correlation plots.

whereas the United States has its smallest $\hat{b}_{jj,s}$ on Friday and largest on Tuesday. However, detailed interpretation is difficult from these coefficients, and hence Figure 1 provides indicative plots of the patterns found in the day-specific effects for the dynamic conditional correlations across markets.

Throughout the sample period, panel (a) shows that the highest conditional correlations between the US and UK markets occur on Thursdays and the lowest on Tuesdays, with the former correlation being, on average, around double the latter, with other days being intermediate between these extremes. The US/France plot, panel (b), shows a broadly similar pattern, as do other plots (not shown) for European markets with the United States. Indeed, with the single exception of Italy, the highest conditional correlations of all European markets with the United States occur on Thursdays, with the lowest being at the beginning of the week

(Monday or Tuesday).[13] The repetition of this daily pattern of correlations across European markets with the United States points to the important role of the latter and implies a systematic source for the high Thursday correlations. Also note that these substantial relative differences in correlations with the United States across the days of the week are largely unchanged from the unconditional returns correlations shown in Table 1.

Interrelationships between stock markets in Europe are of considerable interest in the context of the introduction of the euro currency, and conditional correlations for the important euro area stock markets of Germany and France are shown in panel (c) of Figure 1. While the highest correlations tend to occur on Tuesday in the earlier part of the sample, with lowest correlations on Thursdays, the relative positions alter from around late 1998 and the highest correlations are on Friday toward the end of the sample period. Another notable feature here is that the shock correlations increase to 0.8 or more in the post-euro period, which supports previous studies examining the impact of the euro (see Kim, Moshirian, and Wu, 2005; Savva, Osborn, and Gill, 2005; Cappiello, Engle, and Sheppard, 2006; Bartram, Taylor, and Wang, 2007, among others).

Similar patterns to these apply to other euro area stock market pairs. In particular, these correlations increase in the late 1990s, with the highest correlations subsequently applying at the end of the week (Thursdays or Fridays). Historically high correlations also apply at the end of the sample period. A further illustration of these patterns is given by the Germany/Spain conditional correlations in panel (d), where a relatively low Monday correlations from the late 1990s is particularly evident.

Panels (e) and (f) of Figure 1, relating to UK/Italy and Germany/Switzerland, illustrate the temporal patterns in the conditional correlations between European markets for non-euro with euro area members. Once again, the highest correlations occur at the end of the week in the later part of the sample period. However, in some cases, the periodic pattern remains relatively constant over time, whereas in others it changes, as illustrated by panels (f) and (e), respectively.

To complement the bivariate analyses of Figure 1, Figure 2 shows the determinant of the periodic conditional correlation matrix R_t obtained using the parameter estimates of Table 2.[14] Since perfect correlations between markets will imply a zero determinant, the computed values give a measure of how far the seven markets are from being completely integrated. According to this measure, the markets are closest to being integrated on Wednesdays in the early part of the sample and furthest from integrated on Thursdays, with the latter reflecting the relatively low correlations between European markets on this day in Figure 1. However, the dominant implication of this figure is that the determinant is of a much smaller magnitude in the later part of the sample, reflecting the increase in conditional correlations, especially within Europe, from around 1998.

[13] The highest Italy/US conditional correlations apply on Wednesdays, with the second highest on Thursdays.

[14] We are grateful to a referee for suggesting the inclusion of a multivariate measure of this type.

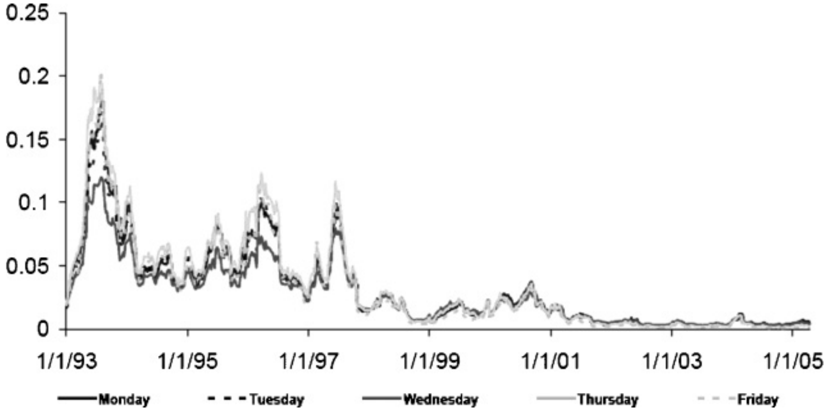

Figure 2 Day-specific determinant of the conditional correlation matrix.

5 CONCLUSIONS

This paper argues that day-of-the-week effects may apply for the conditional correlations of shocks across international stock markets and develops a periodic dynamic conditional correlation model to capture these. The properties of this model are investigated, including the appropriate stationarity condition. Empirically, our analysis applies a periodic DCC model to daily returns data for six European stock markets and the United States from January 1993 to April 2005.

Although we find periodic effects in the mean equations for most stock markets, evidence of periodic variation in the coefficients of the volatility equation is generally stronger. Despite allowing for these effects, we find clear evidence of day-of-the-week patterns in the conditional correlations between markets. For the recent past, such correlations between European markets are highest at the end of the week (Thursdays and Fridays), while correlations of these markets with the United States are often highest on Thursday and lowest at the beginning of the week.

Our hypothesis is that the pattern in conditional correlations through the week may be related to news announcements for important US and European macroeconomic variables. For instance, US employment reports are released on Fridays, producer price index on Thursdays (until 2004) or Fridays (from 2005), while both euro area monetary policy decisions (since 1999) and Bank of England (since 1997) monetary policy decisions are announced on Thursdays.[15] It is also notable that day-of-the-week patterns sometimes change, which may be associated with timings of announcements relevant to the euro area differing from announcements related to individual countries for the earlier subsample. This suggests an important

[15]More details are available on the webpage of the US Bureau of Labor Statistics (http://stats.bls.gov), European Central Bank (www.ecb.int), and Bank of England (www.bankofengland.co.uk).

direction for future research, but it is beyond the scope of the present study to explore these issues in detail.

Received April 24, 2007; revised February 18, 2008; accepted February 27, 2008.

References

Arshanapalli, B., E. d'Ouville, F. Fabozzi, and L. Switzer. (2006). "Macroeconomic News Effects on Conditional Volatilities in the Bond and Stock Markets." *Applied Financial Economics 16*, 377–384.

Asai, M., M. McAleer, and J. Yu. (2006). "Multivariate Stochastic Volatility: A Review." *Econometric Reviews 25*, 145–175.

Bartram, S. M., S. J. Taylor, and Y.-H. Wang. (2007). "The Euro and European Financial Market Integration." *Journal of Banking and Finance 31*, 1461–1481.

Bessembinder, H., and M. G. Hertzel. (1993). "Return Autocorrelations around Non-trading Days." *Review of Financial Studies 6*, 155–189.

Bollerslev, T., and E. Ghysels. (1996). "Periodic Autoregressive Conditional Heteroskedasticity." *Journal of Business and Economic Statistics 14*, 139–151.

Brusa, J., L. Pu, and C. Schulman. (2005). "Weekend Effect, 'Reverse' Weekend Effect, and Investor Trading Activities." *Journal of Business Finance and Accounting 32*, 1495–1517.

Bubak, V., and F. Zikes. (2006). "Seasonality and the Nontrading Effect on Central-European Stock Markets." *The Czech Journal of Economics and Finance 56*, 1–7.

Cappiello, L., R. F. Engle, and K. Sheppard. (2006). "Asymmetric Dynamics in the Correlation of Global Equity and Bond Returns." *Journal of Financial Econometrics 4*, 537–572.

Chandra, M. (2006). "The Day-of-the-Week Effect in Conditional Correlation." *Review of Quantitative Finance and Accounting 27*, 297–310.

Engle, R. F. (2002). "Dynamic Conditional Correlation: A Simple Class of Multivariate GARCH Models." *Journal of Business and Economic Statistics 20*, 339–350.

Engle, R. F. and K. Sheppard. (2001). "Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH." Working Paper 2001–15, Department of Economics, University of California San Diego.

Fantazzini, D., and E. Rossi. (2005). "Asymmetric Periodic Models for High Frequency Data Analysis." *Presented (Poster Session) at the Conference on "Changing Structures in International and Financial Markets and the Effects on Financial Decision Making"*, Venice, Italy, June 2–3, 2005.

Franses, P. H., and R. Paap. (2000). "Modelling Day-of-the-Week Seasonality in the S&P 500 Index." *Applied Financial Economics 10*, 483–488.

Gau, Y.-F., and M. Hau. (2004). "Public Information, Private Information, Inventory Control, and Volatility of Intraday NTD/USD Exchange Rates." *Applied Economics Letters 11*, 263–266.

Ghysels, E., and D. R. Osborn. (2001). "The Econometric Analysis of Seasonal Time Series." Cambridge: Cambridge University Press.

Gibbons, M. R., and P. Hess. (1981). "Day of the Week Effects and Asset Returns." *Journal of Business 54*, 579–596.

Herwartz, H. (2000). "Weekday Dependence of German Stock Market Returns." *Applied Stochastic Models in Business and Industry 16*, 47–71.

Karanasos, M., and J. Kim. (2003). "Moments of the ARMA-EGARCH Model." *Econometrics Journal 6*, 146–166.

Keim, D., and R. Stambaugh. (1984). "A Further Investigation of the Weekend Effect in Stock Returns." *Journal of Finance 39*, 819–835.

Kim, S. J., F. Moshirian, and E. Wu. (2005). "Dynamic Stock Market Integration Driven by the European Monetary Union: An Empirical Analysis." *Journal of Banking and Finance 29*, 2475–2502.

Kiymaz, H., and H. Berument. (2003). "The Day of the Week Effect on Stock Market Volatility and Volume: International Evidence." *Review of Financial Economics 12*, 363–380.

Lakonishok, J., and M. Levi. (1982). "Weekend Effects on Stock Returns: A Note." *Journal of Finance 37*, 883–889.

Longin, F., and B. Solnik. (2001). "Extreme Correlation of International Equity Markets." *Journal of Finance 56*, 649–676.

Martens, M., and S.-H. Poon. (2001). "Returns Synchronization and Daily Correlation Dynamics between International Stock Markets." *Journal of Banking and Finance 25*, 1805–1827.

Nelson, D. B. (1991). "Conditional Heteroskedasticity in Asset Returns: A New Approach." *Econometrica 59*, 347–370.

Osborn, D. R. (1991). "The Implications of Periodically Varying Coefficients for Seasonal Time Series Processes." *Journal of Econometrics 48*, 373–384.

Penman, S. H. (1987). "The Distribution of Earning News over Time and Seasonality in Aggregate Stock Returns." *Journal of Financial Economics 18*, 199–228.

Savva, C. S., D. R. Osborn, and L. Gill. (2005). "Volatility, Spillover Effects and Correlations in U.S. and Major European Markets." Discussion Paper No. 064, Centre for Growth and Business Cycle Research, University of Manchester.

Steely, J. M. (2001). "A Note on Information Seasonality and the Disappearance of the Weekend Effect in the UK Stock Market." *Journal of Banking and Finance 25*, 1941–1956.

Tiao, G. C., and M. R. Grupe. (1980). "Hidden Periodic Autoregressive-Moving Average Models in Time Series Data." *Biometrika 67*, 365–373.

Tsiakis, I. (2006). "Periodic Stochastic Volatility and Fat Tails." *Journal of Financial Econometrics 4*, 90–135.