Rethinking Consumer-Centric Markets Under Uncertainty: A Robust Approach to Community-Based Energy Trades

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***Abstract*—The flexibility of the end users in the electricity markets is becoming more pertinent with the evolution of market mechanisms allowing consumers to participate actively. The advent of Distributed Energy Resources (DERs) and energy storage systems is gradually and continuously changing the roles of the market operators. The impact of the uncertainty of DERs and load demands on community-based market structures has not been fully investigated. In this paper, we propose a robust solution to community-based market operations under uncertainty and compare the optimal decisions on energy trades with deterministic, stochastic, and opportunistic models. Also, we employ the Taguchi's orthogonal array testing (TOAT) to generate proficient scenarios from uncertain variables of prosumers. The proposed method is tested on a community-based microgrid with 15 prosumers assuming a single-rate tariff structure. Simulation results demonstrate the cost of robustness and the impact of uncertainty.**

***Index Terms*—Community-Based Markets, Peer-to-Peer (P2P) Markets, Taguchi orthogonal array testing (TOAT), Uncertainty.**

NOMENCLATURE

Indices

- n Index of prosumers (from 1 to N).
- t Index of hours (from 1 to T).
- w Index of scenarios (from 1 to W).

Parameters

- a_n Quadratic coefficient of φ_n.
- b_n Linear coefficient of φ_n.
- g Cost function for the community manager.
- $s_{\lambda expt}$ single rate export cost by prosumers to the main grid at hour t.
- $s_{\lambda impt}$ single rate import price of prosumers from the main grid at hour t.
- π_w Probability of scenario w.
- ξ_{com} Transaction cost set by the community manager for trading within the community.
- φ_n Cost function for prosumer n.

Sets

- Ω^{U_t} Set of all uncertain parameters at hour t.
- Ω^{Y_t} Feasible space of continuous variables of the community manager at hour t.
- $\Omega^{X_{nt}}$ Feasible space of decision variables of prosumer n at hour t.

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Variables

- α Auxiliary variable.
- δ_{ntw} Energy exported by prosumer n to the grid at hour t in scenario w.
- γ_{ntw} Energy imported by prosumer n from the main grid at hour t in scenario w.
- p_{ntw} Production/consumption of prosumer n at hour t in scenario w.
- q_{exptw} Total energy exported to the main grid at hour t in scenario w.
- q_{ntw} Energy exported by prosumer n to the community at hour t in scenario w.
- r_{imptw} Total energy imported from the main grid at hour t in scenario w.
- r_{ntw} Energy imported by prosumer n from the community at hour t in scenario w.

I. INTRODUCTION

The continuous proliferation of Distributed Energy Resources (DERs) in electricity grids is gradually changing the traditional roles of stakeholders. Consumers become producers, equipped with renewable generation, as well as storage in the form of electric vehicles or wall-mounted batteries. Along with the shift in production and consumption patterns, came the necessity for new, citizen-centred, market mechanisms to allow for the transparent and mutually beneficial exchange of energy between *prosumers*.

The most promising mechanisms in this area are community-based and peer-to-peer (P2P) energy trading markets, where users can own small scale DERs and actively trade their energy resources within a community [1]. Such markets offer novel designs to the restructuring of current electricity markets with regards to prosumers trading their excess/deficit energy with other end-users. The benefits and opportunities arising from such mechanisms are explored in [2]. Moreover, community-based markets can give rise to the sharing of energy resources amongst participating agents in the microgrid [3]. The evolution of these consumer-centric markets has initiated a more proactive role amongst prosumers. They can increase their flexibility and preferences towards the type of DER to meet their load demands. The consumers are no longer restricted to conventional generation, fixed electricity tariffs, and demand-side management initiatives to manage their consumption levels.

Opportunities have been sought to further decentralize the community-based market to full P2P market mechanisms [1], [2], [4]–[9]. There are many methods proposed in the literature for solving an economic dispatch problem within a network through community-based, P2P, or hybrid methods. The majority of these methods rely on formulating an optimization-based market design, including constraints relating to the peers' production and consumption, as well as financial limits. The solution of this problem is then performed, to obtain the optimal bilateral trades and prices [4]. The concept of blockchains is another important aspect of solving decentralized P2P energy trades in a microgrid network of prosumers. Reference [10] has proposed the use of blockchain as an important technology in the operation of microgrid markets, and the Brooklyn microgrid project has used to analyse different market mechanisms. Blockchains and smart contracts are usually implement side by side to facilitate P2P trading amongst prosumers.

There is uncertainty associated with renewable production and electricity consumption due to fluctuations in expected power output. While uncertainty has been tackled in many other optimization problems in power systems [11], it remains a little investigated area for P2P market mechanisms. The majority of the previous methods consider deterministic problems, using forecasted values of renewable generation and load demand. Several techniques have been introduced in the literature which model the uncertainties in renewable generations, load demands, market prices, these include Stochastic Optimization [12], Robust Optimization [13], Taguchi's Orthogonal Array Testing (TOAT) [14], Information-Gap Decision Theory (IGDT) [15].

TOAT [14] is a statistical tool that provides representative testing scenarios which offer possible combinations to a given experimental design or analysis towards achieving desired outcome. The main benefit of TOAT in comparison with Monte Carlo simulation is that TOAT is capable of characterizing different types of uncertain parameter with less deterministic scenario and computation time. TOAT has been previously used to solve different non-deterministic problems in power systems. Reference [16] has presented a transmission expansion planning problem under uncertainty where the optimal solution is robust against the worst-case scenarios generated by TOAT. Also, reference [17] has used TOAT to solve the economic dispatch problem with non-smooth cost function under uncertainty. Reference [18] has introduced a TOAT-based probabilistic load flow model under the uncertainty of renewables and loads. However, to the best of our knowledge, there is no TOAT-based community-based energy trading model under uncertainty in the literature.

In this paper, we have the following main contributions:

- We propose opportunistic (risk-seeker) and robust (risk-averse) community-based market mechanisms under the uncertainty of renewable DERs and load demands. In the proposed approach, uncertainty sources are characterized by the best-case and worst-case scenarios generated by TOAT.
- We evaluate the impact of uncertainties on total costs of community-based energy trading by comparing the deterministic, stochastic, and opportunistic/robust models with best-case/worst-case scenarios.

TABLE I
ORTHOGONAL ARRAY $L_4(2^3)$ WITH FOUR SCENARIOS AND THREE UNCERTAIN PARAMETERS.

Number of Scenarios	α_1	α_2	α_3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

The rest of this paper is organized as follows. In Section II, the proposed scenario generation technique based on the notion of TOAT is presented. In Section III, the mathematical formulations are introduced. In Section IV, the proposed approaches are tested on a microgrid as a community. Finally, Section V concludes the paper.

II. GENERATING ROBUST SCENARIOS USING TOAT

In this paper, TOAT is used to characterize the uncertainty of renewable DERs and load demands by generating appropriate best-case and worst-case scenarios.

Let $Y(X)$ be a function of the vector of uncertain parameters $X(a_1, ..., a_m, ...a_M)$ with M entries. Given N likely future realizations of each uncertain parameter a_m for $m = 1, ..., M$, there are N^M likely scenarios for future realizations of all M uncertain parameters belonging to the vector of uncertain parameters X. Needless to say, including all likely realizations of uncertain parameters in a specific problem may lead to intractability. Therefore, TOAT is used to reduce/increase the complexity/tractability of implementing all likely realizations of uncertain parameters.

In principle, the Taguchi's method is formed on the basis of orthogonal arrays (OA) represented by a general form $L_T(N^M)$, where L represents Latin squares based on TOAT terminology and T represents the number of reduced scenarios. For instance, suppose that there are three uncertain parameters (i.e., $M = 3$, a_1, a_2, and a_3) with two likely future realizations each (i.e., $N = 2$ where 1 and 2 represent the best and worst realization of each uncertain parameter, respectively). In this illustrative example, there are $N^M = 2^3 = 8$ different scenarios including all combinations of likely future realizations. To reduce the number of scenarios, a basic OA with $L_T(N^M) = L_4(2^3)$ can be used as depicted in Table I. Accordingly, the number of scenarios can be reduced from 8 to 4. It is noteworthy to mention that these scenarios are generated in a way that each realization of every uncertain parameter appears an identical number of times in each column of the array in Table I. Therefore, in this $L_4(2^3)$ array, realizations 1 and 2 occur two times [16], [17], and in general, T/N^M times for a generic $L_T(N^M)$ OA. There are available libraries with different OAs to characterize the uncertainty spectrum by means of the TOAT approach. Note that a particular OA may be modified for a specific problem by reducing the number of its uncertain parameters as compared to its original ones in the library.

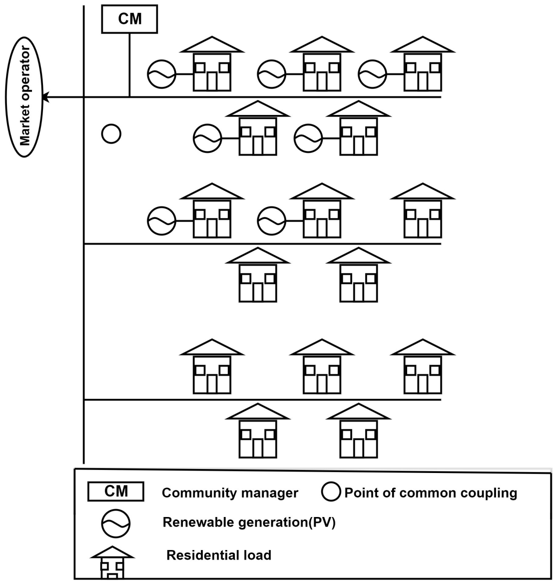

Fig. 1. Schematic representation of the test case.

III. Robust Community-Based Electricity Market

The community-based market structure consists of prosumers involved in energy trading inside and outside the community and a community manager who coordinates the activities of prosumers. In this section, a robust formulation of the community-based market is presented to obtain robust solutions for energy trades withstanding the worst-case realization of uncertain parameters.

The TOAT approach is used to capture the uncertainty of renewable DERs and load demands by means of appropriate scenarios. In this study, a symmetric bounded interval is considered for each uncertain parameter. Also, extreme points of each bounded interval are considered as the likely best-case or worst-case future realizations of each uncertain parameter. Clearly, the upper and lower bounds of the bounded intervals pertaining to the uncertain renewable DERs (i.e., $\tilde{p}_{gt} = [p_{gt}+\sigma_{gt}, p_{gt}-\sigma_{gt}]$, where p_{gt} and σ_{gt} stand for expected value and deviations of renewable DERs at hour t) represent the best-case and worst-case scenarios, respectively, while the lower and upper bounds of the symmetric bounded intervals pertaining to uncertain load demands (i.e., $\tilde{p}_{lt} = [p_{lt}-\sigma_{lt}, p_{lt}+\sigma_{lt}]$, where p_{lt} and σ_{lt} stand for expected value and deviations of load demands at hour t) represent the best-case and worst-case scenarios, respectively. Mainly, the multi-period robust problem can be written as:

$$\max_{U_t\in\Omega^{U_t}} \min_{X_{nt}\in\Omega^{X_{nt}}, Y_t\in\Omega^{Y_t}} \left(\sum_{n=1}^{N}\sum_{t=1}^{T}\varphi_n(X_{nt},U_t)+g\left(Y_t,U_t\right)\right) \tag{1}$$

where Ω^{U_t} represents the set of uncertain parameters at hour t, $\Omega^{X_{nt}}$ represents the feasible space of decision variables of prosumer n at hour t, and Ω^{Y_t} represents the feasible space of continuous variables of the community manager at hour t. The proposed min-max problem in (1) minimizes the total costs of prosumers, i.e., $\varphi_n(X_{nt},U_t)$ where $X_{nt} = \{p_{nt}, r_{nt}, q_{nt}, \gamma_{nt}, \delta_{nt}\}$, and the total costs of the community manager in importing/exporting electricity inside/outside the community, i.e., $g(Y_t,U_t)$ where $Y_t = \{r_{impt}, q_{expt}\}$, under the worst-case realization of uncertain parameters, i.e., $U_t = \{\tilde{p}_{gt}, \tilde{p}_{lt}\}$). Needless to say, this problem cannot be solved for all realizations of uncertain parameters. To obtain a tractable counterpart, first, the orders of minimization and maximization problems are changed based on the classical minimax theorem (Proposition 5.5.4 in [19]), and then, the TOAT approach is used to generate appropriate scenarios capturing the uncertainty spectrum. Therefore, the multi-period robust mathematical formulation can be written as follows:

$$\min\ \alpha \tag{2a}$$

$$\text{s.t.}\quad \alpha \geq \sum_{t=1}^{T}\sum_{n=1}^{N}\varphi_n(p_{ntw})+\xi_{com}(r_{ntw},q_{ntw})+ \\ s_{\lambda impt}(\gamma_{ntw})+s_{\lambda expt}(\delta_{ntw})+g(r_{imptw},q_{exptw})\ \forall w \tag{2b}$$

$$p_{ntw}-r_{ntw}-q_{ntw}-\gamma_{ntw}-\delta_{ntw}=0\ \ \forall n,\forall t,\forall w \tag{2c}$$

$$\sum_{n=1}^{N} r_{ntw}-\sum_{n=1}^{N} q_{ntw}=0\ \ \forall t,\ \forall w \tag{2d}$$

$$\sum_{n=1}^{N}\gamma_{ntw}=r_{imptw}\ \ \forall t,\forall w \tag{2e}$$

$$\sum_{n=1}^{N}\delta_{ntw}=q_{exptw}\ \ \forall t,\forall w \tag{2f}$$

$$\underline{p}_{ntw}\leq p_{ntw}\leq \overline{p}_{ntw}\ \ \forall n,\forall t,\forall w \tag{2g}$$

$$\gamma_{ntw}\geq 0,\delta_{ntw}\geq 0\ \ \forall n,\forall t,\forall w \tag{2h}$$

The objective function (2a) minimizes the total costs of agents, the total transactions costs, and the total costs of the community manager in importing/exporting electricity inside/outside the community under the worst scenario for uncertain parameters obtained from the TOAT approach where the auxiliary variable α is used to find the optimal total cost pertaining to the worst-case scenario. In this study, it is assumed that each agent as a prosumer is equipped with renewable DERs and optimizes its energy production/consumption. With respect to the energy produced/consumed by the prosumer for export/import outside/inside the community, the quadratic cost function $\varphi_n = a_n \cdot p_{ntw}^2 + b_n \cdot p_{ntw}$ is considered here. If the net production of the prosumer is from either renewable or non-renewable DERs, $a_n = 0$ or $a_n > 0$, respectively.

Also, the prosumer assumes a positive/negative cost function while producing/consuming energy. Constraint (2b) ensures that α is greater than or equal to the total costs pertaining to the worst scenario. Constraint (2c) stands for energy balance for each agent. Constraint (2d) represents the total energy traded between all agents within the community where its net value is equal zero. Also, constraint (2e)/(2f) stands for the total energy imported/exported by the community manager inside/outside the community and consumed/produced by different agents. Constraint (2g) limits the power production/consumption of each agent where the lower-bound (i.e., $\underline{p}_{ntw}$) and upper-bound (i.e., $\overline{p}_{ntw}$) parameters for each scenario are obtained from the TOAT approach. Also, constraint (2h) ensures non-negativity of variables indicating imported

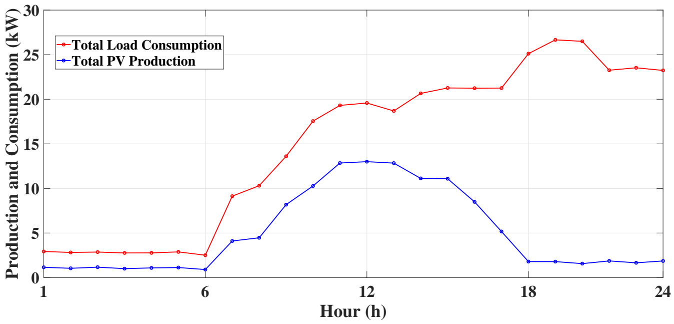

Fig. 2. Total PV production and load consumption.

and exported energy by each agent. It is noteworthy to mention that the proposed model in (2a)-(2h) can find the robust solution by means of the worst-case scenarios and the opportunistic solution by means of the best-case scenarios generated by the TOAT approach. It can also find the deterministic solution by means of only including the forecast values of uncertain parameters. Additionally, a stochastic model can be obtained by modifying the objective function of the proposed model in (2a)-(2h) as follows [12]:

$$\min \sum_{w=1}^{W} \sum_{t=1}^{T} \sum_{n=1}^{N} \pi_w(\varphi_n(p_{nt,w}) + \xi_{com}(r_{nt,w}, q_{nt,w}) + \tag{3a}$$

$$s_{\lambda impt}(\gamma_{nt,w}) + s_{\lambda expt}(\delta_{nt,w}) + g(r_{impt,w}, q_{expt,w}))$$

$$\text{(2c)-(2h)} \tag{3b}$$

Hereafter, the optimal solutions of deterministic, stochastic, robust, opportunistic community-based energy trading models at each hour of the scheduling period are indicated by $DCET_t$, $SCET_t$, $RCET_t$, and $BCET_t$, respectively. Additionally, a similar number of scenarios are considered for all $SCET_t$, $RCET_t$, and $BCET_t$ models to present fair comparisons between different models where all scenarios have similar probabilities in the stochastic model (i.e., $\pi_w = 1/W$). It is worthwhile to note that the robust model solves the problem for different realizations of the uncertain parameters and characterizes different deviations from the nominal estimates of the uncertain parameters while the deterministic model only solves the problem for the nominal estimates of the uncertain parameters. Accordingly, the cost of the robust model is higher than the cost of the deterministic one. The difference between the total costs of the robust and deterministic models is designated as the cost of robustness.

IV. Case Study

A. Data Sets

In this study, the proposed deterministic, stochastic, opportunistic, and robust models are tested on a microgrid as a community with 15 prosumers, including 7 producers and 8 consumers, where it is obtained from the *clean* dataset of 54 customers through removing anomalous data from the Ausgrid dataset [20]. The Ausgrid peak-tariff in 2019 [21] is considered as import prices (i.e., 23.5007 c/kWh) of prosumers for importing energy from outside the community while the solar feed-in-tariff of New South Wales in Australia in 2019 is considered for export prices of prosumers (i.e., 9.5 c/kWh [22]). The transaction cost of the community manager is fixed at 15.5 c/kWh. The total load consumption of prosumers 1-8 and total PV production of prosumers 9-15 are illustrated in Fig. 2. Additionally, 10% symmetric deviations from the forecast values of the uncertain parameters are considered (i.e., $\sigma_{lt} = \sigma_{gt} = 0.1$) for all 24 hours of the scheduling horizon.

As the community has 15 prosumers, there are 15 uncertain parameters with the best and worst realizations at each hour of the scheduling problem pertaining to 7 producers and 8 consumers. In other words, there are $N^M = 2^{15} = 32768$ different scenarios at each hour of the scheduling horizon. Accordingly, the $L_{16}(2^{15})$ OA based on the TOAT approach is used to decrease the number of scenarios from 32768 to 16 and generate appropriate best-case and worst-case scenarios for the opportunistic and robust models, respectively. Also, 16 scenarios with similar probabilities (i.e., $1/16$) are considered for the stochastic model.

TABLE II
Highest Energy Trading Costs for All Models

	Highest Energy Trading Costs (c)			
Time Periods	$RCET_t$	$DCET_t$	$SCET_t$	$BCET_t$
12am-6am	84.0719	78.4517	77.5615	72.8314
6am-12pm	602.4513	568.5300	557.6238	534.6087
12am-6pm	661.3371	603.6658	603.6658	558.4042
6pm-12am	701.3673	640.0523	640.0523	578.7372

B. Discussion

According to Fig. 2, from 12am to 6am, there are low production and consumption in the community. Between 6am-12pm, the load demands of the prosumers increase steadily with an increase in the PV productions of prosumers. Between 12pm-6pm, the load demands of prosumers further increase with a reduction in the PV productions of prosumers. Finally, from 6pm to 12am, the load demands of prosumers decrease with the minimum PV productions of prosumers. The energy trading costs of the deterministic, stochastic, opportunistic, and robust models during different hours of the scheduling horizon are depicted in Fig. 3. The energy trading costs of 16 scenarios pertaining to the robust model are also indicated by CET_w in Fig. 3. Additionally, the highest energy trading costs of these models at different hours are illustrated in Table II.

According to Table II, from 12am to 6am, the energy trading costs are the lowest in all models. As PV production and load consumption gradually increase from 6am to 12pm, the energy trading costs also increase. From 12pm to 6pm, as the PV production declines and the load consumption increases, the energy trading costs remain high. Finally, from 6pm to 12am, although there is a reduction in the load demands of prosumers, the energy trading costs increase further due to the minimum PV productions of prosumers during the entire scheduling horizon leading to more energy imports.

The highest total cost belongs to the energy trading model with the worst-case (robust) TOAT scenarios while the lowest total cost belongs to the energy trading model with

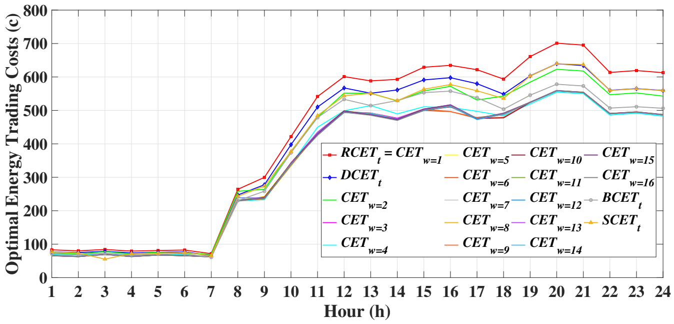

Fig. 3. Optimal energy trading costs of prosumers.

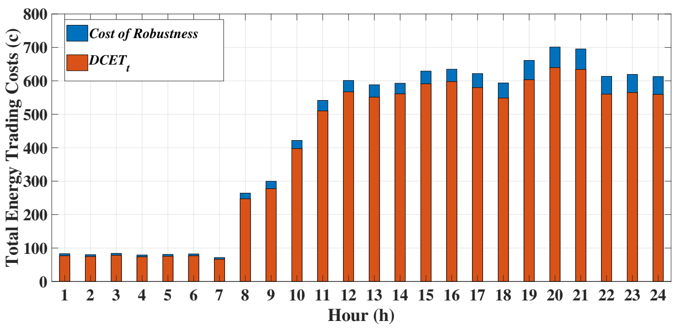

Fig. 4. Cost of robustness in the robust model as compared to the deterministic model.

the best-case (opportunistic) TOAT scenarios during a 24-hour scheduling period as depicted in Table II. Additionally, the total cost of both deterministic and stochastic models is lower/higher than that of the best-case/worst-case model while the total cost of the deterministic model is also higher than that of the stochastic model. Accordingly, as compared to the deterministic model, the opportunistic model is risk-seeker and both stochastic and robust models are risk-averse. However, the conservatism of the robust model is higher than all opportunistic, deterministic, and stochastic models. Consequently, its total costs and the cost of its robustness are higher than that of other models, including the deterministic model, as demonstrated in Fig. 4.

V. Conclusion

In view of the recent trends in current electricity market structures, the evolving consumer-centric market requires more scalability in its integration to the existing markets. Since uncertain parameters can significantly affect consumer-centric energy trades, it is vital to appropriately characterize different types of uncertain parameters in these types of electricity markets. The risk-seeker and risk-averse performances of the consumer-centric markets have been studied here through the TOAT approach generating appropriate best-case and worst-case scenarios. In addition, the total trading costs of these risk-seeker and risk-averse models have been compared with deterministic and stochastic ones. Case studies demonstrate that the risk-averse model has the highest robustness and total costs while the risk-seeker model has the lowest robustness and total costs. In the future research works, the proposed approach can be extended and solved by proficient decomposition approaches to reduce its computation time for a large number of prosumers.

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